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Generalized laws of reflection and refraction from real valued boundary conditions

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ABSTRACT

The generalized Snell's laws are usually derived from phase-matching condition by using harmonic inhomogeneous plane waves. The inhomogeneity makes it difficult to trace curves of energy flow. Here we show that, at a lossy interface, real valued boundary conditions are valid universally. Thus a time-dependent way is developed to directly derive generalized laws of reflection and refraction from real valued boundary conditions by using harmonic homogeneous plane waves. Finally, several novel properties of transmitted wave associated with energy losses are predicted numerically, which may be applied to experimentally test our theoretical analysis.

1. Introduction

Up to now, study on reflection and refraction of electromagnetic wave at a lossy interface is still an active research area [1–22]. It is known early that the usual Snell's law is no longer valid for lossy media. Traditionally, the generalized Snell's laws are derived from phase-matching condition by using harmonic inhomogeneous plane waves (HIPWs) instead of the well known harmonic homogeneous plane waves (HHPWs) [1,4,7]. Then the generalized Fresnel's laws are obtained by combining the complex valued boundary conditions. It is shown that HIPW is usually elliptically polarized [7]. The elliptical polarization leads to oscillations of Poynting vector, making it more difficult to trace the curves of energy flow. Partly due to the complexity of both HIPWs and the generalized Snell's laws, several assumptions and/or choices have to be introduced to further find simple explicit formulas related to energy flow propagation direction, the obtained results are usually correct only under certain conditions [9–12]. Recently, study on negative refractive index materials has attracted a great deal of attention, which offers an opportunity to further reconsider the basic concepts and theorem associated with properties of electromagnetic wave propagating in lossy media [13–22]. It seems that, in these cases, effects of energy losses on properties of wave propagation have not been rigorously treated and fully understood yet [20–23].

In this work, we shall show that, due to oblique propagation of wave (corresponding to Gaussian surface or integrating loop), a term related to wave vector and propagation direction must arise in the integral form of Maxwell's equations, which impact directly on phase-matching condition and boundary conditions. It is demonstrated that, at a lossy interface, phase-matching condition and complex valued boundary conditions are usually valid only for HIPWs. However, the real valued boundary conditions are valid universally for both HHPWs and HIPWs. Thus a time-dependent way is developed to derive laws of reflection and refraction from real valued boundary conditions by using the well known HHPWs. Since the electromagnetic wave may be linearly composited by either HHPWs or HIPWs, the way presented here is equivalent to the previous ones [1,4,7] in principle and does not lose any generality. In addition, our work has following advantages: (a) the adopted HHPWs are simple and well known; (b) the derivation procedure closely corresponds to the well known dynamical process of electric and magnetic fields of HHPWs at every moment, hence the obtained laws have clear physical meaning; (c) in practice, the incident wave is usually taken as a single HHPW or the wave linearly composited by HHPWs. Thus laws obtained in this work are feasible to be applied to rigorously trace curves of energy flow, which may be helpful to further understand the novel properties of electromagnetic waves, such as negative refraction. Our study may also stimulate and urge the reconsideration of properties of electromagnetic fields and electromagnetic waves in the lossy media from a very fundamental viewpoint.

The remainder of the paper is organized as follows: In Section 2, integral form of Maxwell's equations and then the boundary conditions are reconsidered. In Section 3, the generalized laws of reflection and refraction are derived directly from real valued boundary conditions by adopting HHPWs. In Section 4, several novel properties of transmitted wave induced by media losses are predicted. Finally, some conclusions are shown in Section 5.

2. Reconsideration on integral form of Maxwell's equations and boundary conditions

We start our study from reconsidering integral form of Maxwell's equations. It is well known that integral form of Maxwell's equations

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is usually obtained from distribution of the electric field induced by static charges and/or altering magnetic field and the magnetic field produced by conduction and/or displacement current [24–26]. In these cases, effects of the field propagation and energy losses are not considered directly. Here, analogously, we shall attempt to derive integral form of Maxwell's equations according to distribution of electric and magnetic fields of electromagnetic wave (the static (or quasi-static) electric and magnetic fields may be taken as the special cases having the propagation constant tends to be zero), and then address possible effects of energy losses on integral form of Maxwell's equations, phase-matching condition and boundary conditions. In addition, it is emphasized that time domain real valued integral form of Maxwell's equations is basic and universal expression of electromagnetic fields. For harmonic electromagnetic fields, the frequency domain complex valued Maxwell's equations and then the complex valued boundary conditions are introduced [24–26]. Therefore, we shall pay our main attention on the time domain real valued expression of electromagnetic fields.

As a typical example, we study in detail properties of magnetic flux ψ of a beam of harmonic (homogeneous or inhomogeneous) plane wave at the chosen Gaussian surface as shown in Fig. 1. Generally, the true value of magnetic induction intensity $\vec{B}$ of both HHPW and HIPW can be expressed as [7,24–26]

$$\vec{B}\left(\vec{r},t\right) = \mathrm{Re}\left\{\tilde{\vec{B}}_0 \exp\left[-j\omega t + j\tilde{\vec{k}}\cdot\left(\vec{r}-\vec{r}_0\right)\right]\right\}. \tag{1}$$

Where, $\tilde{\vec{B}}_0$ is complex valued magnetic induction intensity of the wave at the moment of $t=0$ and the corresponding position of $\vec{r}=\vec{r}_0$, $\tilde{\vec{k}}=\vec{k}'+j\vec{k}"$ is complex valued propagation constant of the wave in the medium. With the classical formulation, media losses are taken into account by the complex valued propagation constant. For simplicity, the wave-front is assumed to be rectangle-shaped with area of $w\times l$ (where, w is the width of the beam in the direction perpendicular to $x0z$ plane, l size of wave-front in $x0z$ plane). At a moment t, magnetic induction intensity $\vec{B}$ of the wave at the upper surface is

$$\vec{B}\left(r_x+x, r_y, r_z, t\right) = \mathrm{Re}\left\{\tilde{\vec{B}}\left(r_x, r_y, r_z, t\right)\exp\left[j\tilde{k}(x-r_x)\sin\theta\right]\right\}. \tag{2}$$

Here θ is the intersection angle between wave vector and normal of upper surface. To obtain an appropriate integral form of the Maxwell's equation, magnetic induction intensity $\vec{B}$ at the lower surface is taken as the field traveling from the upper surface after Δt time

$$\vec{B}\left(r_x+htg\theta+x, r_y, r_z+h, t+\Delta t\right) = \mathrm{Re}\left[\tilde{\vec{B}}\left(r_x+x, r_y, r_z, t\right)\exp(-k''h/\cos\theta)\right]. \tag{3}$$

Here, $\Delta t = h/v\cos\theta$ is propagation time of the wave insider the Gaussian surface with height of h, v the phase velocity of the wave in the medium. Directly, magnetic flux ψ at the given Gaussian surface is obtained as

$$\begin{aligned}\Psi \equiv & \int_{S_{upper}} \vec{B}\left(\vec{r},t\right)\cdot d\vec{S} + \int_{S_{lower}} \vec{B}\left(\vec{r},t+\Delta t\right)\cdot d\vec{S} \\ = & \int_{r_x}^{r_x+l/\cos\theta} \mathrm{Re}\left\{\tilde{B}_n\left(r_x, r_y, r_z; t\right)\exp\left[j\tilde{k}(x-r_x)\sin\theta\right]\right\} w dx \\ & - \int_{r_x+htg\theta}^{r_x+htg\theta+l/\cos\theta} \mathrm{Re}\left\{\tilde{B}_n\left(r_x+htg\theta, r_y, r_z+h; t+\Delta t\right)\right. \\ & \times \left.\exp\left[j\tilde{k}(x-r_x-htg\theta)\sin\theta\right]\right\} w dx\end{aligned} \tag{4}$$

Note relation of $\tilde{B}_n(r_x+htg\theta, r_y, r_z+h; t+\Delta t) = \tilde{B}_n(r_x, r_y, r_z; t)\exp(-k"h/\cos\theta)$, integrating Eq. (4), we have

$$\Psi = \left[1-\exp(-k"h/\cos\theta)\right]\mathrm{Re}\left[\tilde{B}_n\left(r_x, r_y, r_z, t\right)\frac{\exp\left(j\tilde{k}\,ltg\theta\right)-1}{j\tilde{k}\sin\theta}\right]w. \tag{5}$$

Apparently, Eq. (5) can be rewritten in a complex valued form. It is seen from Eq. (5) that the integral form of Maxwell's equation for static field is recovered as $\lim_{k\to 0}\psi = 0$. In addition, magnetic flux ψ at the given Gaussian surface is always equal to zero when the wave propagating in the lossless medium. However, in the lossy medium, the magnetic flux ψ at the given Gaussian surface is generally not equal to zero (except for $B_n(r_x, r_y, r_z, t)=0$). Of course, $\oint_S \vec{B}\left(\vec{r},t\right)\cdot d\vec{S}\neq 0$ due to attenuation of electromagnetic wave in lossy media is an interesting result [24–26], the detailed discussion are beyond the scope of this paper, we would like to leave it for a further study. Finally, we shall emphasize that a term of $\dfrac{\exp\left(j\tilde{k}\,ltg\theta\right)-1}{j\tilde{k}\sin\theta}$ arises in the integration, which exists also in integral form of Maxwell's other equations. Below, we shall show that effects of energy losses on boundary conditions may be produced by this term.

Let's further investigate magnetic flux ψ of a beam of harmonic plane wave obliquely traveling through an interface as shown in Fig. 2. The wave-fronts are still simply assumed to be rectangle-shaped with areas of $w\times l_\varsigma$, $l_\varsigma(\varsigma=i,r,t)$ are sizes of wave-fronts in $x0z$ plane, the subscript notations of i, r and t are adopted for the incident, reflected and transmitted waves, respectively. The height h_ς and propagation time $\Delta t=\Delta t_1+\Delta t_2$ of the wave insider the Gaussian surface hold following relationships of $h_i=v_1\Delta t_1\cos\theta_i$, $h_r=v_1\Delta t_2\cos\theta_r$, $h_t=v_2\Delta t_2\cos\theta_t$, v_ξ refers phase velocity of the wave in $\xi th(\xi=1,2)$ medium. Analogously, magnetic flux ψ at the given Gaussian surface is directly obtained as

$$\begin{aligned}\Psi = & \mathrm{Re}\left[\tilde{B}_{i,n}\left(0, r_y, 0; t\right)\exp(k_1"h_i/\cos\theta_i)\frac{\exp\left(j\tilde{k}_1 l_i tg\theta_i\right)-1}{j\tilde{k}_1\sin\theta_i}\right]w \\ & + \mathrm{Re}\left[\tilde{B}_{r,n}\left(0, r_y, 0; t\right)\exp(-k_1"h_r/\cos\theta_r)\frac{\exp\left(j\tilde{k}_1 l_r tg\theta_r\right)-1}{j\tilde{k}_1\sin\theta_r}\right]w. \\ & - \mathrm{Re}\left[\tilde{B}_{t,n}\left(0, r_y, 0; t\right)\exp(-k_2"h_t/\cos\theta_t)\frac{\exp\left(j\tilde{k}_2 l_t tg\theta_t\right)-1}{j\tilde{k}_2\sin\theta_t}\right]w\end{aligned} \tag{6}$$

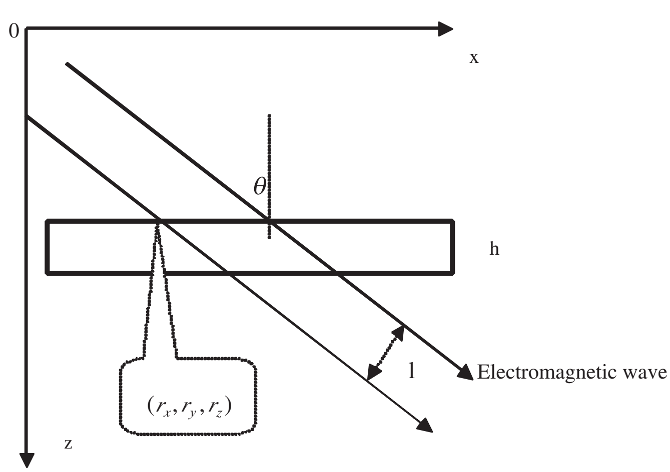

Fig. 1. Sketch of a beam of electromagnetic wave propagating in a homogeneous isotropic medium. The black rectangle refers cross section of the chosen Gaussian surface.

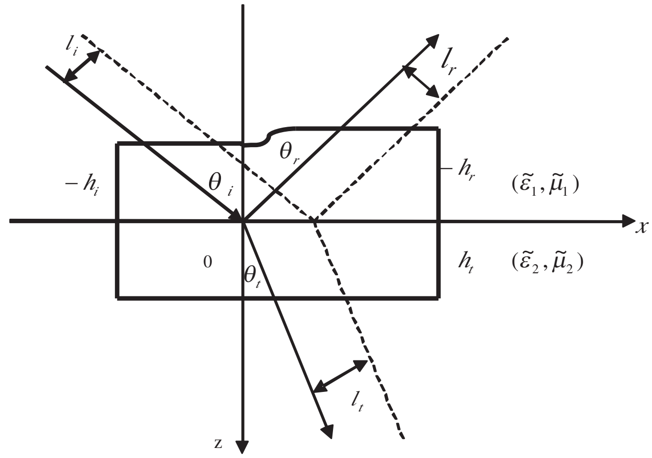

Fig. 2. Reflection and refraction of a beam of electromagnetic wave at an interface. The black rectangle refers cross section of the chosen Gaussian surface.

To address the possible effects of energy losses on phase-matching condition and boundary conditions, let $h_{i,r,t} \to 0$, effects of energy losses on the magnetic flux ψ is neglected, thus

$$\begin{aligned}\Psi = &\operatorname{Re}\left[\tilde{B}_{i,n}\left(0,r_y,0;t\right)\frac{\exp\left(j\tilde{k}_1 l_i tg\theta_i\right)-1}{j\tilde{k}_1\sin\theta_i}\right]w \\ &+\operatorname{Re}\left[\tilde{B}_{r,n}\left(0,r_y,0;t\right)\frac{\exp\left(j\tilde{k}_1 l_r tg\theta_r\right)-1}{j\tilde{k}_1\sin\theta_r}\right]w \\ &-\operatorname{Re}\left[\tilde{B}_{t,n}\left(0,r_y,0;t\right)\frac{\exp\left(j\tilde{k}_2 l_t tg\theta_t\right)-1}{j\tilde{k}_2\sin\theta_t}\right]w \\ =&\ 0\end{aligned} \quad (7)$$

It is clear from Eq. (7) that, due to existence of the terms of $\frac{\exp\left(j\tilde{k}_\xi l_\zeta tg\theta_\zeta\right)-1}{j\tilde{k}_\xi\sin\theta_\zeta}$, none but $\frac{\exp\left(j\tilde{k}_1 l_i tg\theta_i\right)-1}{j\tilde{k}_1\sin\theta_i} = \frac{\exp\left(j\tilde{k}_1 l_r tg\theta_r\right)-1}{j\tilde{k}_1\sin\theta_r} = \frac{\exp\left(j\tilde{k}_2 l_t tg\theta_t\right)-1}{j\tilde{k}_2\sin\theta_t}$, Eq. (7) may be rewritten as $\Psi = \operatorname{Re}\left\{\left[\tilde{B}_{i,n}(0,r_y,0;t) + \tilde{B}_{r,n}(0,r_y,0;t) - \tilde{B}_{t,n}(0,r_y,0;t)\right]\frac{\exp\left(j\tilde{k}_2 l_t tg\theta_t\right)-1}{j\tilde{k}_2\sin\theta_t}\right\}w = 0$. Let $l_{i,r,t} \to 0$, noting the relations of $\exp(x) \approx 1+x$ and $l_i w/\cos\theta_i = l_r w/\cos\theta_r = l_t w/\cos\theta_t = \Delta S$, we can see that phase-matching condition (8) and usual complex valued boundary condition (9) are sufficient, but not necessary, conditions satisfying Eq. (7).

$$\tilde{k}_1\sin\theta_i = \tilde{k}_1\sin\theta_r = \tilde{k}_2\sin\theta_t, \quad (8)$$

$$\tilde{B}_{i,n}\left(0,r_y,0;t\right) + \tilde{B}_{r,n}\left(0_r,r_y,0;t\right) - \tilde{B}_{t,n}\left(0,r_y,0;t\right) = 0. \quad (9)$$

At a lossy interface, HIPWs are traditionally adopted to satisfy the phase-matching condition (8) and then obtain the generalized Snell's laws. It is pointed out that, at a lossy interface, phase-matching condition (8) is generally invalid for HHPW. Thus the complex valued boundary condition (9) is no longer valid for HHPW. On the other hand, let $l_{i,r,t} \to 0$, noting the relations of $\exp(x) \approx 1+x$ and $l_i w/\cos\theta_i = l_r w/\cos\theta_r = l_t w/\cos\theta_t = \Delta S$, a real valued boundary condition for $\vec{B}\left(\vec{r},t\right)$ may be directly obtained from Eq. (7)

$$\operatorname{Re}\left(\tilde{B}_{i,n}(0,0,0;t)\right) + \operatorname{Re}\left(\tilde{B}_{r,n}(0,0,0;t)\right) = \operatorname{Re}\left(\tilde{B}_{t,n}(0,0,0;t)\right). \quad (10)$$

Since the adopted expression (1) is valid for both HHPWs and HIPWs, the condition (10) is also valid for both HHPWs and HIPWs. Analogously, the other universal real valued boundary conditions may be obtained.

3. Laws of reflection and refraction from real valued boundary conditions

It is pointed out that the phase-matching condition may be obtained from the complex valued boundary conditions. Therefore, the laws of reflection and refraction may be given directly from boundary conditions without the phase-matching condition. Below, we shall derive laws of reflection and refraction from the universal real valued boundary conditions by using HHPWs. It is emphasized that, for HHPWs, the planes of constant phase and constant amplitude are coincide, thus the intrinsic propagation constants are adopted directly instead of the effective propagation constants [7,9]. In addition, an incident HHPW may be divided into the usual TM and TE modes, respectively [7].

Consider an obliquely incident TM HHPW traveling from medium 1 into medium 2 as shown in Fig. 3. The real valued boundary conditions are written as

$$b_i\operatorname{Re}\left(\tilde{E}_i\right)\sin\theta_i + b_r\operatorname{Re}\left(\tilde{E}_r\right)\sin\theta_r = b_t\operatorname{Re}\left(\tilde{E}_t\right)\sin\theta_t, \quad (11)$$

$$\operatorname{Re}\left(\tilde{E}_i\right)\cos\theta_i - \operatorname{Re}\left(\tilde{E}_r\right)\cos\theta_r = \operatorname{Re}\left(\tilde{E}_t\right)\cos\theta_t, \quad (12)$$

$$\operatorname{Re}\left(\tilde{E}_i\right)/A_i + \operatorname{Re}\left(\tilde{E}_r\right)/A_r = \operatorname{Re}\left(\tilde{E}_t\right)/A'_t. \quad (13)$$

Where $b_\varsigma = \operatorname{Re}(\tilde{\varepsilon}_\xi) - \operatorname{Im}(\tilde{\varepsilon}_\xi)\operatorname{Im}\left(\tilde{E}_\varsigma\right)/\operatorname{Re}\left(\tilde{E}_\varsigma\right)$, $A_\varsigma = \tilde{\eta}_\xi\tilde{\eta}^*_\xi\left[\operatorname{Re}\left(\tilde{\eta}_\xi\right) + \operatorname{Im}\left(\tilde{\eta}_\xi\right)\operatorname{Im}\left(\tilde{E}_\varsigma\right)/\operatorname{Re}\left(\tilde{E}_\varsigma\right)\right]^{-1}$ and $\tilde{\eta}_\xi = \left(\tilde{\mu}_\xi/\tilde{\varepsilon}_\xi\right)^{1/2}$. In order to keep conservation of energy, A'_t is taken as that $|A'_t| = |A_t|$ and the sign of A'_t is always identical to that of A_i. Solving Eqs. (12) and (13) gives the formulae for transmission and reflection coefficients

$$T_E \equiv \frac{\operatorname{Re}\left(\tilde{E}_t\right)}{\operatorname{Re}\left(\tilde{E}_i\right)} = \frac{A'_t}{A_i}\frac{A_i\cos\theta_i + A_r\cos\theta_r}{A_r\cos\theta_r + A'_t\cos\theta_t}, \quad (14)$$

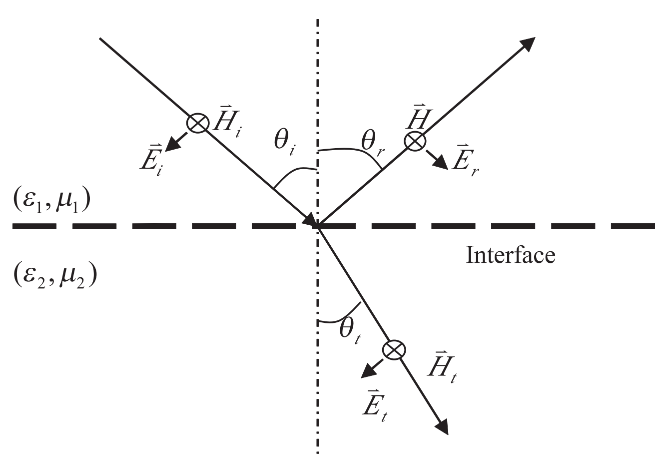

Fig. 3. Sketch of an obliquely incident TM HHPW travels through an interface.

$$\Gamma_E \equiv \frac{\mathrm{Re}\left(\tilde{E}_r\right)}{\mathrm{Re}\left(\tilde{E}_i\right)} = \frac{A_r}{A_i}\frac{A_i\cos\theta_i - A_t'\cos\theta_t}{A_r\cos\theta_r + A_t'\cos\theta_t}. \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (11), we may obtain

$$(b_i A_i \sin\theta_i - b_r A_r \sin\theta_r)A_t'\cos\theta_t = (b_t A_t'\sin\theta_t - b_r A_r \sin\theta_r)A_i\cos\theta_i + (b_t A_t'\sin\theta_t - b_i A_i \sin\theta_i)A_r\cos\theta_r \tag{16}$$

Satisfying the following conditions of (17) and (18), a suitable solution for Eq. (16) may be given.

$$b_i A_i \sin\theta_i - b_r A_r \sin\theta_r = 0, \tag{17}$$

$$b_t A_t' \sin\theta_t - b_r A_r \sin\theta_r = 0. \tag{18}$$

Where $b_\varsigma A_\varsigma = \frac{c}{\omega}\left[\mathrm{Re}\left(\tilde{k}_\xi\right) - \mathrm{Im}\left(\tilde{k}_\xi\right)tg\left(-\omega t + \tilde{\vec{k}}_\xi \cdot \vec{r} - \alpha_{\eta_\xi} + \alpha_{E_\varsigma}\right)\right]$, $\tilde{k}_\xi = \frac{\omega}{c}\left(\tilde{\mu}_\xi\tilde{\varepsilon}_\xi\right)^{1/2}$ the (intrinsic) propagation constant, and α_{E_ς} an initial phase associated with response properties of the media. Since $\mathrm{Re}\left(\tilde{E}_r\right)$ and $\mathrm{Re}\left(\tilde{E}_i\right)$ coexist in the same medium, $\alpha_{E_r} = \alpha_{E_i}$. However, the relation between α_{E_t} and α_{E_i} is determined by the response properties of the two media. In addition, for a given placement vector $\vec{r}$, $\tilde{\vec{k}}_\xi \cdot \vec{r}$ is a constant. Therefore, we can set $b_\varsigma A_\varsigma = \frac{c}{\omega}\left[\mathrm{Re}\left(\tilde{k}_\xi\right) - \mathrm{Im}\left(\tilde{k}_\xi\right)tg\left(-\omega t - \alpha_{\eta_\xi} + \alpha_{E_\varsigma}\right)\right]$. Thus solving Eqs. (17) and (18) gives

$$\sin\theta_i = \sin\theta_r, \tag{19a}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\mathrm{Re}\left(\tilde{k}_1\right)}{\mathrm{Re}\left(\tilde{k}_2\right)}\frac{1 - tg\left(\alpha_{k_1}\right)tg\left(-\omega t - \alpha_{\eta_1} + \alpha_{E_i}\right)}{1 - tg\left(\alpha_{k_2}\right)tg\left(-\omega t - \alpha_{\eta_2} + \alpha_{E_t}\right)}. \tag{19b}$$

According to the uniqueness theorem of domain decomposition method for boundary problem of Helmholtz equation, Eq. (19a), is the only solution.

It is seen from Eq. (19b) that, if all of $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$, $\tilde{\mu}_1$, $\tilde{\mu}_2$ are real numbers, $k_1 \sin\theta_i = k_2 \sin\theta_t$, this is just the normal case and consistent with usual Snell's law. In addition, the coefficients of reflection and refraction expressed as Eqs. (14) and (15) are identical to the usual Fresnel's laws too. However, generally, $\alpha_{k_1} \neq \alpha_{k_2}$, $\alpha_{\eta_1} \neq \alpha_{\eta_2}$ and $\alpha_{E_t} \neq \alpha_{E_i}$, propagation direction of the transmitted wave is a function of time, i.e., the transmitted wave is composited by HHPWs having different propagation directions. Alternation of refraction angle with time leads the coefficients of refraction and reflection to be also the functions of time. Physically, the time-dependent refracted angle, coefficients of refraction and refraction correspond to the fact that parameters of $\vec{E}$ and $\vec{H}$ of HHPWs in the two media closely near the interface alter out of step. Further, the energy balance relation at the interface may be demonstrated as

$$\Gamma_E\Gamma_H + T_E T_H \frac{\cos\theta_t}{\cos\theta_i} = 1. \tag{20}$$

Analogously, the laws of reflection and refraction of an obliquely incident TE HHPW are obtained as follows

$$T_H \equiv \frac{\mathrm{Re}\left(\tilde{H}_t\right)}{\mathrm{Re}\left(\tilde{H}_i\right)} = \frac{A_i\cos\theta_r + A_r\cos\theta_i}{A_t'\cos\theta_r + A_r\cos\theta_t}, \tag{21}$$

$$\Gamma_H \equiv \frac{\mathrm{Re}\left(\tilde{H}_r\right)}{\mathrm{Re}\left(\tilde{H}_i\right)} = \frac{A_t'\cos\theta_i - A_i\cos\theta_t}{A_t'\cos\theta_r + A_r\cos\theta_t}, \tag{22}$$

$$\sin\theta_i = \sin\theta_r, \tag{23a}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\mathrm{Re}(k_1)}{\mathrm{Re}(k_2)}\frac{1 - tg\left(\alpha_{k_1}\right)tg\left(-\omega t + \alpha_{\eta_1} + \alpha_{H_i}\right)}{1 - tg\left(\alpha_{k_2}\right)tg\left(-\omega t + \alpha_{\eta_2} + \alpha_{H_t}\right)}. \tag{23b}$$

Here $A_\xi = \mathrm{Re}\left(\tilde{\eta}_\xi\right) - \mathrm{Im}\left(\tilde{\eta}_\xi\right)\mathrm{Im}\left(\tilde{H}_\varsigma\right) / \mathrm{Re}\left(\tilde{H}_\varsigma\right)$.

4. Properties of transmitted wave induced by media losses and discussions

An experimental verification of the generalized laws can be made under various circumstances. Here, we shall propose some cases that could be realized in practice. We consider the case of an obliquely incident HHPW traveling from a medium 1 $\left(\tilde{\varepsilon}_1, \tilde{\mu}_1\right)$ into free space. For TM wave, the time-dependent field of $E_\varsigma(t)$ is calculated by using Eqs. (14), (15) and (19a), hence $H_\varsigma(t) = E_\varsigma(t)/A_\xi$. And for TE wave, $H_\varsigma(t)$ is obtained by adopting Eqs. (21)–(23a), and then $E_\varsigma(t) = H_\varsigma(t)A_\xi$. Once the fields of $E_\varsigma(t)$ and $H_\varsigma(t)$ are determined, the time-dependent Poynting flow is given as

$$\vec{S}_\varsigma(\theta_t, t) \equiv \vec{E}_\varsigma(t) \times \vec{H}_\varsigma(t). \tag{24}$$

Further, the time-averaged Poynting flow (TAPF) is obtained

$$<\vec{S}_\varsigma(\theta_t)> \equiv \sum_t \vec{S}_\varsigma(\theta_t, t)\Delta t / T_{period}. \tag{25}$$

Some typical results are shown in Figs. 4 and 5, respectively. Firstly, we shall focus on the TAPF distribution of the transmitted wave. Apparently, when parameters of ε_1 and μ_1 are real numbers, direction relationships among incident, reflected and transmitted wave obey the usual Snell's law, and the TAPF of the transmitted wave concentrate in a single direction as shown in Figs. 4(a) and 5(a),

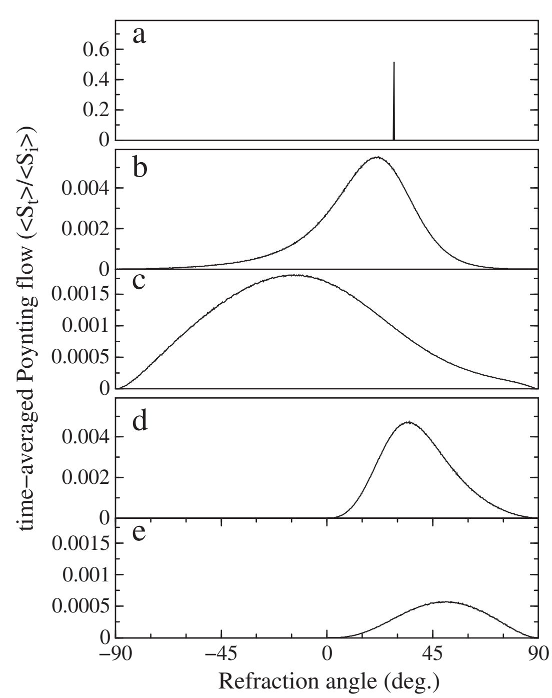

Fig. 4. A TM HHPW travels from medium 1 into free space with incident angle of $\theta_i = 18^0$, the TAPF of transmitted wave versus refraction angle. The permittivity and permeability of medium 1 are taken as (a) $\varepsilon_1 = 1.5, \mu_1 = 1.5$; (b) $\tilde{\varepsilon}_1 = 1.5 + j1.5$, $\mu_1 = 1.5$; (c) $\tilde{\varepsilon}_1 = 1.5 + j4.5, \mu_1 = 1.5$; (d) $\varepsilon_1 = 1.5$, $\tilde{\mu}_1 = 1.5 + j1.5$; and (e) $\varepsilon_1 = 1.5$, $\tilde{\mu}_1 = 1.5 + j4.5$, respectively.

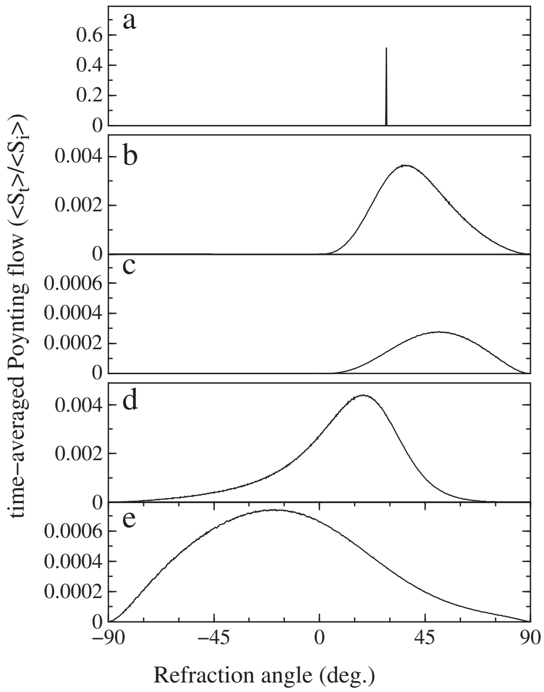

Fig. 5. A TE HHPW travels from medium 1 into free space with incident angle of $\theta_i = 18^0$, the TAPF of transmitted wave versus refraction angle. The permittivity and permeability of medium 1 are taken as (a) $\varepsilon_1 = 1.5, \mu_1 = 1.5$; (b) $\tilde{\varepsilon}_1 = 1.5 + j1.5$, $\mu_1 = 1.5$; (c) $\tilde{\varepsilon}_1 = 1.5 + j4.5, \mu_1 = 1.5$; (d) $\varepsilon_1 = 1.5$, $\tilde{\mu}_1 = 1.5 + j1.5$; and (e) $\varepsilon_1 = 1.5$, $\tilde{\mu}_1 = 1.5 + j4.5$, respectively.

respectively. Increasing value of either ε_1'' or μ_1'', effects of energy losses on TAPF distribution of transmitted wave are presented in Figs. 4(b)–(e) and 5(b)–(e) respectively. It is seen that the TAPF of transmitted wave mainly distribute in a certain range of refraction angle. We note that in some refraction experiments, the energy flow of transmitted wave may distribute in certain angle. Usually, the beam width is set by diffraction at the exit of the incident channel [13,14]. Apparently, here, we give another possible mechanism for the refraction power distribution versus refraction angle when medium losses are not negligible. Physically, this mechanism may be attributed to the fact that parameters of $\vec{E}$ and $\vec{H}$ of HHPWs in the two media closely near the interface alter out of step, which makes the propagation direction of the transmitted wave to be a function of time, i.e., the transmitted wave is composited by HHPWs having different propagation directions. For simplicity, we term the angle corresponding to the peak of the TAPF curve as the refraction angle θ_t^{peak}.

Then we present the properties of θ_t^{peak}. For oblique incident TM wave, it is noted from Fig. 4(b), (c) that, increasing value of ε_1'' only, θ_t^{peak} is smaller than the angle obtained from $\theta_t = \arcsin\left(\frac{\mathrm{Re}(k_1)}{\mathrm{Re}(k_2)}\sin\theta_i\right)$. Especially, when ε_1'' is large sufficiently, θ_t^{peak} may become negative as shown in Fig. 4(c). We noted that the negative refraction produced by heavily lossy wedge has been experimentally demonstrated by Sanz et al. [20] and theoretically addressed by Hansen using a somewhat imprecise approach very recently [21]. On the other hand, increasing value of μ_1'' only, θ_t^{peak} is contrarily bigger than the angle obtained from $\theta_t = \arcsin\left(\frac{\mathrm{Re}(k_1)}{\mathrm{Re}(k_2)}\sin\theta_i\right)$. In addition, the amplitude of the line peak of Fig. 4(d), (e) is significantly smaller then that of Fig. 4(b), (c), which indicates that the magnetic losses may be more effective to decrease the TAPF of transmitted wave than the dielectric losses. For obliquely incident TE HHPW, it is seen from Fig. 5(b)–(e) that the dielectric losses lead θ_t^{peak} to be bigger than angle derived from $\theta_t = \arcsin\left(\frac{\mathrm{Re}(k_1)}{\mathrm{Re}(k_2)}\sin\theta_i\right)$; and the magnetic losses lead θ_t^{peak} to be smaller than that derived from $\theta_t = \arcsin\left(\frac{\mathrm{Re}(k_1)}{\mathrm{Re}(k_2)}\sin\theta_i\right)$, when μ_1'' is large sufficiently, θ_t^{peak} may become negative as shown in Fig. 5(e). In addition, here, the dielectric losses may be more effective to decrease the TAPF of transmitted wave than the magnetic losses. Comparing Fig. 4 with Fig. 5, apparently, an interesting conclusion may be obtained that, for a beam of obliquely incident HHPW, the refraction angle of TE wave is generally different from that of TM wave. Which have been noticed by L.G. Guimaraes and E.E.S. Sampaio recently by a somewhat different theoretical way [12].

Finally, we shall address equivalence between our approach and the previous way in which the generalized laws of reflection and refraction are derived from the phase-matching condition and complex valued boundary conditions. It is noted that there are several traditional expressions for the laws of reflection and refraction, such as the complex Snell's law [1], the Adler–Chu–Fano formulation [4] and the Dupertuis–Proctor–Acklin formulation [7], etc. For the coplanar cases, the three formulations are demonstrated to be equivalent. Generally, the Adler–Chu–Fano formulation and the Dupertuis–Proctor–Acklin formulation may predict that the reflected, transmitted and incident waves are noncoplanar. However, the noncoplanar cases can not be obtained by using the complex Snell's law [7,9]. Here, we shall point out that, on the one hand, either HHPWs or HIPWs may be used to compose the arbitrary electromagnetic wave; on the other hand, either the real valued boundary conditions or the phase-matching and complex valued boundary conditions may satisfy the integral form of Maxwell's equations associated with an interface. Therefore, in principle, our approach may be equivalent to the previous one. In addition, as mentioned above, several similar phenomena may be predicted by the two ways. However, mainly due to the complexity of the HIPWs, a strict confirmation of the equivalence between the two approaches has not been obtained yet. We have to leave it for a further study.

5. Conclusions

In summary, we have shown that, due to oblique propagation of wave (corresponding to Gaussian surface or integrating loop), the term(s) of $\frac{\exp\left(j\tilde{k}\, l\,tg\theta\right)-1}{j\tilde{k}\sin\theta}$ must arise in the integral form of Maxwell's equations, which impacts directly on phase-matching condition and boundary conditions. It is proved that, at a lossy interface, the real valued boundary conditions are valid universally for both HHPWs and HIPWs. A new way is developed to derive laws of reflection and refraction from real valued boundary conditions by using HHPWs. The obtained results show that the usual Snell and Fresnel laws are recovered only in the special case. Several novel properties of transmitted wave induced by the energy losses are predicted numerically. Generally, energy losses produce the TAPF of transmitted wave mainly distribute in certain angle, which does not seem to have been noticed before. In addition, the heavy losses may produce negative refraction without negative index, and the refraction angle of TE wave is not identical to that of TM wave generally.

Our study provides a new angle of view to further understand properties of electromagnetic fields in the lossy media. On the other hand, several issues are required to be further study, such as the equivalence between our approach and the previous one, the dynamical process and physical meanings of the cases having $|\sin\theta_t| > 1$ (which is traditionally taken as the total reflection, however, leads the nonphysical transmitted angle θ_t), and so on. The predicted properties of transmitted wave associated with energy losses may be applied to experimentally test our theoretical analysis. Apparently, confirmations about the expected properties should motivate further theoretical progress.

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