

## Interaction effects on magneto-oscillations in a two-dimensional electron gas

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Motivated by recent experiments, we study the interaction corrections to the damping of magneto-oscillations in a two-dimensional electron gas (2DEG). We identify leading contributions to the interaction-induced damping which are induced by corrections to the effective mass and quantum scattering time. The damping factor is calculated for Coulomb and short-range interaction in the whole range of temperatures, from the ballistic to the diffusive regime. It is shown that the dominant effect is that of the renormalization of the effective electron mass due to the interplay of the interaction and impurity scattering. The results are relevant to the analysis of experiments on magneto-oscillations (in particular, for extracting the value of the effective mass) and are expected to be useful for understanding the physics of a high-mobility 2DEG near the apparent metal-insulator transition.

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### I. INTRODUCTION

The influence of electron-electron interaction on transport properties of low-dimensional disordered conductors at sufficiently low temperatures  $T$  remains one of the central topics of the condensed matter physics. In a seminal set of works (see the review Ref. 1), Altshuler and Aronov studied the effects of the interplay of interaction and disorder on conductivity and tunneling density of states in the diffusive regime characterized by the condition  $2\pi T\tau \ll 1$ , where  $\tau$  is the transport mean free time (we set  $k_B = \hbar = 1$ ). Their results were generalized within the framework of the renormalization group (RG) by Finkelstein.<sup>2</sup> The last decade has witnessed a renewed increase of activity in this field, largely motivated by experiments on an apparent metal-insulator transition in two-dimensional (2D) systems. This interest was triggered by experiments<sup>3</sup> which showed a “metallic” behavior (decrease of resistivity with lowering  $T$ ) in high-mobility Si structures. Later, qualitatively similar behavior was observed in a variety of high-mobility 2D systems, see Refs. 4–7 for reviews.

The metallic behavior has been attributed to the effects of the electron-electron interaction in the ballistic temperature range,  $2\pi T\tau \gg 1$ . These effects were originally considered in the framework of the temperature-dependent screening.<sup>8</sup> More recently, a systematic theory was developed, taking into account also exchange contributions and the effects of both parallel and transverse magnetic fields, and valid in the whole range of  $T$  from the diffusive to the ballistic regime.<sup>9,10</sup> Another mechanism that can explain the metallic behavior of resistivity in an intermediate temperature range in the diffusive regime was studied within the RG framework in Ref. 11. It is applicable to systems with more than one valley, such as silicon metal-oxide-semiconductor field-effect transistor.

Despite these successes of the theory, numerous experimental observations remain puzzling and wait for an explanation. In particular, it was found that the spin susceptibility, proportional to the product  $mg$  of the effective mass  $m$  and

the  $g$ -factor, is strongly growing when the density approaches the value  $n_c$  corresponding to the apparent transition. This conclusion was drawn on the basis of several experimental methods, including the analysis of beating pattern of Shubnikov-de Haas oscillations,<sup>12–15</sup> the study of magnetoresistance<sup>16,17</sup> in the parallel field, and measurement of thermodynamic magnetization;<sup>18</sup> see recent reviews.<sup>19,20</sup>

The enhancement of susceptibility with lowering density, interpreted in a number of papers as its divergence at  $n = n_c$ , has attracted a great deal of attention, since it might be an indicator of some phase transition that the system undergoes with a decrease of density. The interpretation of the data has remained, however, controversial. In particular, it remained unclear whether the strong increase of spin susceptibility should be attributed to that of  $m$  or of  $g$ . This information is of crucial importance for understanding the nature of the possible transition.

Several experimental approaches have been used to separate the behavior of the effective mass from that of the  $g$ -factor. In Refs. 21–23 a fit of the resistivity data to the theoretical formulas of Ref. 9 was used to find the interaction constant  $F_0^\sigma$ , and thus the  $g$ -factor. The accuracy of this procedure is questionable, since the theory of Ref. 9 neglects higher Fermi-liquid interaction constants and assumes isotropic impurity scattering. Another approach is based on thermodynamic measurements in strong magnetic field.<sup>24</sup> However, the authors of this work were able to measure the effective mass in a very narrow interval of electron concentration only, so that the results are not too informative. Also, a strong magnetic field is expected to influence strongly the characteristics of the electron liquid, so that the applicability of such measurements to the low-field properties is questionable. So, while most of the above measurements seem to indicate that it is the effective mass that is responsible for the strongly enhanced susceptibility, an independent verification is clearly needed.

A well-known method for determination of the effective mass is based on the investigation of the temperature dependence of Shubnikov-de Haas oscillations (SdHO). It was ap-

plied to the present problem in Refs. 14 and 25. However, the analysis of the SdHO data is complicated by the fact that both the effective mass and the elastic quantum scattering time  $\tau_q$  are  $T$ -dependent, in view of the combined effect of interaction and disorder. An unambiguous interpretation of experimental data requires<sup>14</sup> a theoretical information on  $T$ -dependence of  $m$  and  $\tau_q$ . A development of the corresponding theory is the aim of the present paper.

In fact, a recent paper<sup>26</sup> has made an important step in this direction. Specifically, it was shown in Ref. 26 that the Lifshitz-Kosevich formalism,<sup>27</sup> originally developed for the analysis of magneto-oscillations in a 3D Fermi liquid, is also applicable in 2D in the regime where the oscillations are exponentially suppressed by temperature smearing or disorder. (In the regime of strong oscillations, the Lifshitz-Kosevich formula in 2D should be modified, as was earlier shown in Ref. 28.) Another result of Ref. 26 is that the inelastic electron-electron relaxation does not contribute to the damping of magneto-oscillations (similarly to the earlier result of Ref. 29 for the case of electron-phonon scattering).

The authors of Ref. 26 then calculated the contribution to the damping induced by the interplay of interaction and disorder. Their theoretical treatment of the problem is, however, far from complete. First, they consider only diagrams for the self-energy with one impurity-ladder vertex correction to the interaction line and discard diagrams with no and with two vertex corrections. Second, they claim that the  $T$ -dependence of the oscillation damping rate can be equivalently attributed either to the correction to the effective mass, or to the quantum scattering rate (Dingle temperature). Furthermore, in the latter case their result for the  $T$ -dependence of  $\tau_q$  is in contradiction with the picture of Friedel oscillations inducing a correction to the relaxation rate, which is linear in  $T$  and is governed by backscattering.<sup>9</sup>

In addition to the above experimental motivation, the development of the theory of interaction effects on magneto-oscillations in a disordered two-dimensional electron gas (2DEG) represents a fundamental theoretical problem. Such a theory should complement the recently developed theory of interaction effects in transport of 2D electrons in zero and nonquantizing magnetic fields.<sup>9,10</sup> Let us emphasize a peculiar aspect of the present problem. The damping of oscillations is governed by the self-energy, which is a single-particle quantity. [Indeed, the relevant diagrams, see Sec. II B below, are reminiscent of those for the tunneling density of states (DOS)]. Generally, the self-energy is not a gauge-invariant object. On the other hand, the magnetization and the conductivity (magneto-oscillations of which we would like to study) are observable (and thus gauge-invariant) quantities. It is well-known that the gauge-invariance is of crucial importance for interaction-induced corrections; a difference between the results for conductivity and for tunneling DOS in the case of Coulomb interaction<sup>1,2</sup> serves as a nice illustration. It is thus a theoretical challenge to see how the gauge invariance manifests itself in the magneto-oscillation problem.

The outline of the paper is as follows. Section II is devoted to presentation of the general formalism. In Sec. III we apply it to calculate the interaction-induced contribution to the damping of magneto-oscillations in the case of short-

range interaction. In Sec. IV we show how to extract from the above result the corrections to the effective mass and the quantum scattering time. We also perform a calculation of the correction to the scattering time based on the picture of Friedel oscillations and demonstrate a complete agreement between the two approaches. In Sec. V we generalize our results to the case of Coulomb interaction. We also perform there a comparison with the calculation of Ref. 26. Section VI summarizes our findings. Some technical details of our calculations are presented in Appendixes.

## II. MAGNETO-OSCILLATIONS: GENERAL FORMALISM

### A. Derivation of the formula for a decay of the oscillations

We begin by calculating the oscillatory part  $\Omega_{\text{osc}}$  of the thermodynamic potential  $\Omega$ . From this quantity one can derive the oscillating contribution to the thermodynamic density of states

$$\frac{\partial n_{\text{osc}}}{\partial \mu} = \frac{\partial^2 \Omega_{\text{osc}}}{\partial \mu^2}, \quad (1)$$

where  $\mu$  is the chemical potential, and de Haas-van Alphen oscillations of magnetic susceptibility

$$\chi_{\text{osc}} = - \frac{\partial^2 \Omega_{\text{osc}}}{\partial B^2}, \quad (2)$$

where  $B$  is a magnetic field. The main subject of our interest is the exponential damping factor of these magneto-oscillations. For noninteracting electrons, the same exponential damping factor governs the magnitude of the Shubnikov-de Haas oscillations of the conductivity for the case of weak disorder potential in sufficiently weak magnetic field,<sup>35,36</sup> where the self-consistent Born approximation<sup>35</sup> (SCBA) is valid. As we are going to show, the interaction-induced correction to the damping factor of the thermodynamic density of states arises due to the renormalization of the effective mass and the quantum scattering time. Therefore these  $T$ -dependent corrections to the damping factor govern the magnitude of the Shubnikov-de Haas oscillations as well, similarly to the noninteracting case.

Our starting point is the expression for the thermodynamic potential derived in the paper by Luttinger and Ward<sup>30</sup>

$$\Omega = -T \text{Tr} \ln(-G^{-1}) - T \text{Tr}(G\Sigma) + \Omega'. \quad (3)$$

Here

$$G(i\varepsilon_n, m\omega_c) = [G_0^{-1}(i\varepsilon_n, m\omega_c) - \Sigma(i\varepsilon_n, m\omega_c)]^{-1} \quad (4)$$

is the dressed Green's function in the Matsubara formalism,  $i\varepsilon_n = (2n+1)i\pi T$  is the Matsubara fermionic energy,  $\omega_c$  is the cyclotron frequency, and  $m$  is the Landau level index. Further,

$$G_0(i\varepsilon_n, m\omega_c) = \frac{1}{i\varepsilon_n + \mu - (m+1/2)\omega_c}$$

is the Green's function in the absence of disorder and interaction, and  $\Sigma(i\varepsilon_n, m\omega_c)$  is a self-energy part of Green's func-

$$-T \text{Tr} \ln(-G^{-1}) = -T \text{Tr} \ln(-G_0^{-1}) + \text{Diagram 1} + \frac{1}{2} \text{Diagram 2} + \frac{1}{3} \text{Diagram 3} + \dots$$

FIG. 1. The logarithmic term  $\tilde{\Omega}$ , Eq. (7), in the thermodynamic potential, Eq. (3), is a sum of closed loop diagrams with self-energy insertions  $\Sigma$ . This term is responsible for magneto-oscillations.

tion which includes all the disorder and interaction effects.

The trace in Eq. (3) implies summation over Matsubara frequencies  $\varepsilon_n$  and over Landau levels  $m$ . The logarithmic term contains all the closed loop diagrams with insertion of self-energy (Fig. 1). The terms  $-T \text{Tr}(G\Sigma)$  and  $\Omega'$  are introduced to avoid double-counting of diagrams.<sup>30,31</sup> The term  $\Omega'$  denotes the sum of all so-called skeleton diagrams with all bare Green's functions replaced by dressed Green's functions (for the recent discussion of Luttinger-Ward formalism in 2D Fermi systems see Refs. 32 and 33).

As shown in Ref. 34, the exponential decay of magneto-oscillations is described by the  $\text{Tr} \ln$  term. The oscillatory parts of the additional terms, which are introduced to fight overcounting, cancel each other. In order to obtain the correction to the thermodynamic potential we need to calculate the self-energy part of the Green's function.

We decompose the self-energy into two parts:

$$\Sigma(i\varepsilon_n, m\omega_c) = \Sigma_{\text{dis}}(i\varepsilon_n, m\omega_c) + \Sigma_{\text{ee}}(i\varepsilon_n, m\omega_c), \quad (5)$$

where  $\Sigma_{\text{dis}}(i\varepsilon_n, m\omega_c)$  denotes the self-energy part due to the scattering on disorder potential with electron-electron interaction switched off and  $\Sigma_{\text{ee}}(i\varepsilon_n, m\omega_c)$  contains all the interaction effects.

In this paper we assume that disorder potential is  $\delta$ -correlated, inducing a large-angle scattering of electrons. The disorder-induced (noninteracting) part of the self-energy for white-noise disorder and weak magnetic field  $\omega_c \tau \ll 1$  is given by

$$\Sigma_{\text{dis}}(i\varepsilon_n, m\omega_c) = -\frac{i \text{sgn } \varepsilon_n}{2\tau}. \quad (6)$$

For stronger magnetic field (i.e., for separated Landau levels), one should employ the SCBA. In this case, both the real and imaginary parts of the self-energy depend on  $i\varepsilon_n$  in a nontrivial way. In this paper, however, we will address only weak magnetic fields when Landau levels overlap.

We thus consider the relevant term  $\tilde{\Omega} = -T \text{Tr} \ln(-G^{-1})$  in the thermodynamic potential,

$$\tilde{\Omega} = -2\nu T \sum_{n=-\infty}^{\infty} \omega_c \sum_{m=0}^{\infty} \ln[\omega_c m - \mu - i\varepsilon_n + \Sigma(i\varepsilon_n, \omega_c m)]. \quad (7)$$

For overlapping Landau levels,

$$\omega_c \tau \ll 1, \quad (8)$$

the  $k$ th harmonics of the magneto-oscillations

$$\Omega_{\text{osc}} = 2\nu \left(\frac{\omega_c}{2\pi}\right)^2 \sum_k A_k \cos \frac{2k\pi^2 n_e}{eB}$$

( $n_e$  is the electron concentration) is damped by disorder even at zero temperature via the standard Dingle factor  $\exp(-\pi k/\omega_c \tau)$ . Therefore we will consider only the first harmonics of the oscillations,  $A_1$ , neglecting all  $A_k$  with  $k > 1$  (whose damping is much stronger).

The oscillatory part of Eq. (7) is calculated in Appendix A:

$$\Omega_{\text{osc}} \simeq 2\nu \left(\frac{\omega_c}{2\pi}\right)^2 A_1 \cos \frac{2\pi^2 n_e}{eB} \quad (9)$$

with the amplitude of the first harmonics of the oscillatory part of the thermodynamic potential given by

$$A_1 \equiv \frac{4\pi^2 T}{\omega_c} \sum_{\varepsilon_n > 0} \exp\left(-\frac{2\pi}{\omega_c^*} \left[\varepsilon_n + \frac{1}{2\tau(1+\alpha_0)} + \frac{i\delta\Sigma(i\varepsilon_n, \xi_0)}{1+\alpha_0}\right]\right). \quad (10)$$

Here  $\delta\Sigma(i\varepsilon_n, \xi_0)$  is the self-energy part related to the interplay of disorder and interaction. It is analytically continued from the points  $m\omega_c$  to the whole complex plane  $\xi$  and taken at  $\xi = \xi_0$ , where  $\xi_0$  [defined in Eqs. (A9) and (A10) of Appendix A] is the pole of the Green's function in the presence of disorder.

The coefficients  $\beta_0$  and  $\alpha_0$  determine the Fermi-liquid (FL) renormalization of the effective mass in a pure system at zero  $T$ ,

$$m^* = m \frac{1+\alpha_0}{1+\beta_0}. \quad (11)$$

The effective mass  $m^*$  in turn governs the expression for the FL-renormalized effective cyclotron frequency,

$$\omega_c^* = \frac{eB}{m^*} = \omega_c \frac{1+\beta_0}{1+\alpha_0}. \quad (12)$$

The coefficient  $\alpha_0$  is related to the FL renormalization of the  $Z$ -factor,

$$Z = \frac{1}{1+\alpha_0}, \quad (13)$$

which is given by the residue of the Green's function.

Depending on the relation between temperature  $T$  and the elastic scattering rate  $1/\tau$ , there are two regimes: ballistic,  $T\tau \gg 1$ , and diffusive,  $T\tau \ll 1$  (more accurately, the relevant dimensionless parameter is  $2\pi T\tau$ ). In the ballistic regime, it follows from Eq. (8) that  $T \gg 1/\tau \gg \omega_c$ . The diffusive regime can be further split into two subregimes: normal diffusive ( $\omega_c \ll T \ll 1/\tau$ ) and ultradiffusive ( $T \ll \omega_c \ll 1/\tau$ ). When  $T \gg \omega_c$ , as in the ballistic and normal diffusive regimes, only the first Matsubara frequency  $\varepsilon_0 = \pi T$  in the sum determining  $A_1$  is relevant, since the contribution of higher Matsubara frequencies are exponentially suppressed. On the other hand, in the ultradiffusive regime  $T \ll \omega_c$  and higher Matsubara frequencies contribute as well.

In what follows we concentrate on the case  $T \gg \omega_c$ . Under this condition, we get

$$A_1 = \frac{4\pi^2 T}{\omega_c} \exp\left[-\frac{2\pi}{\omega_c^*} \{\pi T + iZ\delta\Sigma(i\pi T, \xi_0)\}\right] \exp\left[-\frac{\pi}{\omega_c^* \tau^*}\right], \quad (14)$$

where we introduced the FL-renormalized scattering time

$$\tau^* = \tau(1 + \alpha_0). \quad (15)$$

We note that this renormalization of  $\tau$  is incomplete since it does not include the FL vertex corrections to the impurity scattering line. The corresponding contributions are contained in  $\delta\Sigma(i\pi T, \xi_0)$  [ $T$ -independent terms in Eqs. (41) and (81)] and will be addressed in Secs. III and IV below. We also note that the inelastic contribution to the self-energy  $\propto [(\pi T)^2 - \varepsilon_n^2] \text{sgn } \varepsilon_n$  (see Appendix B) vanishes for  $\varepsilon_n = \pi T$ , and thus does not affect the damping of the magneto-oscillations  $B(T)$  for  $T \gg \omega_c$ , in agreement with Refs. 26 and 29.

Using the renormalized quantities in Eq. (14) we represent  $A_1$  in the form

$$A_1 = A_1^{(0)}(T) \exp[B(T)], \quad (16)$$

where

$$A_1^{(0)}(T) \equiv \frac{4\pi^2 T}{\omega_c} \exp\left[-\frac{2\pi^2 T}{\omega_c^*} - \frac{\pi}{\omega_c^* \tau^*}\right] \quad (17)$$

is the standard FL Lifshitz-Kosevich result and

$$B(T) = -\frac{2\pi i Z \delta\Sigma(i\pi T, \xi_0)}{\omega_c^*}. \quad (18)$$

We thus see that in order to obtain the additional interaction-induced damping factor of magneto-oscillations it is necessary to evaluate  $Z\delta\Sigma(i\pi T, \xi_0)$ .

### B. Self-energy

We begin by considering the interaction-induced self-energy part  $\Sigma_{ee}(i\varepsilon_n, m\omega_c)$  in the lowest order in interaction. This is sufficient in the case of a weak short-range interaction analyzed in Sec. III below. For the more realistic case of the Coulomb interaction (Sec. V), the relevant higher-order terms can be treated using the random-phase approximation (RPA). Higher-order contributions to the  $T$ -dependent part of the self-energy,  $\delta\Sigma(i\pi T, \xi_0)$ , are small in the parameter  $1/E_F\tau$  or  $T/E_F$ .

Let us list important elements which are necessary for calculation of the interaction-induced part of the self-energy (Fig. 2). Each contribution to the self-energy has exchange and Hartree parts. We first address the exchange contribution (the Hartree terms can be written in a similar way). It contains the angle-averaged Green's function covered by the effective interaction line. The corresponding vertices may be dressed by impurity ladders (Fig. 3). Notice that the renormalized vertex includes at least one impurity line. Finally, when the interaction line changes the signs of Matsubara frequencies at vertices, an additional diagram (we term it a

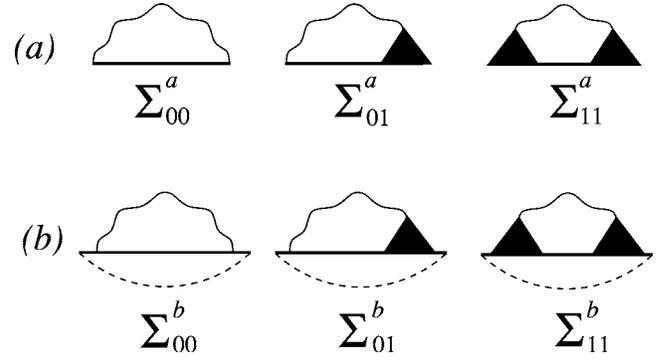


FIG. 2. Self-energy diagrams in the first order in the effective interaction (wavy line). Black triangles denote impurity ladders  $\Gamma$  dressing interaction vertices, Fig. 3. (a) ‘‘Simple’’ self-energies  $\Sigma_{ij}^a$ ; and (b) Hikami-box self-energies  $\Sigma_{ij}^b$  generated by covering each of  $\Sigma_{ij}^a$  by an impurity line (dashed).

‘‘Hikami-box diagram’’ with a single impurity line covering the whole block is to be included [Fig. 2(b)].

We split  $\Sigma_{ee}(i\varepsilon_n, m\omega_c)$  into three contributions, corresponding to different possibilities of dressing the two interaction vertices by impurities,

$$\Sigma_{ee} = \Sigma_{00} + 2\Sigma_{01} + \Sigma_{11}. \quad (19)$$

Here the subscripts  $i, j=0, 1$  indicate whether the corresponding vertex is dressed by an impurity ladder. The factor 2 in front of the  $\Sigma_{01}$  term reflects two possibilities of dressing one of the interaction vertices. Each of the terms  $\Sigma_{ij}$  is a sum of two contributions,  $\Sigma_{ij} = \Sigma_{ij}^a + \Sigma_{ij}^b$ , where  $\Sigma_{ij}^a$  is the ‘‘simple’’ self-energy [Fig. 2(a)] and  $\Sigma_{ij}^b$  is its Hikami-box counterpart [Fig. 2(b)]. We note that in Ref. 26 only one out of six diagrams (namely,  $\Sigma_{01}^a$ ) was taken into account.

The expression for  $\Sigma_{00}^a$  in a finite magnetic field reads

$$\Sigma_{00}^a(i\varepsilon_n, m\omega_c) = -T \sum_{\omega_k} \sum_L \int \frac{d^2 q}{(2\pi)^2} J_{|L|}^2(qR_c) V(i\omega_k, \mathbf{q}) \times G(i\varepsilon_n - i\omega_k, m\omega_c + L\omega_c), \quad (20)$$

where

$$G(i\varepsilon_n, m\omega_c) = \frac{1}{i\varepsilon_n + \mu - (m + 1/2)\omega_c - \Sigma(i\varepsilon_n, m\omega_c)} \quad (21)$$

is the Green's function in Landau levels representation,  $V(i\omega_k, \mathbf{q})$  is the effective interaction, and  $J_{|L|}^2(qR_c)$  describes the bare vertex function connecting Landau levels  $m$  and  $m+L$  in the quasiclassical limit  $m, m+L \gg 1$  (for details see, e.g., Ref. 37 and references therein). In the expressions for



FIG. 3. Interaction vertex renormalized by the impurity ladder,  $\Gamma(i\omega_k, \mathbf{q})$ . Dashed line represents scattering on impurity.

$\Sigma_{01}^a$  and  $\Sigma_{11}^a$  this bare vertex function is multiplied by  $\Gamma(i\omega_k, \mathbf{q})$  and  $\Gamma^2(i\omega_k, \mathbf{q})$ , respectively, where  $\Gamma$  is the impurity ladder (Fig. 3).

In the limit  $B \rightarrow 0$ , the corresponding self-energies depend on the momentum  $\mathbf{p}$  instead of the Landau level index  $m$ . The interaction vertices are dressed only when the Matsubara energies at the vertices have opposite signs, which restricts the summation over  $\omega_k$  in  $\Sigma_{01}$  and  $\Sigma_{11}$  to the domain  $\varepsilon_n(\omega_k - \varepsilon_n) > 0$ :

$$\Sigma_{00}^a(i\varepsilon_n, \xi_p) = -T \sum_{\omega_k} \int \frac{d^2q}{(2\pi)^2} V(i\omega_k, \mathbf{q}) G(i\varepsilon_n - i\omega_k, \mathbf{p} - \mathbf{q}), \quad (22)$$

$$\Sigma_{01}^a(i\varepsilon_n, \xi_p) = -T \sum_{\varepsilon_n(\omega_k - \varepsilon_n) > 0} \int \frac{d^2q}{(2\pi)^2} V(i\omega_k, \mathbf{q}) \Gamma(i\omega_k, \mathbf{q}) \times G(i\varepsilon_n - i\omega_k, \mathbf{p} - \mathbf{q}), \quad (23)$$

$$\Sigma_{11}^a(i\varepsilon_n, \xi_p) = -T \sum_{\varepsilon_n(\omega_k - \varepsilon_n) > 0} \int \frac{d^2q}{(2\pi)^2} V(i\omega_k, \mathbf{q}) \Gamma^2(i\omega_k, \mathbf{q}) \times G(i\varepsilon_n - i\omega_k, \mathbf{p} - \mathbf{q}), \quad (24)$$

where  $G(i\varepsilon_n, \mathbf{p}) = [i\varepsilon_n + \mu - \xi_p + i \operatorname{sgn} \varepsilon_n / 2\tau - \sum_{ee}(i\varepsilon_n, \xi_p)]^{-1}$  with  $\xi_p = p^2/2m$  and the vertex correction (Fig. 3) reads

$$\Gamma(i\omega_k, \mathbf{q}) = \frac{1}{\sqrt{(|\omega_k|\tau + 1)^2 + (qv_F\tau)^2 - 1}}. \quad (25)$$

The expressions for Hikami-box self-energies are obtained in a similar way, see Appendix B.

To calculate the damping factor of the oscillations, we need the self-energy taken at the value of  $\xi$  which is determined by the pole of the Green's function,  $\xi = \xi_0$ , see Appendix B. According to Eq. (18), the self-energy is further multiplied by  $Z$  in the damping exponent. This is equivalent to calculating the following integral:

$$\begin{aligned} Z \Sigma_{ij}^a(i\varepsilon_n, \xi_0) &= \frac{\operatorname{sgn} \varepsilon_n}{2\pi i} \frac{1 + \beta_0}{1 + \alpha_0} \int d\xi_k G(i\varepsilon_n, k) \Sigma_{ij}^a(i\varepsilon_n, \xi_k) \\ &\simeq \operatorname{sgn} \varepsilon_n \frac{v_F^*}{2\pi i} \int dk G(i\varepsilon_n, k) \Sigma_{ij}^a(i\varepsilon_n, \xi_k). \end{aligned} \quad (26)$$

We note that the  $Z$ -factor drops out in the product  $Z\Sigma$ . Indeed, the Green's function under the interaction line in the self-energy contains the  $Z$ -factor in the numerator so that in the numerator of the product  $Z\Sigma$  we get the factor  $Z^2$ . However, the  $Z$ -factor is not a gauge-invariant quantity and therefore should not appear in the expressions for observables, in contrast to the FL-renormalized effective mass. At this point we should take into account the FL renormalization of the two interaction vertices in  $\Sigma$ . Since we are interested in the contribution of relatively slow transferred momenta and frequencies giving rise to the  $T$  dependence of  $B(T)$ ,  $q \ll k_F$  and  $\omega_k \ll E_F$ , we can set them to zero when considering the FL vertex renormalization. Then one can apply the Ward identity for the FL-interaction dressing of the vertices,<sup>39</sup> which amounts to multiplying each vertex by a factor  $1/Z$ . These

vertex factors in the denominator of  $B(T)$  cancel  $Z^2$  in the numerator of  $B(T)$ . This implies that one can simply discard such renormalizations, setting  $Z=1$  everywhere, when the observable quantities are calculated. The FL renormalization then amounts to replacement of the bare band mass  $m$ , Fermi velocity  $v_F$ , and elastic scattering time,  $\tau$ , by the renormalized parameters,  $m^*$ ,  $v_F^*$ , and  $\tau^*$ , respectively. In what follows, we will omit the asterisks, using the notation  $m$ ,  $v_F$ , and  $\tau$  for the renormalized quantities.

The relevant contributions to the self-energy are calculated in Appendix B. Combining all the terms together, we have

$$\delta\Sigma(i\varepsilon_n, \xi_0) = -iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) K(i\omega_m, \mathbf{q}), \quad (27)$$

$$K(i\omega_m, \mathbf{q}) = \frac{[1 + \Gamma(i\omega_m, \mathbf{q})]^2}{S(i\omega_m, \mathbf{q})} \left[ 1 - \frac{W}{\tau S^2(i\omega_m, \mathbf{q})} \right] - \frac{1}{S_0(i\omega_m, \mathbf{q})}, \quad (28)$$

where

$$\begin{aligned} S(i\omega_m, \mathbf{q}) &= \sqrt{(|\omega_m| + 1/\tau)^2 + v_F^2 q^2} = \sqrt{W^2 + v_F^2 q^2}, \\ W &= |\omega_m| + 1/\tau \end{aligned} \quad (29)$$

and

$$S_0(i\omega_m, \mathbf{q}) = \sqrt{|\omega_m|^2 + v_F^2 q^2}. \quad (30)$$

An important feature of the kernel function  $K(i\omega_m, \mathbf{q})$ , Eq. (28), is that it is exactly zero for  $q=0$  for arbitrary  $\omega_m$ . Indeed, using  $S(i\omega_m, q=0) = W$ ,  $S_0(i\omega_m, q=0) = |\omega_m|$ , and  $\Gamma(i\omega_m, q=0) = 1/(W\tau - 1)$ , we get

$$\begin{aligned} K(i\omega_m, q=0) &= \frac{1}{W} \left[ 1 + \frac{1}{W\tau - 1} \right]^2 \left[ 1 - \frac{1}{W\tau} \right] - \frac{1}{|\omega_m|} \\ &= \frac{1}{W - 1/\tau} - \frac{1}{|\omega_m|} = 0. \end{aligned} \quad (31)$$

We stress that this equality only holds when all the contributions to the self-energy are combined together. This property of the kernel function is characteristic for the gauge-invariant quantities in the presence of interaction. Indeed, the interaction at  $q=0$  implies the shift of the chemical potential and hence can be gauged out.<sup>2</sup> Therefore the contribution of small  $q$  to the observables should be suppressed by vanishing of the corresponding kernel function. The same situation is well-known for the interaction-induced correction to the conductivity.<sup>9</sup>

It is worth discussing a peculiarity of the problem of magneto-oscillations with respect to the gauge invariance. The gauge invariance of the oscillatory part of the observables is guaranteed by the fact that the thermodynamic potential is represented by closed loops. Since the characteristic spatial scale for such a loop is cyclotron radius  $R_c$ , the interaction with momenta  $q \ll R_c^{-1}$  should not contribute. So, if we would find that our result does not satisfy this requirement, it would mean that the diagrammatic treatment is not sufficient

and should be complemented by the infrared cutoff at  $q \sim R_c^{-1}$ . On a more rigorous level this could be done by using the real-space path integral approach combining the treatment of magneto-oscillations in the presence of long-range disorder<sup>40</sup> with the quantum kinetic-equation approach to the interaction effects<sup>9,41</sup> and to magnetotransport.<sup>42</sup> In this context, it is instructive to recall the calculation of the dephasing length  $l_\varphi$ , where the infrared cutoff at  $q \sim l_\varphi^{-1}$  or  $q \sim R^{-1}$  arises for the problems of weak localization<sup>43</sup> and Aharonov-Bohm oscillations<sup>44</sup> in a ring of radius  $R$ , respectively.

Since we find, however, that the kernel  $K(i\omega_m, q)$  governing the perturbative self-energy (27) does satisfy the condition  $K(i\omega_m, q=0)=0$ , the above cutoff is irrelevant and the perturbative treatment is sufficient in the considered regime of strongly damped oscillations,  $T \gg \omega_c$ . Indeed, the  $q$ -integral in Eq. (27) is cut off at  $q \sim \omega/v_F \sim T/v_F$  which is much larger than  $R_c^{-1}$  under the above condition.

We also emphasize that the kernel  $K(i\omega_m, q)$  given by Eq. (28) vanishes in the clean limit

$$K(i\omega_m, q)|_{\tau \rightarrow \infty} = 0. \quad (32)$$

This implies, in particular, that the correction to the effective mass found in Refs. 32–34 from the  $\varepsilon$ - and  $p$ -dependence of the self-energy  $\Sigma(\varepsilon, p)$  of a clean system,  $\delta m/m^* \sim T/E_F$  does not show up in the damping of magneto-oscillations, in accordance with the statement made in Ref. 32. In general,  $\Sigma(\varepsilon, p)$  is not an observable (and not gauge invariant) quantity, and thus the above correction  $\delta m$  should be at least treated with caution.

### III. DAMPING OF MAGNETO-OSCILLATIONS: SHORT-RANGE INTERACTION

In this section we evaluate  $\delta\Sigma(i\varepsilon_n, \xi_0)$  in the case of a weak pointlike interaction given by

$$V(i\omega_m, \mathbf{q}) \equiv U_0. \quad (33)$$

We are interested in the correction to the self-energy to the first order in  $\nu U_0$ , where  $\nu = m^*/2\pi$  is the density of states per spin direction. We calculate the contributions  $\Sigma_{ij}^a$  and  $\Sigma_{ij}^b$  starting from Eqs. (B15), (B8), (B9), and (B20)–(B22) derived in Appendix B. Using the notation  $S(i\omega_m, \mathbf{q})$  introduced in Eq. (29), the vertex factor  $\Gamma$  can be presented as

$$\Gamma(i\omega_k, \mathbf{q}) = \frac{1}{S\tau - 1}. \quad (34)$$

Performing the momentum integration for  $0 < q < k_F$ , we obtain

$$\begin{aligned} \Sigma_{00}^a(i\varepsilon_n, \xi_0) &= -iTU_0 \sum_{\omega_m > \varepsilon_n} \int_0^{k_F} \frac{qdq}{2\pi} \left[ \frac{1}{S} - \frac{1}{S_0} \right] \\ &= -\frac{iTU_0}{2\pi\nu_F^2} \sum_{\omega_m > \varepsilon_n} [\sqrt{E_F^2 + W^2} \\ &\quad - W - \sqrt{E_F^2 + \omega_m^2} + \omega_m] \\ &\approx -\frac{iT\nu U_0}{2E_F\tau} \sum_{\omega_m > \varepsilon_n} \left[ \frac{\omega_m}{\sqrt{E_F^2 + \omega_m^2}} - 1 \right], \end{aligned} \quad (35)$$

$$\begin{aligned} &2\Sigma_{01}^a(i\varepsilon_n, \xi_0) + \Sigma_{11}^a(i\varepsilon_n, \xi_0) \\ &= -\frac{iTU_0}{2\pi\nu_F^2\tau} \sum_{\omega_m > \varepsilon_n} \int_{\omega_m+1/\tau}^{\sqrt{E_F^2 + \omega_m^2}} dS \frac{(2S-1/\tau)}{(S-1/\tau)^2} \\ &= -\frac{iT\nu U_0}{2E_F\tau} \sum_{\omega_m > \varepsilon_n} \left( \ln \frac{E_F^2 + \omega_m^2}{\omega_m^2} + \frac{1}{\omega_m\tau} \right). \end{aligned} \quad (36)$$

These expressions are valid independently of the value of the parameter  $T\tau$ , i.e., they describe both the diffusive and the ballistic regimes, as well as the crossover between them. The logarithmic term in Eq. (36) comes from  $\Sigma_{01}^a$ , while the  $1/\omega_m$  term originates from  $\Sigma_{11}^a$ .

The contributions of the Hikami-box diagrams are given by

$$\begin{aligned} \Sigma_{00}^b(i\varepsilon_n) &= \frac{iT\nu U_0}{2E_F\tau} \sum_{\omega_m > \varepsilon_n} \left( \omega_m + \frac{1}{\tau} \right) \int_{\omega_m+1/\tau}^{\sqrt{E_F^2 + \omega_m^2}} \frac{dS}{S^2} \\ &\approx -\frac{iT\nu U_0}{2E_F\tau} \sum_{\omega_m > \varepsilon_n} \left[ \frac{\omega_m}{\sqrt{E_F^2 + \omega_m^2}} - 1 \right], \end{aligned} \quad (37)$$

$$\begin{aligned} &2\Sigma_{01}^b(i\varepsilon_n) + \Sigma_{11}^b(i\varepsilon_n) \\ &= \frac{iTU_0}{2\pi\nu_F^2\tau^2} \sum_{\omega_m > \varepsilon_n} \left( \omega_m + \frac{1}{\tau} \right) \int_{\omega_m+1/\tau}^{\sqrt{E_F^2 + \omega_m^2}} \frac{dS}{S^2} \frac{(2S-1/\tau)}{(S-1/\tau)^2} \\ &= \frac{iT\nu U_0}{2E_F\tau} \sum_{\omega_m > \varepsilon_n} \frac{1}{\omega_m\tau}. \end{aligned} \quad (38)$$

We see that the Hikami-box contribution exactly cancels the second  $(1/\omega_m)$  term in Eq. (36). Thus the total correction to the self-energy reads

$$\begin{aligned} &\delta\Sigma(i\varepsilon_n, \xi_0) \\ &= -\frac{iT\nu U_0}{E_F\tau} \sum_{\omega_m > \varepsilon_n} \left[ \left( \frac{\omega_m}{\sqrt{E_F^2 + \omega_m^2}} - 1 \right) + \ln \frac{\sqrt{E_F^2 + \omega_m^2}}{\omega_m} \right]. \end{aligned} \quad (39)$$

The upper limit of the summation over  $\omega_m$  is effectively given by  $m_{\max} \sim E_F/T$ . The term  $[\omega_m/\sqrt{E_F^2 + \omega_m^2} - 1]$  in Eq. (39), which originates from  $\Sigma_{00}$ , yields after the summation over  $\omega_m > \pi T$  a contribution  $\propto T$  in addition to a large  $T$ -independent contribution, renormalizing  $\tau$ . As we will see below, a term  $\propto T \ln(E_F/T)$  will arise from the contributions of  $\Sigma_{01}$  and  $\Sigma_{11}$ , and hence  $\Sigma_{00}$  yields only a subleading contribution to the damping.

To calculate the sum of logarithms in Eq. (39) we write

$$\sum_{\omega_m > \varepsilon_n} \ln \frac{\sqrt{E_F^2 + \omega_m^2}}{\omega_m} = \sum_{\omega_m > \varepsilon_n} \left[ \ln \frac{E_F}{\omega_m} + \ln \frac{\sqrt{E_F^2 + \omega_m^2}}{E_F} \right]. \quad (40)$$

For  $T \ll E_F$  the second sum in the right-hand side can be replaced by the integral, yielding a  $T$ -independent contribution. We further use the identity

$$\sum_{m=1}^M \ln \frac{N}{m} = M \ln N - \ln \Gamma(M+1),$$

and the Stirling's formula

$$\ln \Gamma(M+1) = M \ln M - M + \frac{1}{2} \ln(2\pi M) + \dots$$

for  $M \gg 1$ , where  $\Gamma(x)$  is the gamma function. As a result, we get for arbitrary positive  $\varepsilon_n$

$$\delta\Sigma(i\varepsilon_n, \xi_0) \simeq -\frac{iT\nu U_0}{E_F\tau} \left\{ \frac{c_1 E_F}{T} - \frac{\varepsilon_n}{2\pi T} \ln \frac{c_2 E_F}{T} + \ln \left[ \frac{1}{\sqrt{2\pi}} \Gamma \left( \frac{\varepsilon_n}{2\pi T} + \frac{1}{2} \right) \right] \right\}, \quad (41)$$

where  $c_{1,2}$  are constants of order unity. We see that the contribution to  $\delta\Sigma(i\pi T, \xi_0)$  containing vertex corrections to the interaction line has an additional factor  $\ln(E_F/T)$  as compared to the  $T$ -dependent part of  $\delta\Sigma_{00}$ . In Eq. (41), we have absorbed the contribution of  $\delta\Sigma_{00}$  into the upper cutoff of the log-term which is given by  $E_F$  up to a factor of order unity. Furthermore, the same can be done with the last term in Eq. (41) which at  $\varepsilon_n = \pi T$  also yields a linear-in- $T$  contribution. Equation (41) thus translates for  $T \gg \omega_c$  into the following expression for the damping exponent  $B(T) = -2\pi i \delta\Sigma(i\pi T, \xi_0)/\omega_c$ :

$$B(T) = -c_1 \nu U_0 \frac{\pi}{\omega_c \tau} + \frac{\pi T \nu U_0}{\omega_c E_F \tau} \ln \frac{c_2 E_F}{T}. \quad (42)$$

The first term in Eq. (42) describes the  $T$ -independent FL-renormalization of  $\tau$  due to vertex corrections and should be included in the effective relaxation time  $\tau^*$ , as was mentioned after Eq. (15) in Sec. II. The second term represents the  $T$ -dependent contribution to the damping factor that we are interested in and is analyzed in the next section.

#### IV. INTERPRETATION: EFFECTIVE MASS VS QUANTUM SCATTERING TIME

The above result (42) can be interpreted in terms of corrections to the effective mass (or  $\omega_c$ ) and the elastic scattering rate entering the standard formula (17). These corrections come from the interplay of disorder and interaction. Writing

$$\begin{aligned} A_1(T) &= \frac{4\pi^2 T}{\omega_c} \exp \left[ -\frac{2\pi^2 T}{(\omega_c + \delta\omega_c)} - \frac{\pi}{(\omega_c + \delta\omega_c)} \frac{1}{(\tau + \delta\tau)} \right] \\ &\simeq A_1^{(0)}(T) \exp \left[ \frac{2\pi^2 T \delta\omega_c}{\omega_c} \right] \exp \left[ \frac{\pi}{\omega_c \tau} \left( \frac{\delta\omega_c}{\omega_c} + \frac{\delta\tau}{\tau} \right) \right] \\ &\simeq A_1^{(0)}(T) \exp \left[ -\frac{2\pi^2 T \delta m}{\omega_c m} \right] \exp \left[ \frac{\pi}{\omega_c \tau} \left( -\frac{\delta m}{m} + \frac{\delta\tau}{\tau} \right) \right], \end{aligned} \quad (43)$$

we conclude that

$$B(T) = -\frac{2\pi^2 T \delta m}{\omega_c m} - \frac{\pi}{\omega_c \tau} \left( \frac{\delta m}{m} - \frac{\delta\tau}{\tau} \right). \quad (44)$$

It is worth noting that the FL-renormalization does not affect the product  $\omega_c m = eB$ .

Comparing Eqs. (42) and (44) [we recall that the first term in Eq. (42) is absorbed in  $\tau$ ], we see that the  $T \ln T$  dependence of the damping factor could in principle originate either from the  $\ln T$  correction to the effective mass, or from the  $T \ln T$ -type correction to  $\tau$ . This led the authors of Ref. 26 to the conclusion that the nonlinear  $T$ -dependence of the damping factor may be equivalently interpreted either as a  $T$ -dependent renormalization of the effective mass or as a  $T$ -dependent Dingle temperature. It is clear, however, that these two possibilities correspond to different physical processes.

#### A. Self-energy at real energies: Analytical continuation

To identify the physical origin of the leading contribution to the damping it is instructive to obtain  $B(T)$  using the expression for the self-energy analytically continued to real values of energies  $\varepsilon_n \rightarrow -i\varepsilon$ . Performing the analytical continuation to real energies  $\varepsilon$  and real frequencies  $\omega$  in Eq. (27), we get

$$\begin{aligned} \delta\Sigma(\varepsilon, \xi_0) &= \frac{\nu U_0}{E_F \tau} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \tanh \left( \frac{\varepsilon - \omega}{2T} \right) \\ &\times \left\{ \ln \left( \frac{\sqrt{E_F^2 - \omega^2}}{-i\omega} \right) + \left( \frac{-i\omega}{\sqrt{E_F^2 - \omega^2}} - 1 \right) \right\}, \end{aligned} \quad (45)$$

so that the real part of the self-energy is given by

$$\begin{aligned} \text{Re } \delta\Sigma(\varepsilon, \xi_0) &= \frac{\nu U_0}{4\pi E_F \tau} \left\{ \int_0^{E_F} d\omega \left[ \tanh \left( \frac{\varepsilon - \omega}{2T} \right) \right. \right. \\ &\quad \left. \left. + \tanh \left( \frac{\varepsilon + \omega}{2T} \right) \right] \left[ \ln \left( \frac{\sqrt{E_F^2 - \omega^2}}{\omega} \right) - 1 \right] \right. \\ &\quad \left. + \int_{E_F}^{\infty} d\omega \left[ \tanh \left( \frac{\varepsilon - \omega}{2T} \right) + \tanh \left( \frac{\varepsilon + \omega}{2T} \right) \right] \right. \\ &\quad \left. \times \left[ \ln \left( \frac{\sqrt{\omega^2 - E_F^2}}{\omega} \right) + \frac{\omega}{\sqrt{\omega^2 - E_F^2}} - 1 \right] \right\} \\ &\simeq \varepsilon \frac{\nu U_0}{2\pi E_F \tau} \ln \frac{E_F}{\max[|\varepsilon|, T]}. \end{aligned} \quad (46)$$

The leading contribution here comes from the term  $\ln(E_F/\omega)$  in the first integral over  $\omega < E_F$  while other terms only rescale the ultraviolet cutoff  $E_F$  of the logarithm by a constant of order unity, which is beyond the accuracy of our quasi-classical approximation.

The imaginary part of  $\delta\Sigma$  reads

$$\begin{aligned} \text{Im } \delta\Sigma(\varepsilon, \xi_0) &= \frac{\nu U_0}{4\pi E_F \tau} \int_0^{E_F} d\omega \left[ \tanh\left(\frac{\varepsilon - \omega}{2T}\right) - \tanh\left(\frac{\varepsilon + \omega}{2T}\right) \right] \\ &\quad \times \left[ \frac{\pi}{2} - \frac{\omega}{\sqrt{E_F^2 - \omega^2}} \right] \\ &\simeq -\text{const} \frac{\nu U_0}{\tau} + \frac{\nu U_0}{2E_F \tau} T \ln \left[ 2 \cosh\left(\frac{\varepsilon}{2T}\right) \right], \end{aligned} \quad (47)$$

where the  $T$ -dependent term has the following asymptotics:

$$T \ln[2 \cosh(\varepsilon/2T)] = \begin{cases} \varepsilon/2, & \varepsilon \gg T, \\ T \ln 2, & \varepsilon \ll T. \end{cases} \quad (48)$$

The contribution of the term  $\omega/\sqrt{E_F^2 - \omega^2}$  to the integral in Eq. (47) is  $T$ -independent up to small corrections of order of  $\nu U_0(T/E_F)^2/\tau$  which are beyond the accuracy of the calculation.

Having calculated  $\text{Re } \Sigma$  and  $\text{Im } \Sigma$  for real energies  $\varepsilon$ , we can determine  $\delta m$  and  $\delta\tau$ . Indeed, the magnitude of the first harmonics of the magneto-oscillations of the thermodynamic density of states is expressed through the real- $\varepsilon$  self-energy  $\delta\Sigma(\varepsilon)$  as follows:

$$A_1(T) = \frac{4\pi^2 T}{\omega_c} \int d\varepsilon \left[ -\frac{\partial n_F(\varepsilon)}{\partial \varepsilon} \right] A_1(\varepsilon, T), \quad (49)$$

$$\begin{aligned} A_1(\varepsilon, T) &= \exp \left\{ \frac{2\pi i}{\omega_c} [\varepsilon - \text{Re } \delta\Sigma(\varepsilon, \xi_0)] \right\} \\ &\quad \times \exp \left\{ -\frac{\pi}{\omega_c \tau} + \frac{2\pi}{\omega_c} \text{Im } \delta\Sigma(\varepsilon, \xi_0) \right\}, \end{aligned} \quad (50)$$

where  $n_F(\varepsilon) = [1 + \exp(\varepsilon/T)]^{-1}$  is the Fermi distribution function.

In analogy with Eq. (43) we represent the energy-dependent amplitude  $A_1(\varepsilon, T)$  in terms of energy- and temperature-dependent corrections to the quantum scattering time and mass,  $\delta\tau(\varepsilon, T)$  and  $m(\varepsilon, T)$ :

$$\begin{aligned} A_1(\varepsilon, T) &= \exp \left\{ \frac{2\pi i \varepsilon}{\omega_c} \left[ 1 + \frac{\delta m(\varepsilon, T)}{m} \right] \right\} \\ &\quad \times \exp \left\{ -\frac{\pi}{\omega_c \tau} \left[ 1 + \frac{\delta m(\varepsilon, T)}{m} - \frac{\delta\tau(\varepsilon, T)}{\tau} \right] \right\}. \end{aligned} \quad (51)$$

Comparing Eq. (51) with Eq. (50), we express  $\delta\tau(\varepsilon, T)$  and  $m(\varepsilon, T)$  through  $\text{Re } \Sigma(\varepsilon)$  and  $\text{Im } \Sigma(\varepsilon)$  as follows:

$$\frac{\delta m(\varepsilon, T)}{m} = -\frac{\text{Re } \delta\Sigma(\varepsilon, T)}{\varepsilon}, \quad (52)$$

$$\frac{\delta\tau(\varepsilon, T)}{\tau} = 2\tau \text{Im } \delta\Sigma(\varepsilon, T) + \frac{\delta m(\varepsilon, T)}{m}. \quad (53)$$

Using Eqs. (46) and (47) in combination with Eqs. (52) and (53), we obtain

$$\frac{\delta m(\varepsilon, T)}{m} = -\frac{\nu U_0}{2\pi E_F \tau} \ln \frac{E_F}{\max[|\varepsilon|, T]}, \quad (54)$$

$$\frac{\delta\tau(\varepsilon, T)}{\tau} = \nu U_0 \frac{T}{E_F} \ln \left[ 2 \cosh\left(\frac{\varepsilon}{2T}\right) \right] - \frac{\nu U_0}{2\pi E_F \tau} \ln \frac{E_F}{\max[|\varepsilon|, T]}. \quad (55)$$

The integration in Eq. (49) sets in effect  $\varepsilon \sim T$  in the above expressions. The  $T$ -dependent corrections to the effective mass and the quantum scattering time extracted experimentally with the help of Eq. (43) are thus given by

$$\frac{\delta m(T)}{m} = -\frac{\nu U_0}{2\pi E_F \tau} \ln \frac{E_F}{T}, \quad (56)$$

$$\frac{\delta\tau(T)}{\tau} = \nu U_0 \frac{T}{E_F} - \frac{\nu U_0}{2\pi E_F \tau} \ln \frac{E_F}{T}. \quad (57)$$

In Eq. (57) we assume that  $\pi\nu U_0 T/E_F \ll \omega_c \tau$ , expand  $\exp[\pi\delta\tau(\varepsilon, T)/\omega_c \tau^2]$ , and then average the term  $\ln[2 \cosh(\varepsilon/2T)]$  [which is a real-energy counterpart of the last term in Eq. (41)] with  $-\partial n_F(\varepsilon)/\partial \varepsilon$ .

It is clear from these results that the leading term in  $B(T)$  [proportional to  $T \ln(E_F/T)$ , Eq. (42)] originates from the real part of the self-energy, Eq. (46), i.e., from renormalization of the effective mass, which affects incommensurability of the oscillations at different values of energy  $\varepsilon$ . The contribution to  $B(T)$  of the imaginary part of the self-energy [corresponding to the last term in Eq. (41)], which is governed in the ballistic regime by the renormalization of the scattering time, is smaller by a factor  $\ln(E_F/T)$ . In the expression for the damping, Eq. (42), this contribution is absorbed in the numerical constant  $c_2$  in the upper cutoff of the logarithm.

The obtained result for the interaction-induced correction to the scattering time  $\tau$ , Eq. (57), agrees, up to a factor  $\frac{1}{2}$ , with the correction to the transport time following from the calculation of conductivity correction in the ballistic regime in Ref. 9. This is exactly what one would expect on physical grounds. Indeed, it is known that the conductivity correction<sup>9</sup> can be understood as governed by an additional, predominantly backscattering, contribution to the scattering cross section related to the dressing of an impurity by Friedel oscillations. Since this contribution is concentrated near the scattering angle  $\phi = \pi$ , the correction to the momentum relaxation rate is larger by the factor  $1 - \cos \phi \approx 2$  than the correction to the total scattering rate. In Sec. IV B we will corroborate the results of this section by an explicit calculation of the contribution to the impurity scattering rate due to Friedel oscillations.

Up to now we calculated the exchange contribution to the self-energy. For the pointlike interaction, the Hartree term has opposite sign and is twice larger in magnitude than the exchange term due to the spin summation. This simply reverses the sign of the corrections to the damping factor.

### B. Calculation of $\delta\tau$ from the scattering off Friedel oscillations

In this section we calculate the correction to the total elastic scattering time  $\delta\tau$  in a different, physically more transparent way, considering the scattering off impurities dressed by Friedel oscillations.<sup>9,38</sup> We will demonstrate how the result (57) is reproduced in this way. In particular, this will confirm once more that there is no  $T \ln T$  term in  $\delta\tau$  and therefore the leading  $T \ln T$  contribution to the damping factor comes from  $\delta m$ .

We start with the expression relating the total elastic scattering rate and the scattering cross section  $S(\phi)$  of a single impurity,

$$\frac{1}{\tau_q(\epsilon, T)} = n_{\text{imp}} v_F \int \frac{d\phi}{2\pi} S(\phi), \quad (58)$$

where  $n_{\text{imp}}$  is the concentration of impurities. The expression for the transport scattering time  $\tau_{\text{tr}}$  determining the conductivity differs from Eq. (58) by a factor  $1 - \cos \phi$  in the integrand:

$$\frac{1}{\tau_{\text{tr}}(\epsilon, T)} = n_{\text{imp}} v_F \int \frac{d\phi}{2\pi} S(\phi) (1 - \cos \phi). \quad (59)$$

We note that the two times,  $\tau_q$  and  $\tau_{\text{tr}}$ , though equal for the pointlike impurities in the noninteracting case [for which  $S(\phi) = (n_{\text{imp}} v_F \tau)^{-1} = \text{const}(\phi)$ ], differ from each other when the scattering off Friedel oscillations is taken into account.

The impurity scattering cross section for dressed impurities reads

$$\begin{aligned} S(\phi) &= \frac{2\pi\nu}{v_F} \left| V\left(2k \sin \frac{\phi}{2}\right) \right|^2 \\ &\approx S_0 + \frac{2\pi\nu V_0}{v_F} 2 \text{Re} \delta V\left(2k \sin \frac{\phi}{2}\right). \end{aligned} \quad (60)$$

Here  $S_0 = 2\pi\nu V_0^2 / v_F$  is the bare impurity scattering cross section and  $V_0 = \int d^2r V_0(r)$ , where  $V_0(r)$  is the bare pointlike impurity potential. The cross section  $S(\phi)$  depends on energy  $\epsilon$  of an electron through  $k = k_F + \epsilon / v_F$  in the Fourier transform  $V(q) = V_0 + \delta V(q)$  of the effective impurity potential, renormalized by the Friedel oscillations of the electron density. For  $r \gg k_F^{-1}$  the oscillatory correction to the electron density reads

$$\delta\rho(r) = -\nu V_0 \frac{(2\pi r T / v_F) \sin 2k_F r}{\sinh(2\pi r T / v_F) \pi r^2}. \quad (61)$$

The correction to the impurity scattering potential  $\delta V(r)$  due to scattering off the Friedel oscillations is proportional, for the short-range interaction, to the electron density at point  $\mathbf{r}$ . Similarly to the consideration of Secs. III and IV A, we will concentrate on the exchange part of this correction,

$$\delta V(r) = -\frac{1}{2} U_0 \delta\rho(r). \quad (62)$$

To calculate the correction to the impurity cross section, we need the Fourier transform of  $\delta V(r)$ ,

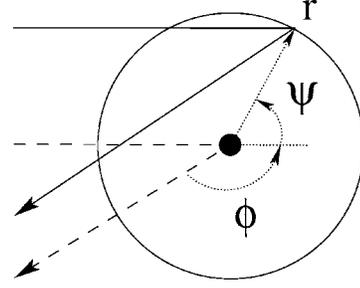


FIG. 4. Schematic illustration of the scattering off Friedel oscillations. The black dot in the middle represents a short-range impurity which creates the oscillatory correction to the electron density around it. The circle represents the equipotential line of the effective impurity potential. The correction to the impurity cross section at the angle  $\phi$  arises due to the interference of two electronic waves (Refs. 9 and 38), one of which (dashed line) scattered by the impurity and another (solid line) by the Friedel oscillations at a point parametrized by the distance  $r$  from the impurity and the angle  $\psi$ .

$$\delta V\left(2k \sin \frac{\phi}{2}\right) = \frac{U_0}{2} \int d^2r \delta\rho(r) \exp\left(2ikr \sin \frac{\phi}{2} \cos \psi\right), \quad (63)$$

where  $\psi$  is the polar angle of  $\mathbf{r}$ , see Fig. 4. Substituting Eqs. (61) and (63) into Eq. (60), we find the interaction-induced correction to the scattering cross section,

$$\begin{aligned} \delta S(\phi) &= \nu U_0 S_0 \int \frac{d\psi}{2\pi} \int dr \frac{(2\pi T / v_F)}{\sinh(2\pi r T / v_F)} \left\{ \sin\left(2r \left[ k_F \right. \right. \right. \\ &\quad \left. \left. \left. + k \cos \psi \sin \frac{\phi}{2} \right] \right) + \sin\left(2r \left[ k_F - k \cos \psi \sin \frac{\phi}{2} \right] \right) \right\}. \end{aligned} \quad (64)$$

Performing the integration over  $r$  we get

$$\delta S(\phi) = S_0 \frac{\nu U_0}{4} \int d\psi [\tanh \beta_+ + \tanh \beta_-], \quad (65)$$

where

$$\begin{aligned} \beta_{\pm} &= \frac{v_F}{2T} \left( k_F \pm k \cos \psi \sin \frac{\phi}{2} \right) \\ &= \frac{E_F}{T} \left[ 1 \pm \left( 1 + \frac{\epsilon}{2E_F} \right) \cos \psi \sin \frac{\phi}{2} \right]. \end{aligned} \quad (66)$$

For  $\beta_{\pm} \gg 1$ , i.e., for most values of the scattering angle  $\phi$  except for those corresponding to the backscattering ( $\phi \approx \pi$ ), we see that the scattering cross section does not depend on  $\phi$  up to exponentially small corrections of order  $\mathcal{O}(\exp[-E_F/T])$ :

$$S(\phi) = S_0 [1 + \pi\nu U_0], \quad |\phi - \pi| \gg (\max[T, \epsilon] / E_F)^{1/2}. \quad (67)$$

On the other hand, in the vicinity of  $\phi = \pi$  the cross section has a ‘‘hump,’’ see Fig. 5, of the width and the height scaling as  $\delta\phi$ ,  $\delta S(\phi) / S_0 \sim (\max[T, \epsilon] / E_F)^{1/2}$ . The explicit expression for  $S(\phi)$  at  $T=0$  can be found in Ref. 9. In Fig. 5 we plotted

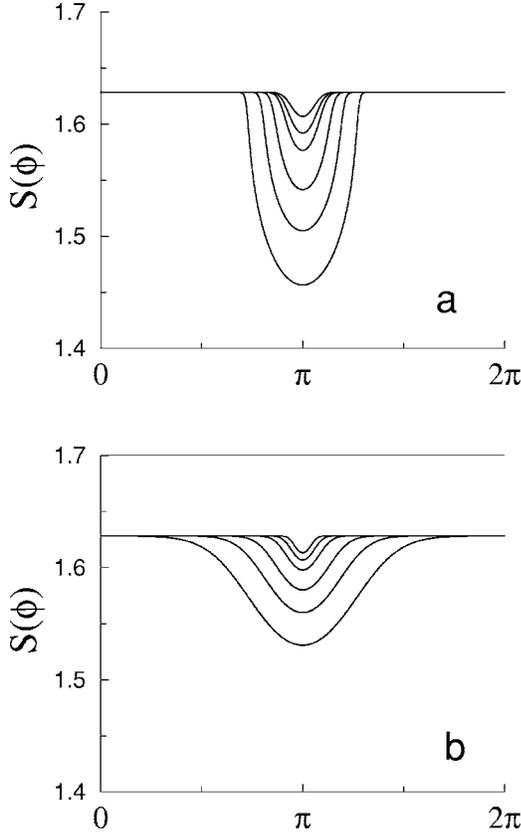


FIG. 5. Differential impurity cross section  $S(\phi)$  (in units of the bare cross section  $S_0$ ) renormalized by the scattering off Friedel oscillations (with only the exchange contribution taken into account) calculated for  $\nu U_0=0.2$  and (a) fixed  $T=0.01E_F$  and  $\epsilon/E_F=0, 0.01, 0.02, 0.05, 0.1$ , and  $0.2$ , from top to bottom; and (b) fixed  $\epsilon=0$  and  $T/E_F=0.005, 0.01, 0.02, 0.05, 0.1$ , and  $0.2$ , from top to bottom.

$S(\phi)$  for fixed  $T$  and several values of  $\epsilon$  [Fig. 5(a)] and for fixed  $\epsilon=0$  and several values of  $T$  [Fig. 5(b)].

To calculate the  $(\epsilon, T)$ -dependent correction to  $1/\tau_q$  (which is determined by the ‘‘hump’’), we expand  $\sin(\phi/2)$  around  $\phi=\pi$ . Furthermore, we expand  $\cos\psi$  around  $\psi=\pi$  and  $\psi=0$  in the expressions for  $\beta_+$  and  $\beta_-$ , respectively. This corresponds to the two possible interfering paths propagating along the horizontal line in Fig. 4 with the scattering off the Friedel oscillation occurring either to the left or to the right from the impurity. Denoting  $\delta\psi=x$ ,  $\delta\phi/2=y$ ,  $z^2=x^2+y^2$ , and  $\omega=z^2E_F$ , we get from Eq. (65)

$$\begin{aligned} \int \frac{d\phi}{2\pi} \delta S(\phi) &\simeq S_0 \frac{\nu U_0}{2\pi} \int dx \int dy \tanh \left\{ \frac{E_F}{T} \left[ 1 - \left( 1 + \frac{\epsilon}{2E_F} \right) \right. \right. \\ &\quad \left. \left. \times \left( 1 - \frac{x^2+y^2}{2} \right) \right] \right\} \\ &\simeq S_0 \nu U_0 \int z dz \tanh \left\{ \frac{E_F}{T} \left[ \frac{z^2}{2} - \frac{\epsilon}{2E_F} \right] \right\} \\ &= S_0 \frac{\nu U_0}{2E_F} \int_0^{\sim E_F} d\omega \tanh \left( \frac{\omega - \epsilon}{2T} \right). \end{aligned} \quad (68)$$

Using Eqs. (58) and (68), we find the total quantum scattering rate,

$$\begin{aligned} \frac{1}{\tau_q(\epsilon, T)} &= \frac{1}{\tau} + \delta \left( \frac{1}{\tau_q} \right), \quad (69) \\ \delta \left( \frac{1}{\tau_q} \right) &= - \frac{\delta \tau_q(\epsilon, T)}{\tau^2} \\ &= \frac{\nu U_0}{\tau} \left[ \text{const} - \frac{T}{E_F} \ln \left( 2 \cosh \frac{\epsilon}{2T} \right) - \frac{\epsilon}{2E_F} \right]. \end{aligned} \quad (70)$$

This result for the interaction-induced correction to  $\tau_q$  agrees (to the leading order in  $1/T\tau$  corresponding to the ballistic regime) with that obtained from the imaginary part of the self-energy, Eqs. (55) and (57). More accurately, Eq. (70) differs from Eq. (55) by the last ( $T$ -independent) term  $\nu U_0 \epsilon / 2E_F \tau$ , which drops out after the thermal averaging with  $-\partial n_F(\epsilon) / \partial \epsilon$  and thus does not contribute to  $\delta \tau_q(T)$ , Eq. (57). This term is in a sense anomalous, since it arises from the ultraviolet limit of the  $\omega$  integration in Eq. (68). In fact, one could question the validity of this contribution, since we used the asymptotic, large- $r$  form of the Friedel oscillations in Eq. (61). One can check, however, that the same result [up to an irrelevant additive constant independent of  $T$  and  $\epsilon$ : in Eq. (67)  $\pi \nu U_0$  is replaced by  $2\nu U_0$ ] is obtained from a calculation using the exact form of the Friedel oscillations.<sup>46</sup> The appearance of this linear-in- $\epsilon$  term is related to the violation of the particle-hole symmetry in the parabolic spectrum; this term did not appear in the diagrammatic calculation of Sec. IV where the spectrum was linearized. What, however, enters the experimental damping of magneto-oscillations is  $1/\tau_q$  integrated over the energy with an even function  $-\partial n_F(\epsilon) / \partial \epsilon$ . Therefore we are in fact interested in the collision rate symmetrized with respect to  $\epsilon \rightarrow -\epsilon$ . Performing this symmetrization in Eq. (64), we get a result determined solely by the infrared scale,  $r \sim \nu F/T$ , yielding Eq. (70) without the last, linear-in- $\epsilon$  term.

Finally, using Eq. (59), we see that the correction to the transport rate is larger than  $\delta(1/\tau_q)$  by a factor of 2. The corresponding correction to the conductivity reproduces the result of Ref. 9 in the ballistic limit.

## V. DAMPING OF MAGNETO-OSCILLATIONS: COULOMB INTERACTION

We turn now to the Coulomb interaction. In the case of Coulomb interaction, one should take into account the dynamical screening of the interaction within the random phase approximation (RPA), see Fig. 6. In what follows we use for simplicity the so-called  $F_0$  approximation,<sup>9</sup> which retains only the zeroth harmonics of the Fermi-liquid constants  $F_0^p$  and  $F_0^\sigma$  in the charge and spin channels, respectively. Then the effective interaction propagator in the charge channel (combining the exchange term and the singlet contribution of the Hartree-type interaction) reads<sup>9,10</sup>

$$V^p(i\omega_m, q) = [(V_0(q) + F_0^p/2\nu)^{-1} + \Pi(i\omega_m, q)]^{-1}, \quad (71)$$

where  $\Pi(i\omega_m, q)$  is the polarization operator

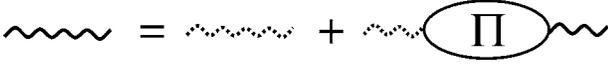


FIG. 6. Diagrammatic equation for the effective interaction line (bold wavy line) in the random-phase approximation, Eqs. (71) and (73). Dotted wavy line represents the bare interaction,  $V_0(q) + F_0^p/2\nu$  in the singlet channel or  $F_0^\sigma/2\nu$  in the triplet channel. The bubble  $\Pi$  is the polarization operator, Eq. (72).

$$\Pi(i\omega_m, q) = 2\nu[1 - |\omega_m|\tau\Gamma(i\omega_m, q)], \quad (72)$$

$\Gamma(i\omega_m, q)$  is the impurity ladder, and  $\nu = m/2\pi$  is the density of states per spin direction. The triplet contribution to the effective interaction arises from the ladder of Hartree-type interaction blocks and reads

$$V^\sigma(i\omega_m, q) = [2\nu/F_0^\sigma + \Pi(i\omega_m, q)]^{-1}. \quad (73)$$

### A. Singlet channel

The main difference as compared to the case of the short-range weak interaction considered in Sec. III is the nontrivial form of the dynamically screened Coulomb interaction (one should take into account the renormalization of the interaction by polarization operator). Using

$$V_0(q) = \frac{2\pi e^2}{q}$$

and neglecting  $F_0^p$ , we get

$$\begin{aligned} V^p(i\omega_m, q) &= \frac{V_0(q)}{1 + V_0(q)\Pi(i\omega_m, q)} \\ &= \frac{2\pi e^2/q}{1 + (2\pi e^2/q)2\nu[1 - |\omega_m|\tau\Gamma(i\omega_m, q)]} \\ &= \frac{1}{2\nu(q + \kappa)(S - 1/\tau) - \kappa(W - 1/\tau)}. \end{aligned} \quad (74)$$

Here we use the standard notation  $\kappa = 4\pi\nu e^2$  for the inverse Thomas-Fermi screening radius and use the shorthand notation  $S$  for  $S(i\omega_m, \mathbf{q}) \equiv \sqrt{W^2 + v_F^2 q^2}$  with  $W = |\omega_m| + 1/\tau$ .

For  $q \ll \kappa$ , neglecting  $q$  in the sum  $q + \kappa$  in the denominator of Eq. (74), one finds that the exchange interaction

$$\tilde{V}^p(i\omega_m, q) = \frac{1}{2\nu} \frac{S - 1/\tau}{S - W} \quad (75)$$

has a singularity  $\propto 1/q^2$  in the limit  $q \rightarrow 0$ :

$$\tilde{V}^p(i\omega_m, q) = \frac{1}{2\nu} \frac{2|\omega_m|(|\omega_m| + 1/\tau)}{q^2 v_F^2}, \quad q \rightarrow 0, \quad (76)$$

so that each separate term  $\Sigma_{ij}^a, \Sigma_{ij}^b$  in the self-energy would diverge. This divergence is analogous to the one encountered in the course of calculation of the tunneling density of states.<sup>1</sup> In that case, one has to keep the  $q$  term in the denominator of Eq. (74), which cuts off the logarithmic divergence. For the present problem, this is, however, not needed. Indeed, as was emphasized in the end of Sec. II, the kernel  $K(i\omega_m, q)$  combining together contributions of all relevant

self-energy diagrams is proportional to  $q^2$  in the limit  $q \rightarrow 0$  and hence cancels the singularity in  $\tilde{V}(i\omega_m, q)$ . In view of this, it is convenient to represent the kernel function (28) in a form which shows explicitly that  $K(i\omega_m, q=0) = 0$  [we recall that  $S_0(q=0) = |\omega_m|$  and  $S(q=0) = W$ ]:

$$K(i\omega_m, q) = \left( -\frac{1}{S_0} + \frac{1}{|\omega_m|} \right) + \frac{S - W}{S - 1/\tau} \left[ \frac{1}{S\tau(S - 1/\tau)} - \frac{1}{|\omega_m|} \right]. \quad (77)$$

Then the product  $\tilde{V}^p(i\omega_m, q)K(i\omega_m, q)$  takes the form

$$2\nu\tilde{V}^p(i\omega_m, q)K(i\omega_m, q) = \frac{S_0 - |\omega_m|}{S_0(S - W)} - \frac{1}{S_0} + \frac{1}{S\tau(S - 1/\tau)}. \quad (78)$$

Performing the integration over the momentum  $q$  in Eq. (27), we get the correction to self-energy in the singlet (charge) channel

$$\begin{aligned} \delta\Sigma^p(i\varepsilon_n, \xi_0) &= -\frac{iT}{2E_F\tau} \sum_{\omega_m > \varepsilon_n}^{\Delta} \left\{ (1 + 2\omega_m\tau) \ln \frac{1 + 2\omega_m\tau}{2\omega_m\tau} - 1 \right. \\ &\quad \left. + \ln \frac{\Delta^2 + \omega_m^2}{\omega_m^2} \right\}, \end{aligned} \quad (79)$$

where we introduced

$$\Delta \equiv \kappa v_F. \quad (80)$$

Comparing Eqs. (79) and (39), we see that the last term in Eq. (79) corresponds to a static short-range interaction with  $\nu U_0 = 1$ .

Setting  $\varepsilon_n = \pi T$  and separating the contributions to the sum (79) governed by the high-energy ( $\varepsilon \sim \Delta$ ) and low-energy ( $\varepsilon \sim T$ ) regions, we can present the result in the following form:

$$\begin{aligned} \delta\Sigma^p(i\pi T, \xi_0) &= -\frac{iT}{2E_F\tau} \left[ \frac{\text{const } \Delta}{T} - \left( 1 - \frac{1}{8\pi T\tau} \right) \ln \frac{\Delta}{T} \right. \\ &\quad \left. - f(4\pi T\tau) \right], \end{aligned} \quad (81)$$

where  $f(x)$  is a parameterless function,

$$\begin{aligned} f(x) &= \sum_{m=1}^{\infty} \left[ 1 - (1 + mx) \ln \frac{1 + mx}{mx} + \frac{1}{2mx} \right] \\ &= \begin{cases} \frac{c_f}{x} + \left[ \frac{1}{2x} + \frac{1}{2} \right] \ln \frac{1}{x}, & x \ll 1, \\ \frac{\pi^2}{36x^2}, & x \gg 1, \end{cases} \end{aligned} \quad (82)$$

with  $c_f = -3/4 - \psi(1)/2 = -0.461392\dots$  [here  $\psi(x)$  is the digamma function]. Thus the dynamical screening of Coulomb interaction leads to different asymptotics of the self-energy in the diffusive and ballistic regimes, in contrast to the case of weak short-range interaction.<sup>47</sup>

The  $T$ -dependence of the leading correction to the magneto-oscillations damping factor due to the interaction in the singlet channel has therefore the form

$$B^p(T) = \frac{\pi}{\omega_c \tau E_F} T \left[ \left( 1 - \frac{1}{8\pi T \tau} \right) \ln \frac{\Delta}{T} + f(4\pi T \tau) \right] \quad (83)$$

$$= \frac{\pi}{\omega_c \tau E_F} T \times \begin{cases} \frac{3}{2} \ln \frac{\Delta}{T} - \frac{1}{2} \left( 1 + \frac{1}{4\pi T \tau} \right) \ln(4\pi \Delta \tau) - \frac{c_f}{4\pi T \tau}, & 4\pi T \tau \ll 1, \\ \left( 1 - \frac{1}{8\pi T \tau} \right) \ln \frac{\Delta}{T}, & 4\pi T \tau \gg 1. \end{cases} \quad (84)$$

This result is illustrated in Fig. 7. Note that the ballistic asymptotics describes the exact result with a remarkable accuracy down to very low temperature,  $T\tau \sim 0.01-0.05$ , see Fig. 7(a). Retaining in Eq. (84) only the leading terms and suppressing the  $T$ -independent contributions which can be absorbed in the FL-renormalized  $\tau$ , we get

$$B^p(T) = \frac{\pi}{\omega_c \tau E_F} T \times \begin{cases} \frac{3}{2} \ln \frac{\Delta}{T} - \frac{1}{2} \ln(4\pi \Delta \tau), & 4\pi T \tau \ll 1, \\ \ln \frac{\Delta}{T}, & 4\pi T \tau \gg 1. \end{cases} \quad (85)$$

### B. Triplet channel

Calculation of the corresponding triplet contribution  $B^\sigma(T)$  is presented in Appendix C and leads to qualitatively similar asymptotics. The leading term in the total correction to the damping factor in the ballistic regime, realized in experiments on low-disorder samples at realistic temperatures, takes the simple form

$$B(T) = B^p(T) + B^\sigma(T) \approx \left( 1 + \frac{3F_0^\sigma}{1 + F_0^\sigma} \right) \frac{\pi}{\omega_c \tau E_F} T \ln \frac{\Delta}{T}. \quad (86)$$

As discussed in Sec. IV, this result arises due to the correction to the effective mass (the consideration of Sec. IV fully applies to the case of the Coulomb interaction as well).

It is worth noting that due to the factor  $(1 + F_0^\sigma)$  in the denominator of  $B^\sigma(T)$ , the damping of magneto-oscillations tends to diverge upon approaching the Stoner ferromagnetic instability. Since the damping is determined by the effective mass  $m^*(T)$  (see Sec. IV), we conclude that  $m^*(T)$  diverges as  $F_0^\sigma \rightarrow -1$  due to the interplay of disorder and interaction. This should be contrasted with the clean case, where the effective mass is solely determined by Fermi-liquid constant in the singlet channel,  $F_1^p$ , and hence is insensitive to the ferromagnetic instability.

### C. Discussion and comparison to earlier work

Let us discuss the obtained results for the damping factor  $B(T)$ . In both the diffusive and ballistic regimes we find the

temperature dependence of the form  $B(T) \propto T \ln T$ . In the diffusive (low- $T$ ) regime the relative correction to the damping factor,  $\omega_c \tau B \sim g^{-1} T \tau \ln T$  is less singular than the known corrections<sup>1</sup> to the conductivity,  $\delta\sigma/\sigma \sim g^{-1} \ln T$ , and the tunneling density of states,  $\delta\rho/\rho \sim g^{-1} \ln^2 T$  (here  $g \sim E_F \tau$  is the dimensionless conductivity). On the technical level, the qualitative difference with the tunneling DOS can be traced back to the contribution of the Hikami box diagrams,  $\Sigma_{ij}^b$ .

As has been already mentioned in Sec. I, the problem of the effect of the interaction on magneto-oscillations was recently addressed in Ref. 26. The result of this work for the damping factor is qualitatively similar to ours,  $B \propto T \ln T$ . However, the crossover function and, in particular, the prefactors in both ballistic and diffusive limits differ from ours [1 instead of 3/2 in the first line of Eq. (85), and 3/2 instead of 1 in the second line]. This difference is not surprising, since the authors of Ref. 26 took into account only one diagram  $\Sigma_{01}^a$  out of six in Fig. 2. Thus even the qualitative agreement may be considered as an accidental coincidence. In fact, there is a conceptual difference between our result and that of Ref. 26. To illustrate this, consider a toy interaction of the type  $V(\omega, \mathbf{q}) = F(\omega) \delta(\mathbf{q})$ . Our result then would be zero, since the kernel function  $K(i\omega_m, \mathbf{q})$ , Eq. (28), satisfies the gauge-invariance constraint [see Eq. (31) and discussion below it],

$$K(i\omega_m, \mathbf{q})|_{\mathbf{q}=0} = 0. \quad (87)$$

In contrast, the formula of Ref. 26 would give a finite result, since their kernel

$$K_{01}^{(a)} = \frac{2}{S} \frac{1}{S\tau - 1} \quad (88)$$

does not satisfy the requirement (87),

$$K_{01}^{(a)}(i\omega_m, q=0) = \frac{2}{|\omega_m|(|\omega_m|\tau + 1)}.$$

For the Coulomb interaction this results in a logarithmic divergence at small  $q$  that is cut off by the plasmon pole at  $q \sim q_{\min}$  with

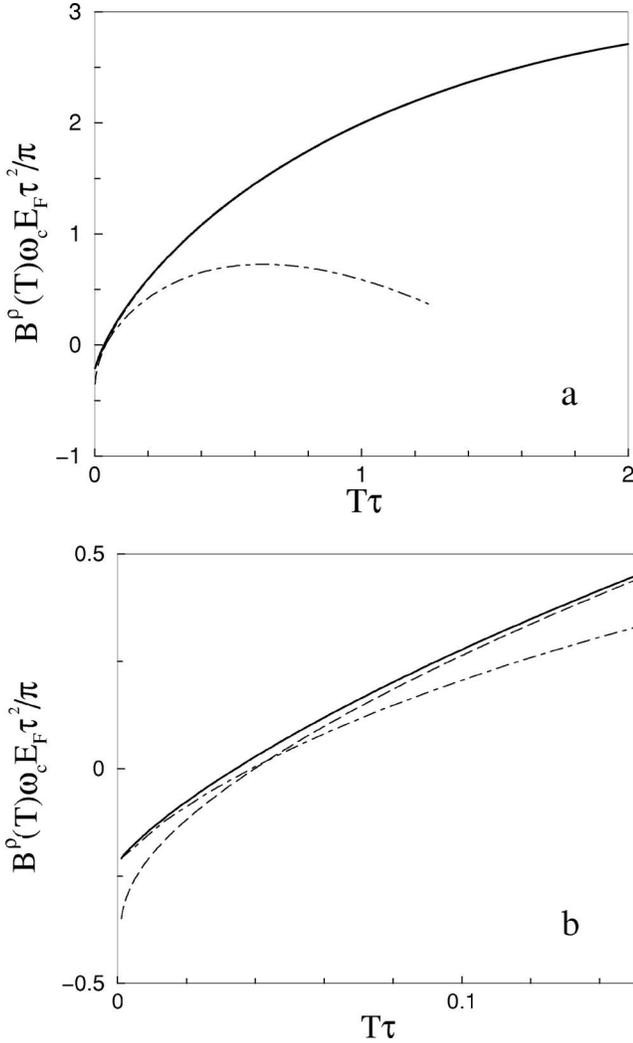


FIG. 7. Temperature dependence of the singlet channel correction to the damping factor  $B^\rho(T)$ , Eq. (83), for  $4\pi\Delta\tau=100$  (solid line) with the low- $T$  (dot-dashed) and high- $T$  (dashed) asymptotics, Eq. (84). (a) Wide temperature range: on this scale  $B^\rho(T)$  is essentially indistinguishable from its high- $T$  asymptotics; and (b) low- $T$  part: the crossover between the two asymptotics occurs at  $T\tau \sim 0.05$ .

$$q_{\min} = \frac{|\omega_m|(|\omega_m| + 1/\tau)}{\kappa v_F^2},$$

similarly to the calculation of the tunneling density of states.<sup>1</sup> As we explained in the end of Sec. II, the contribution of small momenta,  $q < R_c^{-1}$ , should be suppressed for the present problem. Therefore the above small- $q$  divergence should be cut off by the magnetic field, which would partly transform  $\ln T$  of Ref. 26 into  $\ln B$ . However, this problem is in fact spurious: the result of our work does not suffer from any infrared divergencies, since our kernel  $K(i\omega_m, \mathbf{q})$  does satisfy Eq. (87).

Finally, let us briefly comment on the ultradiffusive regime,  $T \ll \omega_c \ll 1/\tau$ . In this regime the summation over Matsubara energies  $\varepsilon_n$  is not restricted to  $n=0$  and  $N_c \sim \omega_c/T \gg 1$  Matsubara harmonics are important. Therefore the

damping due to the inelastic scattering [suppressed only for  $n=0$ , see Ref. 26 and Eq. (B13)] becomes finite. The corresponding contribution to the damping can be roughly estimated using Eq. (B13) taken at relevant  $n \sim N_c$ :

$$\delta B^{\text{inel}}(T) \sim -n^2 \frac{T^2}{\omega_c E_F} \ln(E_F \tau) \Big|_{n \sim N_c} \sim -\frac{\omega_c}{E_F} \ln(E_F \tau), \quad (89)$$

yielding  $|\delta B^{\text{inel}}(T)| \ll 1$ , since in the ultradiffusive regime  $\omega_c \ll 1/\tau$ . Thus the inelastic contribution to the damping factor is always small. Note that the contribution to the damping due to the renormalization of the effective mass in the ultradiffusive regime is also small: at  $n=N_c$  we have  $B(T) = -2\pi\varepsilon_n/\omega_c \delta m/m \sim \delta m/m \ll 1$ .

## VI. CONCLUSIONS

In conclusion, we have studied the  $T$ -dependent interaction corrections to the damping of magneto-oscillations in a two-dimensional electron gas. The damping factor has been calculated for Coulomb and short-range interaction in the whole range of temperatures, from the ballistic to the diffusive regime. While the relevant diagrams are similar to those for the local density of states, the results are essentially different, see Eqs. (42) and (83).

We have identified leading contributions to the damping induced by interplay of interaction and disorder, which can be associated with corrections to the effective mass and the quantum scattering time. It has been shown that in the ballistic regime, which is typically realized in low-disorder samples at realistic temperatures, the dominant effect is that of the renormalization of the effective electron mass due to the interplay of the interaction and impurity scattering. Specifically, the correction to the effective mass is of the form  $\delta m/m \sim 1/(E_F \tau) \ln(E_F/T)$ , Eq. (56). The correction to the impurity scattering time is of the form  $\delta \tau_q/\tau \sim T/E_F$ , Eq. (57), and yields a subleading contribution to the damping. We have confirmed the result for the correction to the quantum scattering time by performing a calculation based on the picture of scattering by impurities dressed by Friedel oscillations. The results of the paper are relevant to the analysis of experiments on magneto-oscillations (in particular, for extracting the value of the effective mass) and are expected to be useful for understanding the physics of a high-mobility 2DEG near the apparent metal-insulator transition.

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### APPENDIX A: LUTTINGER FORMALISM FOR THE THERMODYNAMIC POTENTIAL

In this appendix we derive the oscillatory part of the thermodynamic potential  $\Omega$ , following Ref. 34.

We calculate the sum in Eq. (7), using Poisson's summation formula

$$\sum_{m=0}^{\infty} f(m\omega_c) = \frac{1}{2}f(0) + \int_0^{\infty} \frac{d\xi}{\omega_c} f(\xi) - \sum_{k=1}^{\infty} \frac{1}{2\pi ik} \int_0^{\infty} d\xi f'(\xi) \times (e^{2\pi ik\xi/\omega_c} - e^{-2\pi ik\xi/\omega_c}). \quad (\text{A1})$$

From Eq. (7) we have  $f'(\xi) = -G(i\varepsilon_n, \xi)$ , where  $G(i\varepsilon_n, \xi)$  is the Green's function. Thus to extract the oscillatory contribution to  $\Omega$ , we should calculate the following integral

$$I_{\pm} = \int_0^{\infty} d\xi \frac{e^{\pm 2\pi ik\xi/\omega_c}}{\xi - \mu - i\varepsilon_n + \Sigma_{ee}(i\varepsilon_n, \xi) - i \operatorname{sgn} \varepsilon_n / 2\tau} \times \left[ 1 + \frac{\partial \Sigma_{ee}(i\varepsilon_n, \xi)}{\partial \xi} \right]. \quad (\text{A2})$$

Here we introduce the self-energy  $\Sigma_{ee}(i\varepsilon_n, \xi)$ , which is a function defined in the plane of a complex variable  $\xi$ , such that

$$\Sigma_{ee}(i\varepsilon_n, m\omega_c) = \Sigma_{ee}(i\varepsilon_n, \xi = m\omega_c). \quad (\text{A3})$$

The main contribution comes from the pole  $\xi = \xi_0$ , where  $\xi_0$  obeys the self-consistent equation

$$\xi_0 = \mu + i\varepsilon_n + \frac{i}{2\tau} \operatorname{sgn} \varepsilon_n - \Sigma_{ee}(i\varepsilon_n, \xi_0). \quad (\text{A4})$$

We expand the self-energy (from now on we will skip the subscript "ee" in  $\Sigma_{ee}$ ) in the vicinity of the pole,

$$\Sigma(i\varepsilon_n, \xi) \approx \Sigma(i\varepsilon_n, \xi_0) + (\xi - \xi_0) \left. \frac{\partial \Sigma(i\varepsilon_n, \xi)}{\partial \xi} \right|_{\xi=\xi_0}. \quad (\text{A5})$$

Then the denominator in Eq. (A2) becomes proportional to  $(1 + \partial \Sigma / \partial \xi)$  so that these factors drop out and the integral takes a simple form

$$I_{\pm} = \int_0^{\infty} d\xi \frac{\exp[\pm 2\pi ik\xi/\omega_c]}{\xi - \xi_0}. \quad (\text{A6})$$

We first single out the FL renormalization factors in  $\Sigma$ , i.e., represent the self-energy in the following form (assuming a constant electron concentration):

$$\Sigma(i\varepsilon_n, \xi) \approx \delta\mu + \beta_0(\xi - \tilde{\mu}) - i\alpha_0\varepsilon_n + \delta\Sigma(i\varepsilon_n, \xi), \quad (\text{A7})$$

where

$$\tilde{\mu} = \mu - \delta\mu = \pi n_e / m$$

is the chemical potential for noninteracting electrons ( $n_e$  is the electron concentration):  $\xi - \tilde{\mu} \approx v_F(k - k_F)$ . The correction to the self-energy  $\delta\Sigma(i\varepsilon_n, \xi)$  contains contributions that are smaller than  $\beta_0(\xi - \mu)$  and  $\alpha_0\varepsilon_n$  by either  $T/E_F$  or  $1/E_F\tau$ . These additional contributions are related to the inelastic

processes and to the modification of the pure FL result due to disorder.

In order to solve Eq. (A4), we treat the subleading terms constituting  $\delta\Sigma$  as small corrections. Solving Eq. (A4) by iterations, we first find its solution neglecting these small corrections and expanding  $\Sigma_{00}$  around the mass-shell:

$$\xi_0^{(0)} \approx \tilde{\mu} + i\varepsilon_n + \frac{i}{2\tau} \operatorname{sgn} \varepsilon_n + i\alpha_0\varepsilon_n - \beta_0(\xi_0^{(0)} - \tilde{\mu}), \quad (\text{A8})$$

which yields

$$\xi_0^{(0)} = \frac{\pi n_e}{m} + i\varepsilon_n \frac{1 + \alpha_0}{1 + \beta_0} + \frac{i \operatorname{sgn} \varepsilon_n}{2\tau(1 + \beta_0)}. \quad (\text{A9})$$

Next we use this value of  $\xi_0$  in  $\delta\Sigma$  and solve the self-consistent equation again, now keeping the terms previously neglected. Then we arrive at

$$\xi_0 \approx \xi_0^{(0)} - \delta\Sigma(i\varepsilon_n, \xi_0^{(0)}) / (1 + \beta_0). \quad (\text{A10})$$

Substituting this value of the pole in Eq. (A6), we obtain

$$\Omega = \Omega_{\text{osc}} + \Omega_{\text{Non-osc}}, \quad (\text{A11})$$

where

$$\begin{aligned} \Omega_{\text{Non-osc}} = & -T\nu\omega_c \sum_{n=-\infty}^{\infty} \ln \left( -\mu - i\varepsilon_n + \Sigma(i\varepsilon_n, 0) - \frac{i \operatorname{sgn} \varepsilon_n}{2\tau} \right) \\ & - 2\nu T \sum_{n=-\infty}^{\infty} \int_0^{\infty} d\xi \ln \left( \xi - \mu - i\varepsilon_n + \Sigma(i\varepsilon_n, \xi) - \frac{i \operatorname{sgn} \varepsilon_n}{2\tau} \right) \end{aligned} \quad (\text{A12})$$

and

$$\begin{aligned} \Omega_{\text{osc}} = & \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{T2\nu\omega_c}{k} \exp \left\{ \frac{2\pi k}{\omega_c} \left( i \frac{\pi n_e}{m} \operatorname{sgn} \varepsilon_n - \frac{|\varepsilon_n|(1 + \alpha_0) + 1/2\tau + i\delta\Sigma(i\varepsilon_n, \xi_0)\operatorname{sgn} \varepsilon_n}{1 + \beta_0} \right) \right\} \\ = & \sum_{k=1}^{\infty} \frac{4\nu T\omega_c}{k} \cos \frac{2\pi^2 k n_e}{eB} \sum_{n=0}^{\infty} \exp \left\{ -\frac{2\pi k}{\omega_c} \frac{1 + \alpha_0}{1 + \beta_0} \right. \\ & \left. \times \left[ \varepsilon_n + \frac{1}{2\tau(1 + \alpha_0)} + \frac{i\delta\Sigma(i\varepsilon_n, \xi_0)}{1 + \alpha_0} \right] \right\} \end{aligned} \quad (\text{A13})$$

$$\approx 2\nu \left( \frac{\omega_c}{2\pi} \right)^2 A_1 \cos \frac{2\pi^2 n_e}{eB}. \quad (\text{A14})$$

Here  $A_1$  is the amplitude of the principal harmonics of the oscillations

$$A_1 \equiv \frac{4\pi^2 T}{\omega_c} \sum_{\varepsilon_n > 0} \exp \left( -\frac{2\pi}{\omega_c} \frac{1 + \alpha_0}{1 + \beta_0} \left[ \varepsilon_n + \frac{1}{2\tau(1 + \alpha_0)} + \frac{i\delta\Sigma(i\varepsilon_n, \xi_0)}{1 + \alpha_0} \right] \right). \quad (\text{A15})$$

## APPENDIX B: CALCULATION OF SELF-ENERGIES

In this appendix we calculate the relevant self-energy contributions and derive Eq. (28). The zero- $B$  Green's function is given by

$$\begin{aligned} G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) &= \frac{i}{(\omega_m - \varepsilon_n)(1 + \alpha_0) + \text{sgn}(\omega_m - \varepsilon_n)/2\tau - [i(\xi - \tilde{\mu}) - iv_F q \cos \theta](1 + \beta_0)} \\ &= \frac{i}{\tilde{\omega}_m - \tilde{\varepsilon}_n + \text{sgn}(\omega_m - \varepsilon_n)/2\tau - i(\xi - \tilde{\mu})(1 + \beta_0) + iv_F \tilde{q} \cos \theta}. \end{aligned} \quad (\text{B1})$$

We denote the FL-renormalized energies and momenta as  $\tilde{\omega}_m = (1 + \alpha_0)\omega_m$ ,  $\tilde{\varepsilon}_n = (1 + \alpha_0)\varepsilon_n$ , and  $\tilde{q} = (1 + \beta_0)q$ . As the first approximation, we have set in Eq. (B1)  $p/m = v_F$  in the linear-in- $q$  term  $iv_F q \cos \theta$  and neglected  $q^2/2m$ .

Since the effective interaction [Eqs. (71) and (73)] does not depend on the polar angle of the transferred momentum  $\mathbf{q}$ , we average the FL-dressed Green's function  $G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q})$  over the angle between  $\mathbf{p}$  and  $\mathbf{q}$ . The result of angle-averaging is

$$\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle = \int \frac{d\theta}{2\pi} G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \quad (\text{B2})$$

$$= \frac{i \text{sgn}(\omega_m - \varepsilon_n)}{\sqrt{[\tilde{\omega}_m - \tilde{\varepsilon}_n + \text{sgn}(\omega_m - \varepsilon_n)/2\tau - i(\xi - \tilde{\mu})(1 + \beta_0)]^2 + v_F^2 \tilde{q}^2}}. \quad (\text{B3})$$

We substitute  $\xi = \xi_0 \approx \xi_0^{(0)} = \tilde{\mu} + i\tilde{\varepsilon}_n/(1 + \beta_0) + i \text{sgn} \varepsilon_n/2\tau(1 + \beta_0)$  for  $\xi$  in Eq. (B3) since we are interested in  $\delta\Sigma(i\varepsilon_n, \xi_0)$  (the only place where  $\xi_0$  appears in  $\delta\Sigma$  is the Green's function under the interaction line). Then the denominator in Eq. (B3) for  $q=0$  reads

$$\begin{aligned} &\tilde{\omega}_m - \tilde{\varepsilon}_n + \text{sgn}(\omega_m - \varepsilon_n)/2\tau - i(\xi - \tilde{\mu})(1 + \beta_0) \\ &= \tilde{\omega}_m - \tilde{\varepsilon}_n + \text{sgn}(\omega_m - \varepsilon_n)/2\tau \\ &\quad - i(1 + \beta_0) \left[ i \frac{\tilde{\varepsilon}_n}{1 + \beta_0} + i \frac{\text{sgn} \varepsilon_n}{2\tau(1 + \beta_0)} \right] \\ &= \tilde{\omega}_m + \frac{1}{\tau} \theta(\omega_m - \varepsilon_n) \theta(\varepsilon_n), \end{aligned} \quad (\text{B4})$$

with  $\theta(x)$  the theta function. For definiteness, below we consider  $\varepsilon_n > 0$ .

For  $\Sigma_{01}$  and  $\Sigma_{11}$  at  $\varepsilon_n > 0$  we consider the Green's function at  $\omega_m - \varepsilon_n > 0$  in order to have different signs of Matsubara energies in the Green's functions connected by the interaction vertex. This condition allows us to dress the interaction vertices by impurity ladders.

We see that  $\varepsilon_n$  drops out in the averaged Green's function taken at  $\xi = \xi_0$ , as in two-particle quantities:

$$\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle_{\xi = \xi_0} = \frac{i}{\sqrt{(|\tilde{\omega}_m| + 1/\tau)^2 + v_F^2 \tilde{q}^2}}, \quad \omega_m > \varepsilon_n. \quad (\text{B5})$$

When both  $\varepsilon_n$  and  $\varepsilon_n - \omega_m$  have the same sign (such a contribution appears in the calculation of  $\delta\Sigma_{00}$ ), we find

$$\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle_{\xi = \xi_0} = - \frac{i}{\sqrt{|\tilde{\omega}_m|^2 + v_F^2 \tilde{q}^2}}, \quad \omega_m < \varepsilon_n. \quad (\text{B6})$$

Now we redefine the Fermi velocity to absorb the FL-factors according to

$$v_F^* = v_F \frac{1 + \beta_0}{1 + \alpha_0} = \frac{k_F}{m^*}. \quad (\text{B7})$$

Then we can return from  $\tilde{\omega}_m$  and  $\tilde{q}$  to  $\omega_m$  and  $q$ , expressing the angle-averaged Green's function in terms of FL-renormalized parameters  $Z$ ,  $\tau^*$  [introduced in Eq. (15)] and  $v_F^*$ :

$$\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle_{\xi = \xi_0} = \frac{iZ}{\sqrt{(|\omega_m| + 1/\tau^*)^2 + (v_F^* q)^2}}, \quad \omega_m > \varepsilon_n,$$

$$\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle_{\xi = \xi_0} = - \frac{iZ}{\sqrt{|\omega_m|^2 + (v_F^* q)^2}}, \quad \omega_m < \varepsilon_n.$$

Furthermore, the  $Z$ -factor will be cancelled in the final result, when  $\langle G(i\varepsilon_n - i\omega_m, \mathbf{p} - \mathbf{q}) \rangle$  is used to calculate the correction to the observables, see, e.g., Ref. 39 and discussion in Sec. II B.

Using Eqs. (B5) and (B6), we obtain

$$\begin{aligned}\Sigma_{01}^a(i\varepsilon_n, \xi_0) &= -T \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) \Gamma(i\omega_m, \mathbf{q}) \\ &\quad \times \langle G(i\varepsilon_n, \mathbf{p} - \mathbf{q}) \rangle_{\xi_p = \xi_0} \\ &= -iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q}) \Gamma(i\omega_m, \mathbf{q})}{S(i\omega_m, \mathbf{q})},\end{aligned}\quad (\text{B8})$$

$$\begin{aligned}\Sigma_{11}^a(i\varepsilon_n, \xi_0) &= -T \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) \Gamma^2(i\omega_m, \mathbf{q}) \\ &\quad \times \langle G(i\varepsilon_n, \mathbf{p} - \mathbf{q}) \rangle_{\xi_p = \xi_0} \\ &= -iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q}) \Gamma^2(i\omega_m, \mathbf{q})}{S(i\omega_m, \mathbf{q})},\end{aligned}\quad (\text{B9})$$

where for brevity we introduce new variables  $S(i\omega_m, \mathbf{q}) \equiv \sqrt{(|\omega_m| + 1/\tau)^2 + v_F^2 q^2} = \sqrt{W^2 + v_F^2 q^2}$  and  $W \equiv |\omega_m| + 1/\tau$ .

Now we consider the contribution to the self-energy without vertex corrections,  $\Sigma_{00}^a(i\varepsilon_n, \xi)$ . We recall that in  $\Sigma_{00}^a(i\varepsilon_n, \xi)$  the summation over transferred frequencies is not restricted to  $\omega_m > \varepsilon_n$ . Presenting  $\Sigma_{00}^a(i\varepsilon_n, \xi)$  as

$$\Sigma_{00}^a(i\varepsilon_n, \xi_0) = \Sigma_{00}^{a,+}(i\varepsilon_n, \xi_0) + \Sigma_{00}^{a,-}(i\varepsilon_n, \xi_0), \quad (\text{B10})$$

$$\begin{aligned}\Sigma_{00}^{a,+}(i\varepsilon_n, \xi_0) &= -T \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) \langle G(i\varepsilon_n, \mathbf{p} - \mathbf{q}) \rangle_{\xi_p = \xi_0} \\ &= -iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q})}{S(i\omega_m, \mathbf{q})},\end{aligned}\quad (\text{B11})$$

we further split the contribution  $\Sigma_{00}^{a,+}(i\varepsilon_n, \xi_0)$  corresponding to no change of Matsubara frequencies at the interaction vertices into two parts as follows:

$$\begin{aligned}\Sigma_{00}^{a,+}(i\varepsilon_n, \xi_0) &= -T \sum_{\omega_m < \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) \\ &\quad \times \langle G(i\varepsilon_n, \mathbf{p} - \mathbf{q}) \rangle_{\xi_p = \xi_0} \\ &= iT \int \frac{d^2q}{(2\pi)^2} \left\{ \sum_{|\omega_m| < \varepsilon_n} + \sum_{\omega_m < -\varepsilon_n} \right\} \frac{V(i\omega_m, \mathbf{q})}{\sqrt{|\omega_m|^2 + q^2 v_F^2}}.\end{aligned}\quad (\text{B12})$$

The terms with  $|\omega_m| < \varepsilon_n$  are responsible for the FL-renormalization and for the inelastic (determined by real processes) contribution to the self-energy, yielding the following FL-type term:<sup>26</sup>

$$\Sigma^{\text{FL}}(i\varepsilon_n, \xi_0) = -i\alpha_0 \varepsilon_n - i\gamma(i\varepsilon_n, T) \frac{\varepsilon_n^2 - \pi^2 T^2}{E_F}, \quad (\text{B13})$$

where the function  $\gamma(i\varepsilon_n, T)$  depends logarithmically on  $\max[\varepsilon_n, T, 1/\tau]$ : in particular,  $\gamma(i\varepsilon_n, T) \propto \ln[E_F/(-i\varepsilon_n)]$  for  $\varepsilon_n \gg T, 1/\tau$  and  $\gamma(i\varepsilon_n, T) \propto \ln[iE_F T]$  for  $1/\tau \gg T, \varepsilon_n$ . The first

term in Eq. (B13) determines the FL Z-factor and has been separated from  $\delta\Sigma$  which governs the correction to the damping factor, see Eq. (A7). As for the second term, its imaginary part describes the inelastic electron-electron scattering, while its real part contributes to the renormalization of the effective mass.<sup>32,33,45</sup> However, when taken at  $\varepsilon_0 = \pi T$ , as appropriate for the damping of the magneto-oscillations at  $T \gg \omega_c$ , the second term in Eq. (B13) vanishes, in agreement with Ref. 26. Note that for the case of weak short-range interaction,  $V(i\omega_m, \mathbf{q}) = \text{const}(\omega_m)$ , the inelastic contribution is zero to the first order in  $V$ .

Thus in order to calculate  $\delta\Sigma$ , we shall retain in Eq. (B12) only the term corresponding to the summation over  $\omega_m < -\varepsilon_n$ . In this term we change the sign of  $\omega_m$  and (suppressing the irrelevant inelastic term) obtain

$$\Sigma_{00}^{a,+}(i\varepsilon_n, \xi_0) = iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q})}{\sqrt{|\omega_m|^2 + q^2 v_F^2}}, \quad (\text{B14})$$

thus arriving at

$$\begin{aligned}\Sigma_{00}^a(i\varepsilon_n, \xi_0) &= -iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} V(i\omega_m, \mathbf{q}) \\ &\quad \times \left[ \frac{1}{S(i\omega_m, \mathbf{q})} - \frac{1}{S_0(i\omega_m, \mathbf{q})} \right].\end{aligned}\quad (\text{B15})$$

Here we use the fact that  $V(i\omega_m, \mathbf{q}) = V(-i\omega_m, \mathbf{q})$  as the dynamically screened interactions depends on  $\omega_m$  only through  $|\omega_m|$  and introduced  $S_0(i\omega_m, \mathbf{q}) = \sqrt{\omega_m^2 + q^2 v_F^2}$ .

Let us turn now to the Hikami-box diagrams, shown in Fig. 2(b). We remind the reader that these diagrams are generated by covering each contribution to the self-energy  $\Sigma_{ij}$  from Fig. 2(a) with  $\varepsilon_n(\varepsilon_n - \omega_m) < 0$  by a single impurity line. Therefore for white-noise disorder (addressed in this paper) the Hikami-box contribution to the self-energy is independent of  $\xi_p$  and can be expressed through the corresponding  $\Sigma_{ij}^a$  as

$$\Sigma_{ij}^b(i\varepsilon_n) = \frac{1}{2\pi\nu\tau} \int d\xi_p \nu [G(i\varepsilon_n, \xi_p)]^2 \Sigma_{ij}^a(i\varepsilon_n, \xi_p) \quad (\text{B16})$$

$$= \frac{i}{\tau} \frac{\partial}{\partial \xi_p} \Sigma_{ij}^a(i\varepsilon_n, \xi_p) \Big|_{\xi_p = \xi_0}, \quad (\text{B17})$$

$$\Sigma_{ij}^b(i\varepsilon_n) = \frac{i}{\tau} \frac{\partial}{\partial \xi_p} \Sigma_{ij}^a(i\varepsilon_n, \xi_p) \Big|_{\xi_p = \xi_0} \quad (\text{B18})$$

since the pole of  $G$  is given by  $\xi_0$ , see Eq. (A10). Differentiating Eq. (B3) we get

$$\begin{aligned}\left\langle \frac{\partial}{\partial \xi_p} G(i\varepsilon_n - i\omega_n, \mathbf{p} - \mathbf{q}) \right\rangle \Big|_{\xi_p = \xi_0} \\ = - \frac{|\omega_m| + 1/\tau}{\{(|\omega_m| + 1/\tau)^2 + v_F^2 q^2\}^{3/2}} = - \frac{W}{S^3}.\end{aligned}\quad (\text{B19})$$

This yields

$$\Sigma_{00}^b(i\varepsilon_n, \xi_0) = iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q})W}{\tau S^3(i\omega_m, \mathbf{q})}, \quad (\text{B20})$$

$$\Sigma_{01}^b(i\varepsilon_n, \xi_0) = iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q})\Gamma(i\omega_m, \mathbf{q})W}{\tau S^3(i\omega_m, \mathbf{q})}, \quad (\text{B21})$$

$$\Sigma_{11}^b(i\varepsilon_n, \xi_0) = iT \sum_{\omega_m > \varepsilon_n} \int \frac{d^2q}{(2\pi)^2} \frac{V(i\omega_m, \mathbf{q})\Gamma^2(i\omega_m, \mathbf{q})W}{\tau S^3(i\omega_m, \mathbf{q})}. \quad (\text{B22})$$

Combining all the contributions together, we arrive at Eqs. (27) and (28).

### APPENDIX C: COULOMB INTERACTION: TRIPLET CHANNEL

In this appendix we calculate the contribution of the triplet channel to the damping of magneto-oscillations. The effective interaction in the triplet channel can be found by replacing  $V_0(q) \rightarrow F_0^\sigma/2\nu$  in the expression for the singlet channel:

$$2\nu V^\sigma(i\omega_m, q) = \frac{F_0^\sigma}{1 + F_0^\sigma \Pi(i\omega_m, q)} = \frac{F_0^\sigma}{1 + F_0^\sigma [1 - |\omega_m|/(S - 1/\tau)]} \\ = \frac{F_0^\sigma (S - 1/\tau)}{(1 + F_0^\sigma)(S - 1/\tau) - |\omega_m|F_0^\sigma} = \frac{F_0^\sigma}{1 + F_0^\sigma} \frac{S - 1/\tau}{S - w}, \quad (\text{C1})$$

where we introduced

$$w \equiv |\omega_m| \frac{F_0^\sigma}{1 + F_0^\sigma} + \frac{1}{\tau}. \quad (\text{C2})$$

This yields

$$2\nu V^\sigma(i\omega_m, q)K(i\omega_m, q) \\ = \frac{F_0^\sigma}{1 + F_0^\sigma} \left[ \frac{S_0 - (S - 1/\tau)}{S_0(S - w)} + \frac{S - W}{S - w} \frac{1}{S\tau(S - 1/\tau)} \right]. \quad (\text{C3})$$

Performing the integration over  $q$  in Eq. (27) and taking into account the three triplet terms corresponding to different projection of the total spin on the  $z$  axis  $S_z = 0, \pm 1$ , we obtain

$$\delta\Sigma^\sigma(i\pi T, \xi_0) = -\frac{iT}{2E_F\tau} \frac{3F_0^\sigma}{1 + F_0^\sigma} \sum_{\omega_m = 2\pi T}^\Delta \left\{ \left( \frac{\omega_m}{\sqrt{\Delta^2 + \omega_m^2}} - 1 \right) \right. \\ \left. + \ln \frac{\Delta^2 + \omega_m^2}{\omega_m^2} + h(\omega_m\tau, F_0^\sigma) \right\}, \quad (\text{C4})$$

$$h(z, y) \simeq \left( 1 - \frac{1}{y} \right) \ln[1 + y] + \frac{y}{1 + y} z h_1(x, y), \quad (\text{C5})$$

$$h_1(z, y) = \ln \frac{(1 + 2z)(1 + y)}{2z} + \frac{zy + (1 + y)}{\sqrt{|2(1 + y)z - (zy)^2|}} \begin{cases} \left[ \arcsin \frac{zy - 1}{\sqrt{2z + 1}} - \arcsin \frac{zy + (1 + y)}{(1 + y)\sqrt{2z + 1}} \right], & z < \frac{2(1 + y)}{y^2}, \\ \left[ \ln \left( 1 + \frac{2 + y}{zy - 1 + \sqrt{(zy)^2 - 2(1 + y)z}} \right) - \ln(1 + y) \right], & z > \frac{2(1 + y)}{y^2}. \end{cases} \quad (\text{C6})$$

We see that the first two terms in Eq. (C4) correspond to the pointlike interaction with  $\nu U_0 \rightarrow 3F_0^\sigma/(1 + F_0^\sigma)$ , see Eq. (39). The term  $h(\omega_m, F_0^\sigma)$  corresponds to the crossover function  $f(x)$  in the singlet channel, see Eqs. (81) and (82). The result for the singlet channel is reproduced in the limit  $F_0^\sigma \rightarrow \infty$  (cf. Ref. 9).

The summation over Matsubara frequencies leads to

$$\delta\Sigma^\sigma(i\pi T, \xi_0) = -\frac{iT}{2E_F\tau} \frac{3F_0^\sigma}{1 + F_0^\sigma} \left\{ \frac{\text{const } \Delta}{T} - \left[ 1 - \frac{\lambda(F_0^\sigma)}{8\pi T\tau} \right] \ln \frac{\Delta}{T} \right. \\ \left. - f_\sigma(4\pi T\tau, F_0^\sigma) \right\}, \quad (\text{C7})$$

where

$$\lambda(y) = \frac{1 + y}{y^3} [y(6 + y) - 2(3 + 2y)\ln(1 + y)] \quad (\text{C8})$$

and

$$f_\sigma(x, y) = \sum_{m=1}^{\infty} \left\{ -h\left(\frac{mx}{2}, y\right) + \left[ 1 - \frac{2\ln(1 + y)}{y} \right] + \frac{\lambda(y)}{2mx} \right\}. \quad (\text{C9})$$

As a result we obtain the following  $T$  dependence of the triplet contribution to the damping exponent

$$B^\sigma(T) = \frac{\pi}{\omega_c \tau} \frac{3F_0^\sigma}{1 + F_0^\sigma} \frac{T}{E_F} \left\{ \left[ 1 - \frac{\lambda(F_0^\sigma)}{8\pi T \tau} \right] \ln \frac{\Delta}{T} + f_\sigma(4\pi T \tau, F_0^\sigma) \right\}. \quad (\text{C10})$$

For not too strong interaction,  $(F_0^\sigma)^2/(1+F_0^\sigma) \lesssim 1$  (i.e., for

$|F_0^\sigma| \lesssim 0.6$ , which is typically met in experiments, see, e.g., Ref. 48) the crossover function  $f_\sigma(x, y)$  only yields the sub-leading  $T$ -dependence<sup>49</sup> of  $B^\sigma(T)$ , so that the leading contribution to the damping is given by Eq. (42) for the short-range interaction with  $\nu U_0$  replaced by  $3F_0^\sigma/(1+F_0^\sigma)$ .

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