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Exactness of modern quantum-mechanical effects

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The arguments for the exactness of the quantization effects of modern physics are discussed. Some comments are made on past and possible experiments which combine these quantum effects.

I. QUANTUM-MECHANICAL EFFECTS FROM THE SINGLE-VALUEDNESS OF WAVE FUNCTIONS

The many "coherent" quantum-mechanical effects which have been observed invariably have led physicists to ask if the quantization conditions are exact or only approximate.

(i) Quantized flux. The flux quantum $(hc/2e)$ trapped in a superconducting ring was first predicted by London[1] and Onsager[2] without[3] the factor 2. It was later observed by Deaver and Fairbank[4] and Doll and Näbauer.[5] Understanding of its exactness was based on the realizations that the factor 2 is due to the pairing of electrons[3,6,7] in a superconductor, the superconductor allowing the effect itself to come from the continuity (here single-valuedness) of wave functions.[1,2,6,7] As will come up again in this Brief Report, there is a relationship between this (flux) quantization and the properties of the gauge group involved.[8]

(ii) Aharonov-Bohm effect. After the discovery of flux quantization, it was immediately emphasized[7,9] that it is related to the (then unconfirmed) predictions of the (Ehrenberg-Siday[10]) Aharonov-Bohm (AB) effect,[11] except for the factor 2. Here the factor 2 disappears because one is only dealing with single electrons and the shift of their interference fringes.

Amusingly, the Aharonov-Bohm effect has been subject to much controversy on two (sometimes not uncoupled) fronts. The first question is whether the effect actually exists, even though the theoretical arguments for its existence are as fundamental as our interpretations of quantum mechanics.[12] Ever since the first reported observations of Chambers[13] were questioned on grounds of possible flux leakage,[14] later experiments have also invariably been questioned. There have been a number of experiments, and I list some of the most pertinent.[15–18]

Recently there has been a renewed criticism of the existence of the AB effect both on grounds of flux leakage and also on grounds of theoretical misinterpretations.[19,20] These claims have been vehemently contested.[21–25]

Independent of whether or not the effect has been observed in the past, it certainly is fair to say that the weight of opinion is that the effect exists, especially given its relation to quantized flux. Be this as it may, the recent electron holography experiments of Tonomura *et al.*[26] will hopefully overcome experimental objections and finally put to rest the question of whether or not the AB effect exists and has been observed. This is especially true since the extremely detailed and perceptive analysis[27] of Greenberger and Overhauser show that the controversial early Marton experiment was consistent both with the existence of the AB effect and also with the gravity experiments we shall come to later.

The more penetrating theoretical question is whether one needs the vector potential to describe the AB effect. In their original papers,[11] AB came to the conclusion that the effect is a true physical manifestation of the vector potential in quantum mechanics. (The only other commonly discussed possibility of a physical manifestation of the vector potential is if, in electrodynamics, the photon has a rest mass.[28]) This conclusion has been challenged from the beginning, not by doubters of the effect's existence, but by doubters of the necessity of using the vector potential to describe it.[29,30] I give here a representative list of papers arguing against the necessity of using the vector potential.[31–34]

Perhaps the best way to look at it is from the discussion of Wu and Yang,[35] who point out that, in gauge theories, a discussion of the AB effect is possible with the standard electromagnetic fields plus the nonintegrable phase factor P:

$$P=\exp(i\phi) \ , \tag{1.1}$$

$$\phi=\frac{e}{\hbar c}\oint A_\mu dx^\mu \ . \tag{1.2}$$

They emphasize that it is only the phase factor P that is required, *not* the phase ϕ. This is crucial, since if only the phase factor P is "physically meaningful," Strocchi and Wightman[31] would argue that P can be written in terms of electromagnetic fields:

$$\oint \vec{A}\cdot d\vec{l}=\int \vec{B}\cdot d\vec{s} \ . \tag{1.3}$$

In any event, what is needed for the AB effect to exist is a continuous wave function.[29,36] As before, this can be thought of in more general terms as an application of charge superselection rules[31] and of global formulations of gauge fields.[35]

(iii) Quantized vortices. The single-valuedness of wave functions was also the basis of the prediction of quantized vortices in liquid helium by Onsager,[37,38] as elucidated by Feynman.[39] The unit of circulation h/m_{He} was observed both in irrotational flow in multiply connected regions and in vortex rings,[40] and later in arrays.[41] Once again, arguments for exactness on the basis of gauge theories can be applied.[42]

II. DOUBLE-VALUED WAVE FUNCTIONS: ROTATIONS OF 4π

All of the discussions in Sec. I ultimately can be thought of as the prediction of quantum mechanics (in the boson mode) that wave functions are single valued, even when they are multiply connected.[29,36]

In hindsight, one can realize that spinor quantum mechanics requires that it be double-valued (a rotation of

2π multiplies the wave function by a minus sign).[43] It can be demonstrated that a rotation of a solid body (in distinction to a rigid body) by 2π is *not* equivalent to the indentity, but that a rotation by 4π *is* equivalent to the identity. This demonstration has been attributed to Dirac himself,[44] and is formalized in the theory of braids.[44]

Even so, it was somewhat striking when Bernstein[45] and Aharonov and Susskind[46] realized that the mathematical structure of quantum mechanics which allowed double-valued wave functions for fermions could, in principle,[47] have real physical consequences. These observations[45–47] led to interference experiments[48–50] which split a beam of neutrons into two paths, rotated one path magnetically, and recombined the paths of neutrons into an interference pattern. It was verified that a rotation of 2π changed an interference fringe from maximum to minimum, i.e., introduced a relative minus sign. That is, a fermion needs to be rotated by 4π to return to its original phase.

Indeed, this spinor effect has been demonstrated in pseudo-two-level systems in molecular-beam resonance experiments[51] and NMR interferometer experiments.[52] Stoll[53] and co-workers have even demonstrated a transition from 4π to 2π rotational symmetry in a boson system. They started with a pseudo-two-level system in the deuteron and showed the transition from 4π to 2π rotational symmetry as they slowly went to a three-level system. This was done by irradiating the deuterons with two different, continuously adjustable, asynchronous rf fields.

III. AC JOSEPHSON EFFECT

Perhaps the most famous, and certainly so far the most useful, of the quantum-mechanical effects discussed here is the ac Josephson effect. Soon after the successful prediction[54] and experimental verification[55] of the ac-Josephson-effect relation

$$\nu = \frac{2eV}{h} , \tag{3.1}$$

the effect became[56] the best method[57] of determining the fine-structure constant

$$\alpha = e^2/\hbar c . \tag{3.2}$$

In fact, use of it allowed discrepancies in the values of the fundamental constants to be eliminated.[58]

The question arose, then, as to how exact the Josephson relation of Eq. (3.1) is. In particular, are there renormalizations of the electron charge which affect the result?[59] The answer is "No," on grounds of gauge invariance,[60–63] the properties of quantum electrodynamics,[61,62] and single-valuedness of wave functions,[63] as a number of people have determined.[60–63]

IV. QUANTIZED HALL EFFECT

In 1975, Ando, Matsumoto, and Uemura[64] published a study of the Hall conductivity in a low-temperature two-dimensional (x-y plane) electron system experiencing a strong magnetic field perpendicular to it in the z direction. They predicted that in certain substances, when a strong magnetic field breaks up the electron degenerate ground state into separated Landau levels, the current (I) is essentially resistanceless, since the electrons do not have the energy to jump to the next level. This implies that the Hall voltage (V_0) perpendicular to the current is quantized, according to

$$I = n(e^2/h)V_0 . \tag{4.1}$$

It still was a surprise, however, when the effect was experimentally[65] observed to be exact to better than a part in 10^5. It was clear that here was a new method for determining the fine-structure constant. This has now been done[66] to a precision comparable with the ac Josephson effect. (In fact, the ac Josephson and Hall methods of determining α are now in slight disagreement with the quantum electrodynamic value obtained from the anomalous magnetic moment $(g-2)$ of the electron. Therefore, Kinoshita[67] views this subject as a possible way to study the exactness of many-body quantum mechanics from first principles.)

It was in this context that Laughlin[68] gave a simple and elegant explanation, further elucidated by Halperin,[68] of why the quantized Hall effect should be exact.[68–71] Laughlin's geometry was a metallic ribbon loop of length L with the current going around the loop, the Hall voltage across the loop, and the magnetic field through it.

Laughlin pointed out that in this model the current is proportional to the adiabatic derivative of the total electronic energy (U) of the system around the loop with respect to the flux through the loop, which, in turn, is related to the uniform vector potential around the loop of circumference L,

$$I = c\frac{\partial U}{\partial \Phi} = \frac{c}{L}\frac{\partial U}{\partial A} . \tag{4.2}$$

The effect of A is to multiply the wave function by a gauge transformation

$$\Lambda = \exp(ieAr\theta/\hbar c) . \tag{4.3}$$

However, if the states are extended, as is necessary in the coherent quantized Hall effect, then this transformation must have the properties $(2\pi r = L)$

$$A = n\frac{hc}{eL} . \tag{4.4}$$

To make an aside, we already see that the quantized Hall effect has the same mathematical basis as the other effects we have discussed. There is single-valuedness of the wave function, the properties of gauge invariance, and, since it needs a magnetic field, a direct relation to quantized flux.

Laughlin observed that phase coherence allows a shift in the vector potential to change the total energy by making the filled states go to one side of the ribbon. He then showed that the shift in energy caused by a shift in the vector potential is linear in A, and, in particular, is

$$\Delta E = -\frac{\Delta A}{H_0}(eE_0) . \tag{4.5}$$

Because of the gauge condition of Eq. (4.4) this means that the transfer of n electrons from one side to the other gives a current of

$$I = c\frac{neV_0}{\Delta\Phi} = n\left(\frac{e^2}{h}\right)V_0 . \tag{4.6}$$

It has recently been shown[72,73] that the above derivation goes through analogously with the relativistic Dirac problem of crossed constant electric and magnetic fields. This is because the exact Dirac solution to the relativistic problem has energy eigenvalues which are proportional to the momentum,[74] just as in the nonrelativistic case.

We also note that the original derivation of the quantized Hall effect used a bulk analysis.[64,71] Recently Ramal, Toulouse, Jaekel, and Halperin[75] have shown how this result can be connected with that of the Laughlin-Halperin[68] approach. The bulk formula can be explained in terms of a special gauge-invariant property of the edge states. Finally, there are new, exciting results on the fractionally quantized Hall effect.[76]

V. EXPERIMENTAL COMMENTS

Experiments combining two of the effects we have discussed, or one of them with another physical phenomenon, have now been done a number of times. The most well known is the superconducting quantum interference device (SQUID), which combines two (ac) Josephson-(effect) junctions with quantized flux.[77] These have developed into very useful measuring devices. In fact, since quantized flux and the AB effect are related,[7,9] after the original measurements[77] the same group[78] slightly modified their experiment to study the Aharonov-Bohm effect in the experiment we have already referred to.[17] We recommend the review by Mercereau[79] on all these phenomena.

One of the neutron interferometers that was used to verify the rotation of 4π was also used to verify the quantum-mechanical phase shift of a neutron when it changes its gravitational potential[80] and rotates[81] with the Earth.[82,83] These experiments have led to a number of discussions[82-85] on coupling quantum-mechanical interference (devices) with gravity.

Other experiments are also possible. Aharonov and Vardi[86] have proposed combining the interference of a split beam of polarized electrons with an AB solenoid in the middle.

We would point out that a rotation experiment combining the rotation of 4π with the quantized flux and/or AB effect has not yet been attempted. However, as this article was completed, Silverman[87] proposed such an experiment as a test against half-integer eigenvalues for angular momentum.

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