Low-density finite-temperature apparent insulating phase in two-dimensional semiconductor systems

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We propose that the observed low-density ''insulating'' phase of a two-dimensional (2D) semiconductor system, with the carrier density being just below ($n<n_c$) the so-called critical density where the derivative of resistivity changes sign at low temperatures [i.e., resistivity $\rho(T)$ increases with increasing T for $n>n_c$ whereas it decreases with increasing T for $n<n_c$], is in fact a high-temperature crossover version of the same effective metallic phase seen at higher densities ($n>n_c$). This low density ($n<n_c$) finite-temperature crossover 2D effective insulating phase is characterized by $\rho(T)$ with power-law temperature dependence in contrast to the truly insulating state (occurring at still lower densities) whose resistivity increases exponentially with decreasing temperature.

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The so-called two-dimensional (2D) metallic phase, identified[1] in *n*-Si metal-oxide-semiconductor field-effect transistor (MOSFET's) by Kravchenko and collaborators, is still a subject of considerable interest and controversy.[2] Although the effectively metallic character (albeit at finite temperatures) of this phase is well established experimentally in many 2D systems, it is still not known whether a true $T=0$ 2D metal exists. The nature of the associated 2D metal-insulator transition (2D MIT) as a function of carrier density (n) is also a mystery (particularly since a noninteracting disordered 2D system at $T=0$ is known on firm theoretical grounds not to undergo any MIT as all states in two dimensions are localized in the presence of finite disorder[3]).

There is increasing evidence that the putative 2D ''metallic'' phase, while being a perfectly good ''effective'' metal down to $T=50$–100 mK where experiments have so far been carried out, is essentially a finite-temperature effective metal with screening[4] and related electron-electron interaction effects[5] playing important quantitative roles in determining the temperature-dependent resistivity (and producing the effective metallic behavior). It should also be emphasized that no reasonable model or theory for a $T=0$ strongly correlated 2D metal (stabilized presumably by electron-electron interaction in the presence of disorder) has been put forward in the literature.[2] (Such a true 2D metal, if it exists at $T=0$, cannot be a Fermi liquid since the corresponding noninteracting 2D system is an insulator at $T=0$.[3]) The observed 2D metallic phase shows strong temperature dependence ($d\rho/dT>0$) which is nominally unexpected in a metal at low temperatures where phonons freeze out in the low-temperature Bloch-Grüneisen regime. We have argued, on the basis of concrete calculations, that the strong temperature dependence in the resistivity in the effective metallic phase is caused by a temperature-dependent effective disorder arising from the strong temperature dependence of low-density 2D screening.[4] This viewpoint of the metallic temperature dependence has received further support from recent theoretical calculations of interaction corrections to 2D resistivity generalizing the screening theory.[5] Unlike 3D metals, where $q_{TF}/2k_F \sim 1$ and $T/T_F \ll 1$ (where q_{TF}, k_F, T_F are the Thomas-Fermi screening wave vector, the Fermi wave vector, and the Fermi temperature, respectively), the semiconductor-based 2D systems at low carrier densities could typically have $q_{TF}/2k_F \gg 1$ and $T/T_F \sim 1$, engendering strong screening-induced temperature dependence in the resistivity arising from the strong temperature dependence in the screened impurity disorder scattering in contrast to 3D metals where any temperature dependence of disorder scattering is exponentially suppressed at low temperatures.

In this paper we extend our screening calculations to very low densities (below n_c, the apparent critical density for the MIT), where the experimental resistivity increases with decreasing temperature ($d\rho/dT<0$) indicating the existence of a nominally insulating phase. Quite surprisingly we predict the existence of an effective ''insulating'' phase defined by $d\rho/dT<0$ at intermediate T and *low n*, arising from the same temperature-dependent disorder effect which produces the effective metallic phase ($d\rho/dT>0$) at *low T* and *high n*. We find that for a range of densities below n_c our calculated resistivity $\rho(T,n)$, as a function of temperature (T) and density (n), exhibits striking qualitative similarity to the experimentally observed $\rho(T, n \leqslant n_c)$,[6–10] where the resistivity increases with decreasing T (i.e., $d\rho/dT<0$) in a rather slowly rising approximate power-law manner in contrast to the thermal exponential (either Mott variable range hopping or activated) behavior expected of a true $T=0$ insulator. The strongly insulating phase (which is beyond the scope of our theory) may occur at still lower densities where $\rho(T)$ would rise exponentially with decreasing temperature. Our theory applies to all 2D systems where the 2D MIT has been reported, and most significantly, in some systems [such as high mobility (low disorder) 2D *n*- and *p*-GaAs] the observed 2D insulating phase seems to be entirely the interesting low-density effective insulating phase identified in this paper as the exponential insulating behavior seems not to manifest itself in high quality GaAs systems except perhaps at the lowest densities and temperatures. An important salient feature of this low-density novel phase (with $d\rho/dT<0$) is that it is purely a temperature-induced crossover behavior, and as such the critical density n_c at which the 2D system crosses

over to this phase (as n is decreased from above n_c to below) is completely nonuniversal, being a function of temperature itself, i.e., $n_c \equiv n_c(T)$, with the effective critical density n_c progressively decreasing with decreasing carrier temperature. The experimentally determined ''critical'' density is then $n_c(T=T_{\min})$ where $T_{\min}(\approx 50-300$ mK depending on the experiment) is the effective minimum temperature to which the 2D carrier system (and *not* the background bath) can be cooled down (typically $T_{\min} \gg T_{\text{bath}}$ in 2D semiconductor systems for low values of T_{bath}). Such a nonuniversal behavior for the critical density defining 2D MIT, namely that $n_c \equiv n_c(T)$ with the observed effective n_c decreasing with decreasing temperature, has been pointed out experimentally.[9–11] In particular, our theory specifically rules out a sharp (temperature- independent) separatrix delineating the effective metallic ($d\rho/dT>0$) and the effective insulating ($d\rho/dT<0$) phase in the 2D MIT phenomenon. We emphasize that the experimental evidence for a sharp separatrix in the 2D MIT is extremely sparse, and most 2D MIT experiments observe a temperature-dependent crossover density $n_c(T)$ separating $d\rho/dT>0$ for $n>n_c$ and $d\rho/dT<0$ for $n<n_c$ in agreement with our theory. Recently, Punnoose and Finkel'stein[12] have shown that within the renormalization-group analysis the interplay of interaction and disorder may give rise to the crossover behavior near the critical regime in the two valley Si-MOSFET systems. But this model is irrelevant to the GaAs material which is a single-valley system.

We refer to this interesting (low n/intermediate$T$) crossover ''insulating'' ($d\rho/dT<0$) phase as an ''apparent'' insulating phase in order to emphasize the fact that this phase is *not* a true insulating phase at $T=0$, but is rather a low-density (and ''high''-temperature) metallic phase where $\rho(T)$ decreases with increasing temperature in a power-law fashion. This effective insulating phase (i.e., $d\rho/dT<0$) is only an apparent phase since lowering temperature further (at a fixed $n<n_c$) will eventually cause a temperature-induced reentrance into the effective metallic phase at sufficiently low temperatures which, however, may not be accessible experimentally making the apparent insulating phase to appear to be a true insulating phase due to this low-T cutoff. We emphasize that the hallmark of our proposed apparent insulating phase is the approximate power-law temperature dependence of the resistivity in contrast to a true insulating phase where $\rho(T)$ diverges exponentially with decreasing temperature. All localization effects, either weak or strong, are neglected in our theory. This is entirely consistent with our motivation of trying to understand carrier transport in a ''high-temperature phase'' where quantum interference effects should be small. The Boltzmann theory, described below, should be well valid for this effective high-temperature semiclassical phase independent of the actual value of the resistivity.

We calculate the resistivity of a 2D carrier system scattering off random charged impurity centers with a screened Coulomb impurity scattering potential. Within the Drude-Boltzmann semiclassical transport theory with the carrier-impurity interaction treated in the standard ensemble averaged Born approximation, we can express the 2D resistivity ρ as[13]

$$\rho^{-1} = ne^2\langle\tau\rangle/m, \tag{1}$$

where m is the carrier effective-mass assuming a parabolic band effective mass approximation for the semiconductor band structure with n being the 2D carrier density. The thermally averaged scattering time $\langle\tau\rangle$ can be written, within the Drude-Boltzmann semiclassical theory (and the standard relaxation time approximation), as

$$\langle\tau\rangle \equiv \langle\tau\rangle_E = \frac{\int dE\,\tau(E)E\left(-\frac{\partial f}{\partial E}\right)}{\int dE E\left(-\frac{\partial f}{\partial E}\right)}, \tag{2}$$

where $\langle\cdots\rangle_E$ indicates an energy averaging over the carrier thermal distribution function $f(E)=\{1+\exp[\beta(E-\mu)]\}^{-1}$ with $\mu \equiv \mu(T,n)$ as the finite-temperature chemical potential ($\beta = 1/k_B T$) given by $\mu = (1/\beta)\ln[-1+\exp(\beta E_F)]$ for a 2D system (note that $\mu = E_F$ at $T=0$, but μ at finite T could be substantially different and is negative for $T>1.4T_F$). The central quantity of interest is therefore the energy-dependent transport relaxation time (i.e., the scattering time) $\tau(E)$, which in the Born approximation is given by

$$\tau^{-1}(E)|_{E=\epsilon_{\mathbf{k}}} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}'}\sum_{\alpha}\int_{-\infty}^{\infty} dz N_i^{(\alpha)}(z)|u_{\mathbf{k}-\mathbf{k}'}^{(\alpha)}(z)|^2 \times(1-\cos\theta_{\mathbf{k}\mathbf{k}'})\,\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'}), \tag{3}$$

where $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ is the effective-mass band energy of the 2D carriers with $k \equiv |\mathbf{k}|$ as the 2D wave vector, and N_i is the three-dimensional volume density of the random charged impurity centers of the αth kind (we allow, in general, for different kinds of quenched disorder in the problem arising from the distributions of different types of random charged impurity centers in the 2D system) — we take z direction to be the confinement direction perpendicular to the relevant 2D plane of confinement of the carrier system. In Eq. (3) $u_q^{(\alpha)}(z)$ is the 2D (in the x-y plane) Fourier transform of the 3D screened electron-impurity interaction in the system defined to be

$$u_q^{(\alpha)}(z_i) = \frac{2\pi Z^{(\alpha)} e^2 F(q,z_i)}{\bar{\kappa} q\,\epsilon(q)}, \tag{4}$$

where the effective dielectric function $\epsilon(q)$ is given by

$$\epsilon(q) = 1 - \frac{2\pi e^2}{\bar{\kappa} q} f(q)\Pi(q), \tag{5}$$

where $\Pi(q)$ is the finite temperature static 2D noninteracting polarizability function and $\bar{\kappa}$ as the background static lattice dielectric constant. (We have carried out calculations also by including several different local field corrections in the 2D polarizability, but our qualitative results remain unaffected and therefore we will only present here random-phase ap-

proximation (RPA) screening results where Π is the 2D noninteracting static polarizability.) In Eqs. (4) and (5) $F(q,z_i)$ and $f(q)$ are, respectively, the 2D subband form factors for electron-impurity and electron-electron interactions, associated with the finite width of carrier wave functions in the z direction. These form factors are essential for quantitative accuracy, most particularly at the very low carrier densities of our interest where the quasi-2D electron layer spreads considerably in the transverse direction making the strict 2D approximation highly unreliable. We calculate the form factors by using the Fang-Howard-Stern variational wave functions for the quasi-2D layer.[13] For more details on the theory see Refs. 13 and 14.

We have carried out extensive calculations of dc transport, based on the above-described Boltzmann theory, in several different 2D systems of current interest (i.e., systems where an apparent MIT, as reflected in the change of sign in $d\rho/dT$ at some low density, has been experimentally observed) at low temperatures ($T=50$ mK-5 K) and densities (n around, $n\sim n_c$, the reported critical densities being $n_c \approx 10^{11},10^{10},10^{9}$ cm^{-2} for typical high quality n-Si-MOS, p-GaAs, and n-GaAs systems, respectively). Before presenting our numerical results for $\rho(T,n)$ in Figs. 1–3, we first discuss qualitatively the theoretical results expected on the basis of Eqs. (1)–(5) in various temperature and density regimes. The dimensionless temperature, $T/T_F \sim n^{-1}$, plays an important role in our calculated temperature and density dependence 2D resistivity $\rho(T,n)$.

First, a straightforward asymptotic expansion of the Boltzmann transport equation shows that, for $T/T_F \ll 1$, the resistivity can be written as $\rho=\rho_0+\Delta\rho(T)$, where $\rho_0=\rho(T=0)$ and $\Delta\rho$ is given at very low temperature by[14,15]

$$\frac{\Delta\rho(T\ll T_F)}{\rho_0}\approx A_1\left(\frac{T}{T_F}\right)+A_{3/2}\left(\frac{T}{T_F}\right)^{3/2}-B_2\left(\frac{T}{T_F}\right)^2, \quad (6)$$

where $A_1(>0)$, $A_{3/2}(>0)$ arise from the thermal smearing of the electronic polarizability at $2k_F$ wave vector, and B_2 (>0) arises from the thermal energy averaging over the Fermi surface. Note that the first two screening terms in Eq. (6) are both positive, indicating an increasing resistivity with increasing temperature (the so-called 2D metallic behavior) due to the suppression of screening at finite temperatures whereas the third term, arising from the thermal energy averaging in Eq. (2), is a negative correction to resistivity (i.e., a positive correction to conductivity) due to the increased thermal velocity of the carriers at finite temperatures. The expansions for A_1, $A_{3/2}$, and B_2 have earlier been derived in the literature.[5,14,15]

$$A_1=2\left(1+\frac{1}{fq_0}\right)^{-1}, \quad (7)$$

$$A_{3/2}=2.646\left(1+\frac{1}{fq_0}\right)^{-2}, \quad (8)$$

$$B_2=\frac{\pi^2}{6}p(p+1), \quad (9)$$

where $q_0=q_{TF}/2k_F$, $f\equiv f(q=2k_F)$ is the 2D subband form factor (defining the deviation from the strict two dimensionality in the layer) at wave vector $2k_F$, and p (~ 1) gives the energy dependence of the scattering time in Eq. (3), i.e., $\tau(E)\sim E^p$ for $E\sim E_F$. Three important feature of the asymptotic expansion given in Eq. (6) need to be emphasized: (i) The A and the B terms in Eq. (6) arise from totally different physics, and therefore we have neglected a possible $A_2(T/T_F)^2$ term in the screening expansion in Eq. (6) which may very well modify the B_2 term to $B_2'\equiv(B_2-A_2)$. We use Eq. (6) only for the purpose of qualitative discussion, and all our results are based on the full numerical calculation of the semiclassical transport equations [Eqs. (1)–(5)] making any qualitative difference between B_2 and B_2' irrelevant for our results. (ii) There are higher-order interaction corrections for all the terms in Eq. (6), but these corrections are quantitatively unknown and are therefore left out of our discussion completely. (iii) The asymptotic expansion only holds at very low temperature $(T/T_F\ll 1)$ which may not be of relevance to the 2D MIT phenomena.[14] Details on the low-temperature transport behavior can be found in our recent work.[14] Here we are interested in the high-temperature resistivity, which is manifestly not given by Eq. (6).

It is possible to obtain the asymptotic high-temperature result for the classical limit $(T\gg T_F)$ of the 2D resistivity defined through Eqs. (1)–(6), and the result is[4]

$$\rho(T\gg T_F)\sim(T_F/T), \quad (10)$$

indicating that the high temperature resistivity decreases with increasing temperature in an apparently insulatinglike manner (i.e., $d\rho/dT<0$ for $T\gg T_F$). This can be rather easily understood physically based on the fact that at high temperatures the main effect arises from the thermal increase in the average carrier velocity which obviously leads to a decreasing resistivity. Thus the linear power-law resistivity, $\rho \sim T^{-1}$, in Eq. (10) arises from the energy averaging in Eq. (2).

Even before carrying out the full numerical calculations it is obvious that any smooth interpolation between the low-temperature behavior of Eq. (6) and the high-temperature behavior of Eq. (10) would exhibit nonmonotonic temperature-dependent resistivity[4] associated with this quantum-classical crossover, which has been seen [6–10] experimentally. This leads to the following generic temperature dependence of $\rho(T/T_F)$: At very low temperatures ρ is linear in T/T_F with $d\rho/dT(>0)$being determined by the absolute value of the coefficient A_1; with increasing T/T_F (but with $T/T_F<1$) it is possible (but *not* necessarily essential) for $d\rho/dT$ to change sign becoming negative, $d\rho/dT<0$, if the B_2 term can overcome the effects of the first two positive terms within the constraint $T/T_F<1$. This obviously depends on the relative values of coefficients A_1, $A_{3/2}$, and B_2. In Si (100) inversion layer and in the p-GaAs system screening effects are strong leading to $A_1, A_{3/2}\gg B_2$ so that at low T/T_F the metallic resistivity, $d\rho/dT>0$, dominates. In the n-GaAs 2D system, on the other hand, screening effects are weak (since the density of states is small) and for modest values of T/T_F, the B_2 term is larger than the first two terms,

particularly at high densities where $2k_F \gg q_{TF}$, leading to $d\rho/dT<0$ for moderate values of T/T_F. (In fact, this is also expected in highly disordered systems where screening effects may be cut off by impurity scattering at a characteristic temperature T_D which may not be too low.) The message from the analytical results of Eqs. (6)–(9) is therefore clear: A 2D system would always exhibit ''metallic'' resistivity (defined as $d\rho/dT>0$) for the lowest value of the dimensionless temperature T/T_F (ignoring all weak localization effects which we neglect throughout this paper) crossing over to an apparent insulating (defined as $d\rho/dT<0$, but *not* with an exponential temperature dependence) resistivity at some intermediate temperature $T_{\rm cross}$, where metallic ($d\rho/dT>0$) screening effects are overwhelmed by the insulating ($d\rho/dT<0$) thermal averaging effects. If the carrier density of the 2D system is effectively low so that $T_{\rm cross}$ is lower than the lowest achievable carrier temperature ($T_{\rm min}$) in the system, then the 2D system will manifest an apparent insulating phase ($d\rho/dT<0$) down to the lowest experimental temperature $T(>T_{\rm cross})$ since the low temperatures ($T<T_{\rm cross}$) where the screening effect dominates for a crossover to the metallic behavior ($d\rho/dT>0$) cannot be achieved. We note that it is possible in many situations, depending on carrier densities and system parameters (e.g., effective mass, type of impurity scattering, etc.), for $T_{\rm cross}<T_F$. Remembering that $T_F \approx 0.7\tilde{n}$ K (Si and $p-$GaAs) and $4\tilde{n}$ K ($n-$GaAs) where $\tilde{n}$ is the 2D carrier density measured in the units of 10^{10} cm^{-2}, we conclude that $T_{\rm cross}$ could be as low as 100 mK (in some situations) for 2D carrier densities around 10^{10} cm^{-2} for n-Si (and p-GaAs) and 10^9cm^{-2} for n-GaAs systems. A characteristic fundamental feature for the crossover we are discussing here is the strong dependence of the crossover temperature on the carrier density with $T_{\rm cross}(n)$ increasing strongly with carrier density. The other important feature is the nonexistence of the apparent insulating ($d\rho/dT<0$) phase at low enough temperature ($T<T_{\rm cross}$), i.e., if the carrier temperature of a 2D system can be decreased arbitrarily then (leaving out all localization effects) the system would always become metallic ($d\rho/dT>0$) at low enough temperatures (defined as $T \ll T_{\rm cross}$).

Our numerical transport results shown in Figs. 1–3 for p-GaAs and n-GaAs, respectively, bear out the theoretical considerations outlined above. In particular, the calculated resistivity is always metallic at the lowest temperatures, manifesting a clear crossover to an insulating behavior at $T_{\rm cross}(n)$ with a change of sign in $d\rho/dT$ (i.e., $d\rho/dT>0$ for $T<T_{\rm cross}$ and $d\rho/dT<0$ for $T>T_{\rm cross}$) as described above. We note that $T_{\rm cross}$ could be quite low for low values of carrier density, and therefore it is possible, in principle, for the apparent insulating behavior to arise entirely from this crossover effect. In particular, in Fig. 1 we choose (rather arbitrarily) the lowest possible achievable carrier temperature to be 300−500 mK (for the purpose of illustration) and show what a typical 2D MIT transport data set (in the $T=300$ mK–3 K range) would look like based entirely on this finite-temperature crossover phenomenon. The overall qualitative consistency between our results and many of the existing 2D MIT experimental results in the literature suggests that the simple (and physically appealing) crossover scenario presented in this paper cannot be entirely ruled out as the cause for the 2D MIT behavior observed experimentally at least in some situations. We note a quantitative discrepancy between our theory and experiment: in general, we find $T_{\rm cross}$ in our theory to be higher than the corresponding crossover temperature in the experiment. (This discrepancy can be corrected by introducing a cutoff T_D in the screening as discussed later in this paper.)

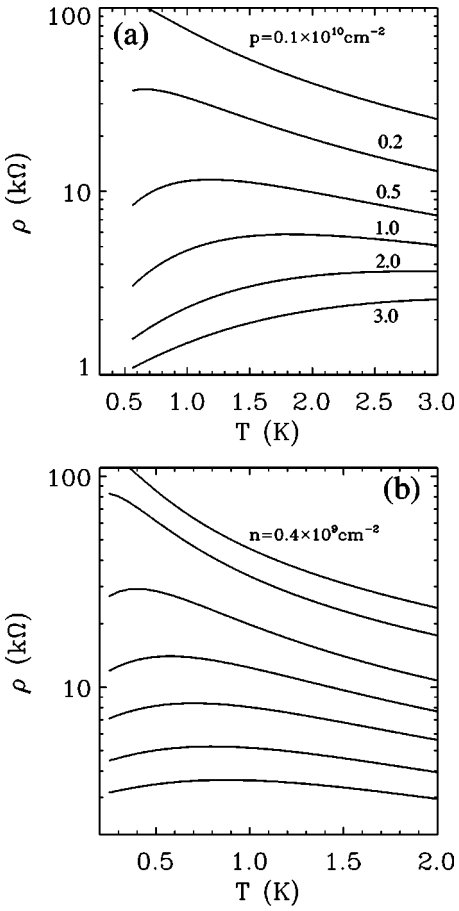

FIG. 1. (a) Calculated 2D resistivities for p-GaAs system for various hole densities $p=0.1,0.2,0.5,1.0,2.0,3.0\times10^{10}$ cm^{-2} (from the top) as a function of temperature. In this figure we use the charged bulk impurities inside the quasi-2D systems. (b) Calculated 2D resistivities for n-GaAs system for various electron densities $n=0.4,0.5,1.0,2.0,3.0,4.0,5.0\times10^{9}$ cm^{-2} (from the top) with the charged interface impurities as a function of temperature. ($T_D=0$ is used for these results.)

It is essential to emphasize two points: (i) The effective insulating phase is only an apparent insulating phase by virtue of the insulator being defined through the negative temperature derivative of resistivity, i.e., $d\rho/dT<0$. This apparent insulating phase is really a metallic (i.e., extended or delocalized electron wave function) electron liquid phase (interacting with random charged impurity centers) exhibiting $d\rho/dT<0$ down to a crossover temperature scale ($T>T_{\rm cross}$) with the lowest temperature ($T \ll T_{\rm cross}$) phase being always a true metal with $d\rho/dT>0$. (ii) The temperature dependence of $\rho(T)$ in this apparently insulating phase simu-

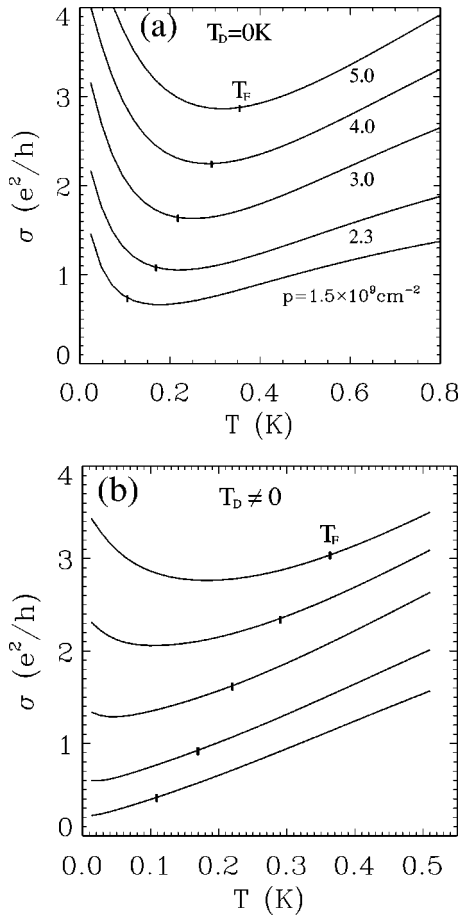

FIG. 2. Calculated conductivities for p-GaAs system (Ref. 7) for various hole densities $p=5.0,4.0,3.0,2.3,1.5\times10^9$ cm^{-2} (from the top) with the remote charged impurities. In (a) we calculate conductivities without level broadening in the polarizability function, and in (b) we include the level broadening in the screening, following Ref. 4, with $T_D=0.2,0.25,0.3,0.4,0.5$ K (from the top). The vertical bars indicate the Fermi temperature T_F.

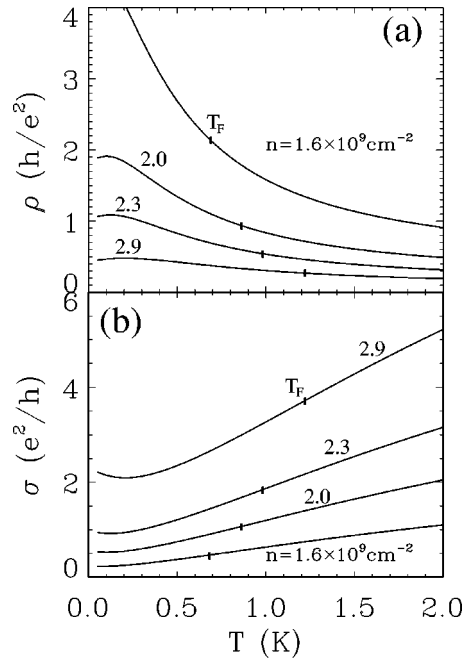

FIG. 3. Calculated (a) resistivities and (b) conductivities for n-GaAs system for various electron densities $n=1.6,2.0,2.3,2.9\times10^9$ cm^{-2} with the remote charged impurities and a fixed Dingle temperature $T_D=0.75$ K as a function of temperature. The vertical bars indicate the Fermi temperature T_F.

lates a power law (with $\rho\sim1/T$ in the $T>T_F$ high-temperature region) rather than the exponential temperature dependence typical of a true insulating phase. The finite-temperature crossover insulating phase (i.e., $d\rho/dT<0$) is, by definition, a high-temperature phase in our Drude-Boltzmann transport model with the true low-temperature ($T\rightarrow0$) phase in our theory (which manifestly excludes weak and strong localization effects) being always a metallic (i.e., $d\rho/dT>0$) phase. The crucial point of physics is, however, the fact that experimentally there is always a low-temperature cutoff $T_{\min}$, below which carriers simply cannot be cooled down, and for $T_{\rm cross}<T_{\min}$, the 2D apparent insulating phase discussed in this paper would, for all particular purposes, behave like a ''real'' insulating phase. The apparent 2D metal-insulator transition in this crossover scenario will therefore appear to occur at the density n_c defined by $T_{\rm cross}(n_c)\approx T_{\min}$.

In comparing our theoretical results with the existing 2D MIT data in the experimental literature, we mention that a large number of reported data look qualitatively similar to the results shown in Figs. 1–3 of this paper. In particular, the following features of our theoretical results are often observed experimentally: (i) the low-density–low-temperature 2D insulating phase (for $n<n_c$) is often seen to have an approximate power-law temperature-dependence resistivity; (ii) the low-temperature extrapolated ($T\rightarrow0$) resistivity does not seem to be divergent in many cases; (iii) the experimentally observed critical density n_c has been reported to be a function of the lowest measurement temperature with $n_c(T)$ decreasing monotonically with decreasing temperature. For low enough carrier densities, most 2D systems should eventually enter a strongly localized transport regime where the low-temperature resistivity diverges in an exponential manner, but in high quality (i.e., low disorder) systems of interest in the 2D MIT phenomenon such exponentially divergent resistivity seems to be always preceded by a density regime (below n_c) where $\rho(T)$ increases smoothly in a power-law manner with decreasing temperature. This intermediate density regime of apparent insulating behavior is most strongly manifested in 2D p-GaAs and n-GaAs systems, but it has also been seen in n-SiMOS systems. A possible reason for the prevalence of the apparent insulating phase in 2D GaAs systems could be the much higher quality (i.e., much lower disorder) for GaAs-based 2D systems compared with 2D Si structure, making it possible for the system not to manifest localization down to rather low carrier densities, thus enabling the crossover insulating phase to show up more easily without being preempted by localization.

In discussing experimental data, it is important to realize that the measured 2D resistivity $\rho(T)$ typically shows a saturation at low enough temperature, i.e., below some saturation temperature ($T_s\sim50-300$ mK, depending on the experi-

ment) $\rho(T<T_s)$ becomes essentially a constant completely independent of temperature. Although an underlying (unknown) fundamental cause for this resistivity saturation behavior cannot be definitively ruled out, the saturation is generally believed to arise from a saturation in the carrier temperature due to the inevitable carrier heating problems in semiconductors at low temperatures and densities. This possibility of carrier temperature saturation at low temperatures (where further cryogenic cooling only lowers the temperature of the surrounding bath, not the 2D electrons themselves) is further supported by the fact that the resistivity saturation is also seen in the insulating system at low enough carrier temperatures. We have completely ignored the low-temperature resistivity saturation problem in our analysis, assuming, perhaps somewhat uncritically, but in agreement with all existing theoretical analyses in the subject, that the resistivity saturation arises from temperature saturation, i.e., $T_s \equiv T_{\min}$. Within the screening theory, in fact, thermal suppression of screening will be cut off at temperatures ($T<T_D$) lower than the temperature scale of the disorder scattering (with $T_D \geqslant \hbar/k_B\tau$), and we have taken this disorder broadening effect approximately into account in the results presented in Figs. 2 and 3 of this paper. Another comment in the context of 2D MIT experiments is the issue of the so-called separatrix, where the critical density n_c separating the metallic and the insulating phase is claimed to manifest a completely temperature-independent resistivity sharply delineating the metallic phase ($d\rho/dT>0$) from the insulating phase ($d\rho/dT<0$) and thus defining a quantum phase transition. Such a sharp separatrix cannot be explained at all within our Drude-Boltzmann theory since all electronic states are by definition extended in our semiclassical theory. As has been emphasized above, the apparent critical density n_c in our theory is strongly temperature dependent with n_c delineating $d\rho/dT>0$ (for $n>n_c$) and $d\rho/dT<0$ (for $n<n_c$) decreasing with the lowest measurement temperature $T_{\min}$ with $n_c \to 0$ as $T_{\min} \to 0$ since a true insulator does not exist in our theory. An experimental separatrix defining the 2D MIT has, however, been rarely reported, and in that sense our theory is consistent with a large body of the existing 2D MIT experimental data. We do emphasize, however, that at low enough densities all real 2D systems should eventually exhibit exponential localization behavior (at densities below the "power law" apparent localization being discussed in this paper) which is beyond the scope of our work. We believe that Si MOS systems, typically being more disordered than 2D GaAs systems, exhibit much more of the exponential-type true insulating behavior than the power-law apparent insulating behavior being discussed in this paper.

In a very recent paper Noh *et al.*[7] have reported an anomalous power-law insulating behavior in a 2D p-GaAs systems at low densities (and intermediate temperature), which we believe to be the apparent insulating phase induced by quantum-classical crossover being predicted in this paper. The r_s values in the samples of Ref. 7 are extremely high ($r_s \sim 44$–80), bringing up the possibility of Wigner crystal (perhaps in a strongly correlated molten phase[16]) physics playing a role in this experiment. We believe that it is unlikely that Wigner crystal (WC) physics has much to do with this anomalous insulating behavior. First, the experimental temperature and density regime where the anomalous insulating behavior is observed is well outside the WC phase boundary (see Fig. 4). Second, the transport data plotted as $\sigma(T) \equiv \rho^{-1}$ for various densities in Ref. 7 show absolutely no indication of a critical density separating metallic and insulating phases — the data at all densities look essentially the same at high temperatures where $\sigma(T) \sim T$ whereas at low temperatures the relatively high-density conductivity shows an upward bending below some density-dependent temperature $T_{\text{cross}}(n)$ (i.e., $d\rho/dT$ changes sign from being insulating-like, $d\rho/dT<0$, at higher temperatures to being metallic-like, $d\rho/dT>0$, at lower temperatures). We emphasize that for $T>T_{\text{cross}}(n)$ there is absolutely no difference in

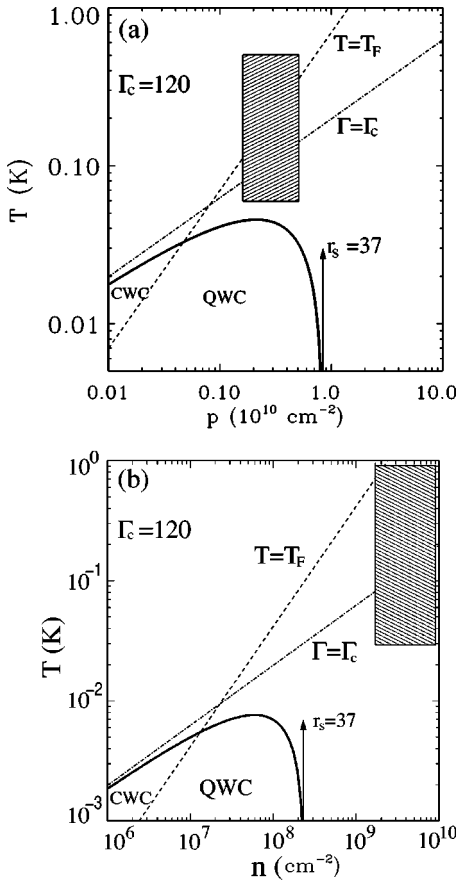

FIG. 4. Calculated 2D hole/electron liquid-Wigner crystal phase diagram in the density-temperature (n,T) plot for (a) p-GaAs and (b) for n-GaAs systems. The shaded regions indicate the experimental samples corresponding to Ref. 7 for p-GaAs and Ref. 8 for n-GaAs. Here T_F is the Fermi temperature and $\Gamma = \langle V \rangle / \langle T \rangle$ is the ratio of the classical mean potential energy to the mean classical kinetic energy. The classical WC-electron liquid (first order) phase transition line is defined by $\Gamma = \Gamma_c$, which is found from numerical (molecular dynamics) simulations (Ref. 4) to be $\Gamma_c \approx 120$. At $T=0$ a 2D system is expected to become a quantum WC phase at $r_s \geqslant 37$ (Ref. 18). The heavy solid line, indicating the liquid-solid phase boundary, is obtained from the interpolating scheme described in Ref. 16. The region above (below) the $T=T_F$ line is the classical (quantum) region with CWC (QWC) denoting the classical (quantum) Wigner crystal phase.

the observed qualitative behavior of $\sigma(T)$ at different densities. Since $T_{\rm cross}(n)$ decreases with carrier density n, it is impossible to rule out the possibility that the experimental low-density $\sigma(T)$ plots would actually also bend upward at temperatures lower than the lowest hole temperature ($\sim$60 mK) achieved in these experiments. In fact, the experimental data of Noh *et al.* are entirely consistent with this scenario. For example, if the experimental temperature cutoff is taken to be 200 mK in the data of Noh *et al.*, then all the plots shown in Ref. 7 would appear to be insulating (up to the hole density of 5×10^9 cm^{-2}) since all the experimental data exhibit $d\rho/dT<0$ (i.e., $d\sigma/dT>0$) down to 200 mK.

To emphasize the similarity between the experimental data of Noh *et al.* and our predicted crossover apparent insulating phase we show in Fig. 2 our calculated low-density transport results for the Noh *et al.* sample. Remembering that phonon scattering (neglected in our theory) becomes important for GaAs holes already at 500 mK,[6] the results of Fig. 2 are remarkably similar to the experimental data presented in Ref. 7. This is particularly true for Fig. 2(b), where we include a Dingle temperature cutoff in the screening function in order to approximately simulate the effect of collisional broadening (due to impurity scattering). The basic idea[17] is to include in the finite wave-vector static polarizability the level broadening (parametrized by a Dingle temperature $T_D=\Gamma/k_B$ where Γ is the collisional damping) arising from the impurity scattering. As discussed earlier in the literature[17] such a collisional damping term, parametrized by the level broadening parameter T_D, acts to suppress the thermal effects on screening at low temperatures, $T\ll T_D$, while leaving the screening function essentially unaffected at high temperatures $T>T_D$. Since the level broadening parameter in the screening, T_D, is unknown (within our approximation scheme), we do not attach any great quantitative significance to our approximation except to note that this zeroth-order scheme of incorporating collisional damping in screening is physically meaningful since impurity scattering should tend to suppress finite wave-vector static screening of an electron gas. We emphasize that the parameter T_D used in our theory should not be construed to be $T_D\approx\hbar/k_B\tau$ (which would be far too small), but should be considered an effective level broadening parameter which cuts off the strong temperature dependence of screening for $T<T_D$, with T_D being a parameter of the theory (increasing with decreasing conductivity).

It is therefore worthwhile to emphasize that our results (with $T_D\neq0$ in screening) shown in Fig. 2(b) are qualitatively (and even semiquantitatively) in good agreement with the approximately linearly rising, $\sigma\propto T$, conductivity at higher temperatures found by Noh *et al.* In fact, even our $T_D=0$ static RPA screening transport results are in qualitative agreement with experiment except for the fact that the crossover temperature $T_{\rm cross}$ for $d\sigma/dT$ to change sign is consistently much higher in Fig. 2(a) with $T_D=0$ than in our Fig. 2(b) with $T_D\neq0$ or in the experiment. The basic behavior of an approximately linear temperature dependence of conductivity, $\sigma\propto T$, for $T>T_{\rm cross}(n)$ with $T_{\rm cross}(n)$ decreasing with decreasing density [and $d\sigma/dT<0$, i.e., $\sigma(T)$ increasing with decreasing temperature for $T<T_{\rm cross}(n)$] applies to Figs. 2(a) and (b) and to the experimental data. Inclusion of phonon scattering (neglected in our calculations here) is likely to further improve the good agreement between experiment and theory.

In the context of comparison with the experimental data we show in Fig. 3 our calculated low-temperature transport results for the 2D GaAs electron system corresponding to the recent measurement of Lilly *et al.*[8] carried out in a high quality low-density n-GaAs gated heterostructure system. We have kept the level broadening parameter $T_D=0.75$ K fixed in the results shown in Fig. 3 and have shown the behavior of both the resistivity $\rho(T)$ and conductivity $\sigma(T)$ for several low densities. Again, phonon-scattering effects, possibly of some importance here for $T>1$ K,[8] have been neglected. These theoretical results are again in striking qualitative and semiquantitative agreement with the experimental data of Lilly *et al.*[8] demonstrating that the experimentally observed anomalous insulating phase in high quality (i.e., low-disorder) 2D semiconductor systems may very well be the apparent crossover insulating phase [which would manifest metallic behavior for $T<T_{\rm cross}(n)$ except that $T_{\rm cross}$ may be too low to be experimentally accessible at low carrier densities] predicted by our theory.

The striking qualitative agreement between our theory and experiment for the anomalous insulating phase encourages us to predict the following scenario for high quality GaAs-based 2D (both electron and hole) systems. As 2D carrier density is lowered the high-temperature crossover insulating phase ($T>T_{\rm cross}$) extends to lower temperatures as $T_{\rm cross}(n)$ goes down, and eventually when $T_{\rm cross}$ goes below the lowest accessible experimental temperature, the system appears to be insulating with an anomalous (i.e., power law) temperature dependence characterizing the apparent insulating phase. Since our predicted scenario is in excellent qualitative agreement with the available experimental data in GaAs-based high-mobility (low disorder) 2D holes and electrons, we propose the following direct experimental test for its verification (or falsification): One should lower the temperature to check whether the observed anomalous temperature dependence of the putative insulating phase remains insulating or bends upward (i.e., $d\sigma/dT$ goes through zero changing its sign from positive to negative as it does at higher densities) at lower temperatures. Since lowering carrier temperature arbitrarily is typically difficult to achieve in 2D semiconductor systems, a relatively easy way of testing our proposed scenario will be to directly plot the experimentally measured $d\sigma/dT$ (or $d\rho/dT$) as a function of temperature at low densities in the anomalous insulating phase. If the magnitude of this derivative is decreasing with decreasing temperature indicating that σ (or ρ) may be approaching a minimum (or maximum), then this is a rather strong verification of our proposed scenario. We mention in this context that the experimental data of Refs. 7 and 8 are qualitatively consistent with our proposed scenario of a decreasing magnitude of $d\sigma/dT$ as T decreases. We believe that there are simply two possibilities (which are consistent with our prediction of the anomalous power-law insulating phase being a crossover phase): Either the magnitude of $d\rho/dT$ will decrease indicating a transition to an effective metallic phase or $d\rho/dT$ will diverge exponentially indicating a true insulating

phase. We suggest that experiments be carried out in the anomalous power-law insulating phase (with careful measurements of $d\rho/dT$ as a function of T and n) to verify our prediction.

Finally, we discuss the possibility of WC physics playing a role in the observed anomalous insulating behavior of Refs. 7 and 8. In Fig. 4 we show our calculated approximate WC $(T-n)$ phase diagram for both 2D electrons and holes in GaAs, combining both the $T=0$ quantum Wigner crystallization at low carrier densities and the classical Wigner crystallization. The calculation of this approximate WC phase diagram follows our recent work in Ref. 18 where we show how one can implement an interpolation scheme to obtain the full phase diagram in the density-temperature plot by combining the known classical[19] and quantum[21] WC limits. We also show as shaded regimes the experimental samples corresponding to Refs. 7 and 8 in Figs. 3(a) and (b), respectively. Both the shaded regions are well outside the WC phase in the phase diagram indicating that WC physics is unlikely to be playing a dominant role here. The more important reason for ruling out the WC scenario is, however, experimental — the experimental data of Noh *et al.*[7] are completely smooth and the conductivity behavior in the metallic (higher density) and the insulating (lower density) phase is identical at higher temperatures, the only difference being the upward curvature in $\sigma(T)$ in the higher density data which moves to lower temperatures in the lower density plots and may have simply moved to temperatures below the measurement temperatures at the lowest densities making it experimentally invisible.

In summary, we have identified an apparent high-temperature insulating phase in 2D systems at intermediate carrier densities where the conductivity increases with temperature in a power-law fashion (approximately linearly). This apparent or effective insulating phase is a temperature-induced quantum-classical crossover phase which exists only for $T>T_{\rm cross}(n)$ where $T_{\rm cross}$ decreases with decreasing carrier density. We have shown that, although typically $T_{\rm cross} \sim T_F(\propto n)$, it is possible for $T_{\rm cross}$ to be substantially below the Fermi temperature if collisional broadening effects are included in the carrier screening function. We have shown that our calculated transport behavior in this effective insulating phase agrees qualitatively very well with recent observations in high quality 2D electrons and holes in GaAs heterostructures. We have not discussed the 2D Si-MOS system in this paper because the disorder effects in Si MOSFET's are substantially higher (mobilities are typically factors of 10–1000 lower in Si MOS systems than in GaAs systems), and the Si MOS systems make transitions to the strongly localized phase (with exponential temperature dependence) much sooner — this intermediate crossover insulating phase therefore exists only in a very narrow range of densities and temperatures in Si MOS systems.

The weakest empirical point in our quantitative comparison with the experimental data is the fact that our theoretically obtained $T^{th}_{\rm cross}(n)$ seems to be consistently higher than the experimental crossover temperature $T^{ex}_{\rm cross}(n)$ where $T_{\rm cross}$ at a particular density could, for example, be defined by the condition $d\sigma/dT=0$ at $T=T_{\rm cross}$. We have tentatively fixed this problem in this work by introducing the collisional damping induced screening cutoff through the Dingle temperature parameter T_D. The other possibility is to take the *real* Fermi temperature T_F to be smaller than the nominal $T_F(\sim n/m)$ given by the carrier density n and the band effective mass m which could happen if the actual effective mass is larger than the band mass and/or the actual free carrier density is smaller (e.g., due to trapping of carriers). The crossover temperature can be reduced if we use the temperature dependence of the effective mass, which is recently investigated in Ref. 20. In Ref. 20 it is shown that the low-density effective mass has a peak at $T\approx 0.5T_F$ and shows strong temperature dependence. Since the larger effective mass gives rise to higher resistivity the decrease of effective mass above $T\approx 0.5T_F$ can reduce the crossover temperature bringing our results in better agreement with experiment. This is, however, a subtle issue since effective-mass renormalization would not ordinarily affect transport in a translationally invariant system, and further work is needed to settle this point. While in the GaAs-based low disorder systems studied in this paper, the discrepancy between $T^{th}_{\rm cross}(n)$ and $T^{ex}_{\rm cross}(n)$ is relatively small even for the $T_D=0$ theory, this discrepancy is very large in Si MOS based 2D systems where disorder is high. A straightforward application of our $T_D=0$ theory for Si MOS systems indicates $T_{\rm cross}>1$ K for the $n=5\times 10^{10}-10^{11}$ cm^{-2} density range of relevance to Si 2D systems. To reduce $T_{\rm cross}$ substantially one needs to introduce large values of T_D where the theory is not meaningful. The other possibility is that the effective mass in Si may be enhanced from its band value and/or the effective carrier density may be substantially lower than the nominal n. It is therefore interesting to note that there is no clearcut signature of this anomalous insulating phase in 2D Si MOS systems where the insulating phase, for $T<1$ K or so, indicates exponentially rising resistivity with decreasing temperature. This is consistent with our finding of rather large $T^{th}_{\rm cross}$ in Si MOS systems implying that this apparent insulating phase in Si systems remains only a high-temperature phase since the electrons can always be cooled below $T_{\rm cross}$ in the 2D Si samples.

We note finally that the validity of the semiclassical Drude-Boltzmann theory for studying the anomalous insulating phase is not a crucial issue in this context. The reason is that Boltzmann theory, being semiclassical, becomes increasingly valid at higher temperatures, and therefore should be a reasonable qualitative description for the high-temperature effective insulating phase being discussed here. The precise value of the resistivity ρ and the corresponding localization parameter $k_F l$ (where l is the transport mean-free path), i.e., whether ρ is smaller than the quantum of resistance h/e^2 or $k_F l$ is larger than unity, are important issues for $T=0$ considerations (where quantum localization effects are important) but not for finite T/T_F (≈ 0.1–1) case which is of interest here. In fact, within our semiclassical Boltzmann transport theory the behavior shown in Figs. 1–3 are *generic* behavior determined entirely by n and T, completely independent of the actual resistivity values of the 2D system, i.e., one should think of the ordinate (ρ or σ in our figures) to

have completely arbitrary units — the actual values of ρ $(=\sigma^{-1})$ in our theoretical results being determined by the unknown charged impurity density (N_i) in the system which we use as an adjustable parameter to set the resistivity scale. It is more appropriate to think of ordinate at Figs. 1 and 2 as ρ/ρ_0 or σ/σ_0 where ρ_0 or σ_0 are the $T=0$ Drude values of resistivity or conductivity. These considerations make it clear that our qualitative results are valid (at finite T/T_F) irrespective of the actual sample resistivities as long as one can neglect quantum localization effects which seems to be the case for the GaAs experiments in Refs. 7 and 8, but perhaps not for Si MOS samples. For similar reasons, RPA becomes a better approximation at larger values of T/T_F since RPA is essentially exact in the $T/T_F \gg 1$ limit. Thus finite-T and large-r_s systems (which have relatively larger values of T/T_F) may be well described by our RPA theory.

In our conclusion, we discuss critically the approximations of our theory which may limit its applicability to real 2D systems. Our most important approximations are the Drude-Boltzmann scattering theory (due to charged impurity scattering) and RPA screening by the electron liquid. Both of these approximations are simplistic at the low carrier density phenomena of interest here. But there are good reasons to believe that our theory is sound in explaining the qualitative behavior of 2D semiconductor systems at densities and temperatures above the true localization regime. First, recent systematic diagrammatic calculations[5] of higher-order interaction corrections to the 2D resistivity show that the basic picture of an effective 2D metallic behavior with $d\rho/dT > 0$ (with a leading order linear temperature coefficient of resistivity) applies well in the ballistic transport regime $(\hbar/k_B\tau \ll T \ll T_F)$ provided weak localization effects are negligible. Weak localization effects may dominate at very low temperatures and provide a different crossover of resistivity from metal to insulator as temperature decreases.[22] Second, the results for $\rho(T,n)$ obtained in the 2D metallic regime within the RPA-Drude-Boltzmann theory agree well with existing experimental results. Third, RPA becomes quantitatively accurate at high temperatures (and our inclusion of local-field correlation corrections going beyond RPA gives qualitatively very similar results). We therefore believe that our description of the high-temperature apparent insulating phase based on the semiclassical Drude-Boltzmann-RPA transport theory may have considerable theoretical validity.

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