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Oscillations conditions in a gain grating in the Bragg diffraction regime

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ABSTRACT

Coupled-wave analysis is applied to the determination of an oscillation threshold in thick periodic grating with gain (distributed-feedback oscillator). Pure phase (refractive index modulation), amplitude (gain modulation) and mixed-type gratings are considered. The origin of oscillation modes and their position on the diffraction efficiency contour is analyzed. The threshold conditions are determined on a basis of analytical expressions.

1. Introduction

The interest to distributed feedback (DFB) oscillators started in 70s of the last century [1] is renewed now. Particularly, in recent years there is great attention paid to the problem of new (organic) materials as components for optoelectronic devices and suitable for DFB lasers creation [2–6]. So called soft materials are promising in a view of their easy manufacturing and ability of periodic structures recording. A simple example is a holographic grating recorded in a dye-doped polymer material, with periodic refractive index modulation and gain due to the optical external pumping. The grating spacing and the average refractive index determine the conditions of Bragg resonances for the waves propagating in the grating. Distributed Bragg reflection is responsible for the feedback, and lasing occurs when the gain reaches the threshold value. However, the spatial and spectral structure of the laser emission is not evident in advance. In this view, an analysis of the general properties of volume gratings with uniform or distributed gain is of importance.

The goal of the present paper is to apply well-known coupled-wave theory analysis to bulk Bragg gratings with gain and determine the conditions for self-starting oscillations. Kogelnik's approach [7] with some modification is used for description of the field in the grating below the oscillation threshold. Similar formulae as was derived by Kogelnik for absorption modulation are obtained for gain gratings. On the basis of analytical expression for diffraction efficiency contours (angular or spectral selectivity of a grating) formation of pre-oscillating peaks is shown. Threshold gain dependence on the modulation of the refractive index and gain spatial modulation is determined.

2. Coupled-wave analysis

The method of the analysis is to calculate probe plane wave diffraction whose wavelength and propagation angle are arbitrary, but related each other with the resonance conditions as described below. The geometry of waves interaction within a spatially modulated medium is shown in Fig. 1. The law of the dielectric permittivity modulation in a slab has the form

$$\varepsilon(z) = \varepsilon_0 + \Delta\varepsilon\cos(Kz), \tag{1}$$

where ε_0 is the unperturbed dielectric permittivity, $\Delta\varepsilon$ is the magnitude of modulation and K is the grating vector modulus. The propagation angle θ is assumed to be close to the Bragg angle θ_B satisfying the condition $2n\Lambda\sin\theta_B = \lambda$, where Λ is the grating spacing, n is the average refractive index and λ is the vacuum wavelength. We also assume the presence of the gain α in the medium, which may possess uniform component and spatial modulation with the grating periodicity. The origin of the gain is the result of external optical pumping.

The grating is supposed to be recorded in a thin film between the substrates, thus producing a kind of waveguide. The thickness of the film substantially exceeds Λ. Below the attention will be concentrated mainly on the DFB properties and the analysis of the waveguide effects is let for further consideration.

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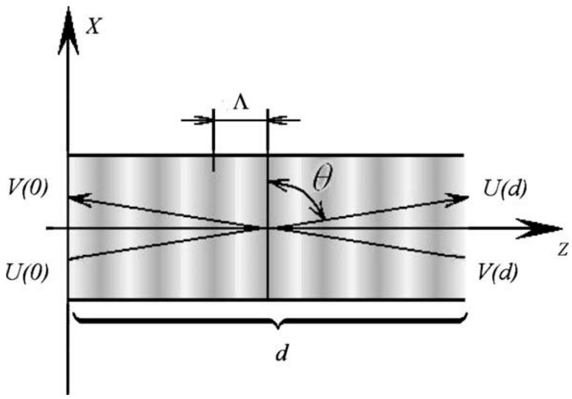

Fig. 1. Schematic of the waves in the modulated medium: U is the incident wave, V is the diffracted wave, θ is the propagation angle, Λ is the grating spacing, d is the thickness of the grating.

For the field $E(x,z)$ in the medium being a superposition of incident wave

$$U(x,z) = U(z)\exp\left(ik_z z + \frac{\alpha k_z z}{k}\right)\exp\left(ik_x x + \frac{\alpha k_x x}{k}\right), \tag{2}$$

and reflected wave

$$V(x,z) = V(z)\exp\left(-ik_z z - \frac{\alpha k_z z}{k}\right)\exp\left(ik_x x + \frac{\alpha k_x x}{k}\right), \tag{3}$$

we write an expression

$$E(x,z) = \left[U(z)\exp\left(ik_z z + \frac{\alpha k_z z}{k}\right) + V(z)\exp\left(-ik_z z - \frac{\alpha k_z z}{k}\right)\right] \times \exp\left(ik_x x + \frac{\alpha k_x x}{k}\right), \tag{4}$$

where positive α is responsible for the gain, k_x and k_z are components of the wave vectors in the medium, $k_x^2 + k_z^2 = k^2 = k_0^2 n^2$ and k_0 is the wave number in free space, $k_0 = 2\pi/\lambda$. The first-order Bragg condition corresponds to equality $k_z = K/2$.

The insertion of Eq. (4) into scalar wave equation leads to the expression

$$\frac{\partial^2 E(x,z)}{\partial x^2} + \frac{\partial^2 E(x,z)}{\partial z^2} = -k_0^2 \varepsilon(z) E(x,z), \tag{5}$$

where $\varepsilon(z)$ is the dielectric permittivity, which with the presence of the spatial modulation and uniform gain in the medium has a complex value. The right side of Eq. (5) in the case of a modulated medium with uniform gain attains a form [8]

$$k_0^2 \varepsilon(z) E(x,z) = \left[k^2 - \alpha^2 - 2ik\alpha + k_0^2 \Delta\varepsilon \cos(Kz)\right] E(x,z). \tag{6}$$

Neglecting the terms corresponding to higher orders of diffraction, we combine the terms obtained after the differentiation in Eq. (5) into the system

$$\begin{cases} \frac{\partial^2 U(z)}{\partial z^2} e^{\frac{\alpha k_z}{k} z} + 2ik_z\left(1 - \frac{i\alpha}{k}\right) \frac{\partial U(z)}{\partial z} e^{\frac{\alpha k_z}{k} z} = -\frac{k_0^2 \Delta\varepsilon}{2} V(z) e^{-\frac{\alpha k_z}{k} z - i\delta z}, \\ \frac{\partial^2 V(z)}{\partial z^2} e^{-\frac{\alpha k_z}{k} z} - 2ik_z\left(1 - \frac{i\alpha}{k}\right) \frac{\partial V(z)}{\partial z} e^{-\frac{\alpha k_z}{k} z} = -\frac{k_0^2 \Delta\varepsilon}{2} U(z) e^{\frac{\alpha k_z}{k} z + i\delta z}, \end{cases} \tag{7}$$

where δ is the measure of detuning from exact Bragg resonance: $\delta = 2k_z - K$. This general detuning factor includes angular deviation from exact Bragg angle, wavelength deviation, or both.

With the assumption of slowly varying amplitudes $U(z)$ and $V(z)$, we omit second derivative terms in Eq. (7) and will seek the solution in a form

$$U(z) = u\exp\left(\sigma z - \frac{\alpha k_z}{k} z - i\frac{\delta z}{2}\right), \tag{8}$$

$$V(z) = v\exp\left(\sigma z + \frac{\alpha k_z}{k} z + i\frac{\delta z}{2}\right). \tag{9}$$

The system of Eq. (7) thus attains the form

$$\begin{cases} 2ik_z\left(1 - \frac{i\alpha}{k}\right)\left(\sigma - \frac{\alpha k_z}{k} - i\frac{\delta}{2}\right) u = -\frac{k_0^2 \Delta\varepsilon}{2} v, \\ 2ik_z\left(1 - \frac{i\alpha}{k}\right)\left(\sigma + \frac{\alpha k_z}{k} + i\frac{\delta}{2}\right) v = \frac{k_0^2 \Delta\varepsilon}{2} u. \end{cases} \tag{10}$$

Multiplication of the equations of the system (10) gives

$$\sigma_{1,2} = \pm\left[\left(\frac{\alpha k_z}{k} + i\frac{\delta}{2}\right)^2 + \frac{\left(\frac{k_0^2 \Delta\varepsilon}{4k_z}\right)^2}{\left(1 - \frac{i\alpha}{k}\right)^2}\right]^{1/2}. \tag{11}$$

The resulting solution is

$$U(z) = [u_1 \exp(\sigma_1 z) + u_2 \exp(\sigma_2 z)]\exp\left(-\frac{\alpha k_z z}{k} - i\frac{\delta z}{2}\right), \tag{12}$$

$$V(z) = [v_1 \exp(\sigma_1 z) + v_2 \exp(\sigma_2 z)]\exp\left(\frac{\alpha k_z z}{k} + i\frac{\delta z}{2}\right). \tag{13}$$

To find the field in the grating, we have to substitute Eqs. (12) and (13) into Eq. (4). For the sake to determine the reflected wave amplitude $V(0)$ it is enough to use the simplified boundary conditions $U(0) = U_0$ and $V(d) = 0$:

$$\begin{aligned} &u_1 + u_2 = U_0, \\ &v_1 \exp(\sigma_1 d) + u_2 \exp(\sigma_2 d) = 0. \end{aligned} \tag{14}$$

Combination of the Eqs. (10) and (11) with the boundary conditions (14) gives the solution for the reflected wave amplitude (the gain due to the x-dependence in Eq. (3) is also taken into account)

$$V(0) = iU_0 \frac{\frac{k_0^2 \Delta\varepsilon}{4k_z}}{\left(1 - i\frac{\alpha}{k}\right)} \frac{\sinh(\sigma_1 d)}{\sigma_1 \cosh(\sigma_1 d) - \left(\frac{\alpha k_z}{k} + i\frac{\delta}{2}\right)\sinh(\sigma_1 d)} \times \exp\left(\alpha \frac{k^2 - k_z^2}{k k_z} d\right). \tag{15}$$

For a pure phase grating $\Delta\varepsilon \approx 2n\Delta n$, and the modulation term is $\frac{k_0^2 \Delta\varepsilon}{4k_z} = \frac{k\Delta n}{2n\sin\theta}$. An illustration of the diffraction efficiency contour $\eta = V(0)V^*(0)/U_0^2$ is shown in Fig. 2. In calculations we used nor-

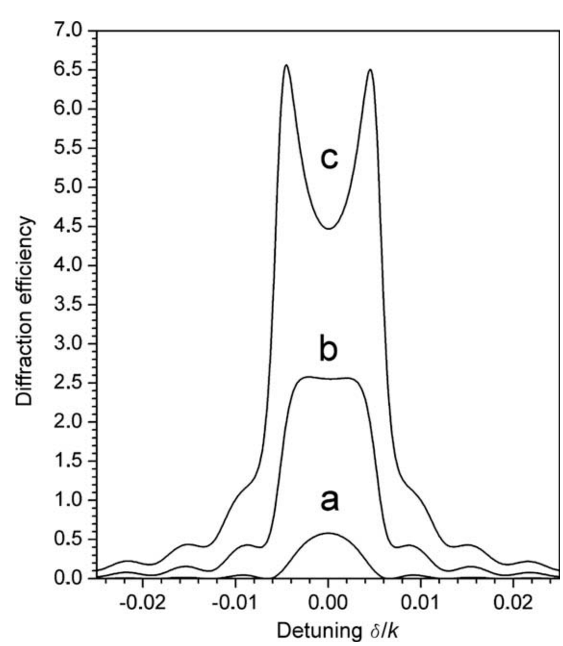

Fig. 2. Diffraction efficiency contours calculated for pure phase grating with uniform gain. (a) Zero gain, $\Delta n/n = 1.8 \times 10^{-3}$. (b) The same contour calculated with the presence of the uniform gain $\alpha/k = 8 \times 10^{-4}$. (c) The same with $\alpha/k = 1.1 \times 10^{-3}$.

malized value $k = 1$ to generalize the results. Fig. 2a shows the calculated contour as a function of the detuning δ/k for the grating parameters $d = 10^3/k$ (for comparison with realistic $k_0 = 10\ \mu m^{-1}$ and $n = 1.6d$ will amount 62.5 μm), $K/2k = \sin\theta_B = 0.9$ and $\Delta n/n = 1.8 \times 10^{-3}$. Then, the same contour is calculated with nonzero gain $\alpha/k = 8 \times 10^{-4}$ (Fig. 2b) and for $\alpha/k = 1.1 \times 10^{-3}$ (Fig. 2c).

The influence of a gain results in the growth of the diffracted wave intensity which overcomes unity (Fig. 2b). The transformation of the contour with the gain increase is accomplished with the formation of narrow pre-oscillating peaks (Fig. 2c). The phase shift $\pi/2$ inherent to the diffraction by pure phase grating prevents the oscillation mode appearance at the exact Bragg resonance, and "edge modes" are located at the both sides of the angular (or spectral) selectivity contour.

For mixed-type gratings the modulation of dielectric permittivity $\Delta\varepsilon$ combines the refractive index and the gain modulation:

$$\Delta\varepsilon \approx 2n\Delta n \pm i\frac{2n^2\Delta\alpha}{k}. \tag{16}$$

Below we shall use the notifications

$$\chi = \frac{k_0^2\Delta\varepsilon}{4k_z} = \kappa_n \pm i\kappa_\alpha,$$

where

$$\kappa_n = \frac{k_0\Delta n}{2\sin\theta},$$
$$\kappa_\alpha = \frac{\Delta\alpha}{2\sin\theta},$$

and the sign ± in Eq. (16) indicates the mutual orientation of the gain grating and refractive index grating: (+) corresponds to in-phase matching, and vice versa.

The presence of the amplitude (gain) modulation discriminates one edge peak. Depending on the mutual overlapping of the periodical gratings of refractive index and gain a peak will be suppressed at one or another side of the selectivity contour. Fig. 3 gives an example of the diffraction efficiency contour for the grating parameters $\alpha/k = 10^{-3}$, $\kappa_n = 10^{-3}$, $\kappa_\alpha = 10^{-4}$.

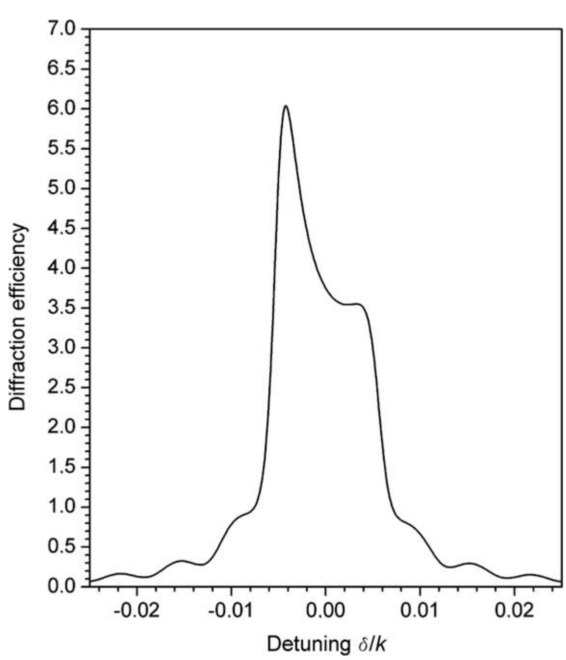

Fig. 3. Diffraction efficiency contour calculated for mixed-type grating with gain $\alpha k = 10^{-3}$, $\kappa_n = 10^{-3}$, $\kappa_\alpha = 10^{-4}$, $d = 1000/k$.

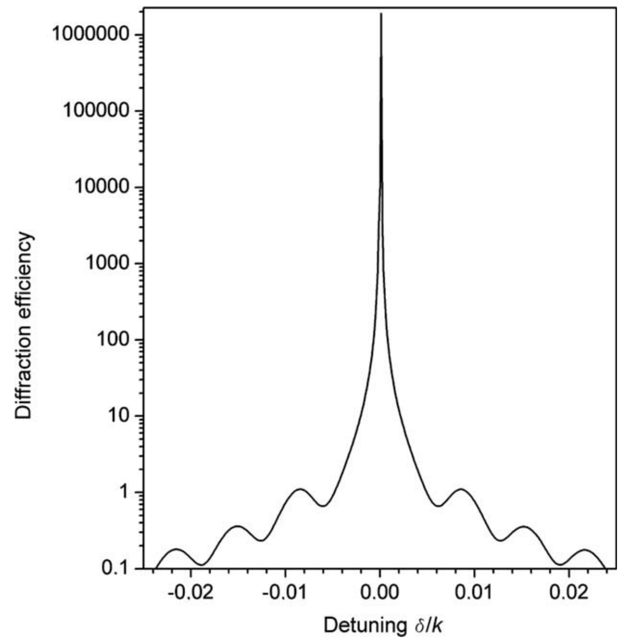

Fig. 4. Calculated pre-oscillations diffraction efficiency contour for a gain grating. The grating parameters are $\alpha/k = 10^{-3}$, $\kappa_n = 0$, $\kappa\alpha = 1.05 \times 10^{-3}$, $d = 1000/k$. Note the logarithmic scale.

Pure gain modulation seems to be preferable for the first-order Bragg DFB oscillator. Due to the absence of the $\pi/2$ phase shift for Bragg diffraction at pure amplitude grating, pre-oscillations peak appears at the center of the grating selectivity contour, as shown in Fig. 4. (Actually, a small detuning is seen in Fig. 4 which is caused by the phase shift due to the uniform gain).

3. Threshold conditions

Self-oscillations (lasing) occur in the grating when the gain (or distributed gain) reaches a threshold value. Even in the absence of an input wave, the diffracted wave will exist. An equation for the threshold determination can be derived from Eq. (15), in the condition of the equality

$$\sigma_1\cosh(\sigma_1 d) = \left(\frac{\alpha k_z}{k} + i\frac{\delta}{2}\right)\sinh(\sigma_1 d), \tag{17}$$

which turns the denominator of Eq. (15) to zero. Assuming $\alpha \ll k$, we write Eq. (11) as

$$\sigma_1 = \left[\left(\frac{\alpha k_z}{k} + i\frac{\delta}{2}\right)^2 + \chi^2\right]^{1/2}. \tag{18}$$

Further, close to the Bragg resonance angle, we can let $\frac{\alpha k_z}{k} = \frac{\alpha K}{2k}$, $\kappa_n = \frac{k^2\Delta n}{nK}$ and $\kappa_\alpha = \frac{k\Delta\alpha}{K}$.

While in general case Eqs. (17) and (18) hardly can be solved analytically, some combinations of the parameters δ, α and χ permit to find easy the threshold dependence between the modulation and the gain.

For instance, pure gain grating starts to oscillate with zero uniform gain and zero detuning at $\kappa_\alpha d = \pi/2$ (this value turns $\sigma_1 d$ to $i\pi/2$). This surprising feature, however, directly follows from Kogelnik's coupled-wave theory (Eq. (67) in Ref. [7]), but was not recognized before.

Implicit solution of Eqs. (17) and (18) (the computation method is described in Ref. [8]) is presented in Fig. 5 in a form of the threshold gain dependence on the modulation factor $\kappa_\alpha d = k\Delta\alpha d/K = \Delta\alpha d/2\sin\theta_B$ for pure gain-modulated grating. It is seen that

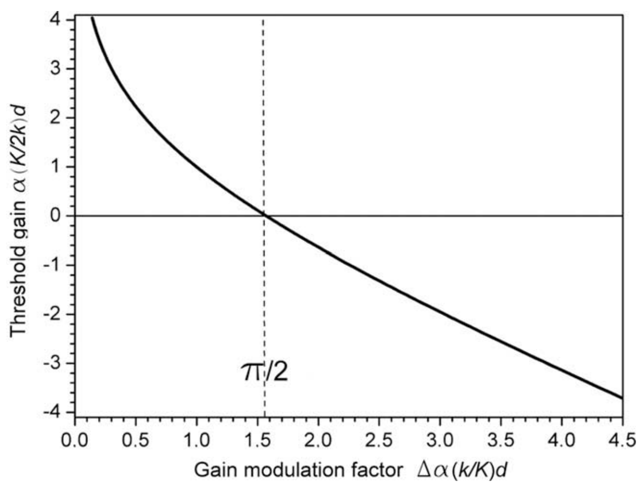

Fig. 5. Calculated threshold gain $\alpha(K/2k)d)$ as a function of the modulation factor $\Delta\alpha d/2\sin\theta_B$ for a pure gain grating.

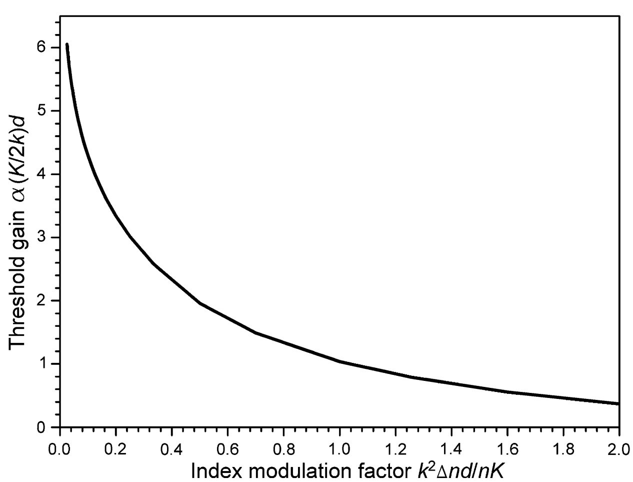

Fig. 6. Calculated threshold gain $\alpha(K/2k)d)$ as a function of the modulation factor $k^2\Delta nd/nK$=$k\Delta nd/2\sin\theta_B$ for a pure phase grating with uniform gain.

even with dominating absorption (negative gain) distributed gain is able to switch on the oscillations. This feature, first emphasized by Kogelnik in Ref [9] appears due to the overlapping of a field which forms a kind of standing wave within the grating with zones of the maximum gain.

To calculate the threshold gain factor in general case, we combine Eqs. (17) and (18) into the "threshold" one:

$$\left[\frac{\sinh(\sigma_1 d)}{\sigma_1 d}\right]^2 = -\frac{1}{(\chi d)^2}. \tag{19}$$

Numerical solution of Eq. (19) for the threshold gain factor in the case of pure phase grating (real χ) with uniform gain is presented in Fig. 6. As seen, the threshold gain exceeds the corresponding value for a pure gain grating (Fig. 5). In the case of a mixed grating, the threshold gain value will appear between these limit dependences.

There is also possibility to reach the threshold for high-order grating modes with the increase of gain [9], however, in the case when DFB structure overcomes the threshold, another physics will rule its behavior.

The role of the waveguide is to select from the whole angular and wavelength spectrum of the gain grating narrow oscillating modes which correspond to the waveguide resonances. An important conclusion which follows from the dependences shown in Figs. 5 and 6 is that for a grating with given parameters d, K, $\Delta\alpha$ or Δn the threshold gain will decrease with the growth of k. This obviously corresponds to the smaller wavelengths and angles θ_B: DFB oscillations will preferably occur as far as possible from z-axis direction. The resulting spatial shape and wavelength are the result of the balance between the master DFB gain, spectral contour of the gain and quality factor of the waveguide modes.

4. Conclusions

The performed coupled-wave analysis of Bragg diffraction in a grating with gain describes the main features of the DFB oscillating modes formation. Pure gain modulation results in the appearance of the oscillation mode exactly at the Bragg resonance center, in contrast to the slightly off-Bragg edge modes in the case of phase grating. We note the results obtained are in agreement with those reported for a one-dimension model [9]. Mixed-type grating has single off-Bragg oscillation mode, which position depends on the ratio of index and gain modulation magnitudes.

The used approach explains the tendency of DFB oscillator to choose higher-order waveguide modes [10] due to the lower threshold gain providing by Bragg diffraction properties.

Another important merit of the used approach is that the Bragg resonances influence only on the z-components of the wave vectors of the coupled waves, remaining the x-component intact, whereas the waveguide resonances are determined by the x-component of the wave vector of the propagating field inside the cavity. This peculiarity permits to describe these two resonance effects separately. In other words, we can consider the waveguide modes for unperturbed waveguide. We note also that the resonance conditions for s and p polarized waveguide modes are slightly different, and therefore we can expect splitting of the oscillation wavelength [10].

The used model can be easy applied to for inspection of a technique for a fine tuning of the oscillation wavelength.

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