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Stabilization of a non-planar optical cavity using its polarization property

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This paper describes new scheme for obtaining a differential signal to lock an optical cavity at a resonance peak. This scheme utilizes a unique property of non-planar cavities; due to an additional phase factor originated from the geometric configuration of the optical path, the two circular polarizations produce two resonance peaks. An ideal signal that crosses zero at the resonance peaks can be obtained using a simple setup consisting of a polarizing beam splitter and photo-diodes. The principle and the results of an experiment are also presented.

1. Introduction

High-finesse optical cavities have been used in various scientific fields. By injecting a laser beam from a laser oscillator into an optical cavity, the beam's intensity can be raised by several orders of magnitude. In the field of accelerator physics, this technique is necessary for developing compact X-ray light sources based on the laser-Compton scattering scheme [1,2].

The cavity's resonance must be maintained at the sharp peak. Various schemes have been developed to obtain a differential signal from the resonance curve that can be used to control the servo system's direction of movement. One widely used scheme is the Pound–Drever–Hall method [3], which introduces frequency side-bands outside the resonance peak and measures the phase shift of the reflected wave. Another scheme, the Tilt-locking method [4] utilizes interference between the fundamental mode and a higher transverse mode to detect the phase shift of the reflected wave. The Hänsch–Couillaud (HC) method [5,6] involves placing a birefringent material inside the cavity in order to make the cavity resonance polarization-dependent and measures the variations in the reflected wave's polarization that result from the phase shift experienced at the resonance. Since linear polarization-dependent property can arise in a multi-mirror cavity system that uses mirrors with an angle, or even in a two-mirror system due to stresses in the mirrors, a variation of HC method can be realized without an additional material [7,8].

A four-mirror ring cavity with a three-dimensional (non-planar) configuration [9] can produce a small spot at a point inside the cavity [10]. When used with laser-Compton X-ray sources, the four-mirror ring cavity provides efficient laser-electron crossing and improved flux performance.

Non-planar optical cavities generally have a circular-polarization dependent property due to the rotation of the image in the three-dimensional optical path [11]. We propose a new method utilizing this property to obtain a differential signal from the cavity resonance. This method is a variation of HC method with circular polarization dependence.

Section 2 of this paper describes our optical cavity. Section 3 presents the proposed scheme and its verification by experiment. Section 4 is for conclusions.

2. Experimental setup

2.1. Schematic of the non-planar cavity

Fig. 1 shows the configuration of our non-planar four-mirror ring cavity. The optical path follows the sides of a symmetrical

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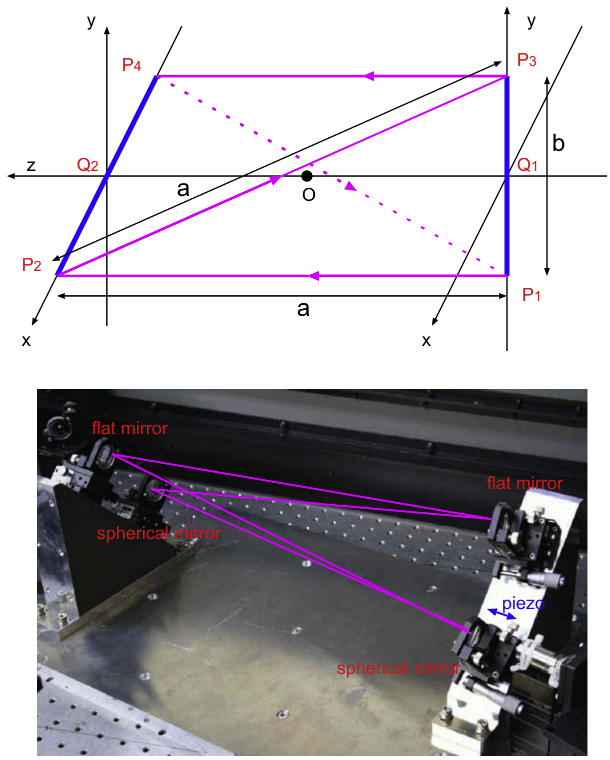

Fig. 1. Schematic of the non-planar optical cavity.

tetrahedron whose surfaces are isosceles triangles with their sides a, a, b. We designed a to be 420 mm and b to be 100 mm. The mirrors located at P_1, P_2, P_3 and P_4 form a closed cavity in this order. The mirrors at P_1 and P_2 are planar, while those at P_3 and P_4 are concave. All of the cavity mirrors used in this experiment have 99% reflectance and 1% transmittance. Q_1 is the center point between P_1 and P_3. Q_2 is the center point between P_2 and P_4.

2.2. Effect of the geometric phase

The uniqueness of the non-planar cavity lies in the degeneration splitting of the resonance of the circular polarizations. In this paper, we calculate the effect for our configuration. A more detailed explanation can be found in [9,11].

For the effect at P_2, unit vectors $\widehat{\boldsymbol{k}_1}$ and $\widehat{\boldsymbol{k}_2}$ describe the rays from P_1 to P_2 and from P_2 to P_3 as follows:

$$\widehat{\boldsymbol{k}_1} = \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|}, \quad \widehat{\boldsymbol{k}_2} = \frac{\overrightarrow{P_2P_3}}{|\overrightarrow{P_2P_3}|}. \tag{1}$$

The normal vectors of the reflection at P_2 and P_3, $\widehat{\boldsymbol{n}_1}$ and $\widehat{\boldsymbol{n}_2}$, are written as follows:

$$\widehat{\boldsymbol{n}_1} = \frac{\overrightarrow{P_2Q_1}}{|\overrightarrow{P_2Q_1}|}, \quad \widehat{\boldsymbol{n}_2} = \frac{\overrightarrow{P_3Q_2}}{|\overrightarrow{P_3Q_2}|}. \tag{2}$$

The plane that includes the input and reflected rays can be defined by the $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{n}}$, and described by $\hat{\boldsymbol{a}}$ as follows:

$$\widehat{\boldsymbol{a}_1} = \frac{\widehat{\boldsymbol{n}_1} \times \widehat{\boldsymbol{k}_1}}{|\widehat{\boldsymbol{n}_1} \times \widehat{\boldsymbol{k}_1}|}, \quad \widehat{\boldsymbol{a}_2} = \frac{\widehat{\boldsymbol{n}_2} \times \widehat{\boldsymbol{k}_2}}{|\widehat{\boldsymbol{n}_2} \times \widehat{\boldsymbol{k}_2}|}. \tag{3}$$

The angle between $\widehat{\boldsymbol{a}_1}$ and $\widehat{\boldsymbol{a}_2}$, α_{12}, is the rotation of the image experienced at one side of the four-mirrors cavity. α_{12} can be calculated from the following equation:

$$\sin \alpha_{12} = \pm|\widehat{\boldsymbol{a}_1} \times \widehat{\boldsymbol{a}_2}|. \tag{4}$$

Since the image rotation at each reflection is cumulative, the overall effect for one complete circuit is $4\alpha_{12}$. The image rotation corresponds to the phase shift in the case of circular-polarization waves. Because the sign of the phase shift for the right- and left-polarization is opposite, it splits the degeneration of the resonance between the two circular polarizations.

For example, $4\alpha_{12}$ is calculated to be -0.0575 rad (The integer of 2π is neglected) in our cavity. $4\alpha_{12}$ is called the geometric phase (ϕ_{geo}) in a later section.

2.3. Measurement of the cavity resonance

We have performed a preliminary experimental test to show the polarization property of our optical cavity. The system layout is depicted in Fig. 2. A single mode *cw* laser (Innolight Prometheus model) was used as the light source. A polarization beam splitter (PBS) determined the polarization of the incoming laser light. The lens pair was placed so that the incoming laser was matched to the cavity eigen mode. The position and the angle at cavity injection was adjusted by a pair of mirrors. One of the cavity mirrors was mounted on a piezo-controlled stage so that the length of the cavity could be varied. The cavity's resonance was determined by measuring the cavity's transmission power with a photo-diode while applying a ramping voltage to the piezo.

Fig. 3 shows the typical signal observed when the injected wave has linear polarization. The resonance peaks have a double peak structure, with one peak corresponds to right-polarization and the other to left-polarization. Because the linear polarization of the injected wave contains equal amounts of the two circular polarizations, the cavity resonates at both of the resonance conditions at slightly different phases.

3. Scheme of the locking technique

3.1. Calculation

3.1.1. Description of the reflection wave

The complex amplitude of the reflection wave, E^r, can be written as follows [12]:

$$\begin{aligned} E^r &= E^i\left[\sqrt{R_1} - \frac{T_1}{\sqrt{R_1}}\frac{Re^{i\delta}}{1 - Re^{i\delta}}\right] \\ &= E^i\left[\sqrt{R_1} - \frac{T_1R}{\sqrt{R_1}}\frac{\cos\delta - R + i\sin\delta}{(1-R)^2 + 4R\sin^2(\delta/2)}\right] \equiv E^iF(\delta), \end{aligned} \tag{5}$$

where E^i is the complex amplitude of the injection wave, R_1 and T_1 are the reflectivity and transmissivity of the entrance mirror. R relates the cavity finesse (F) by $F = \pi/(1-R)$. R can be understood as mirror reflectivity of a two-mirror cavity which has equivalent

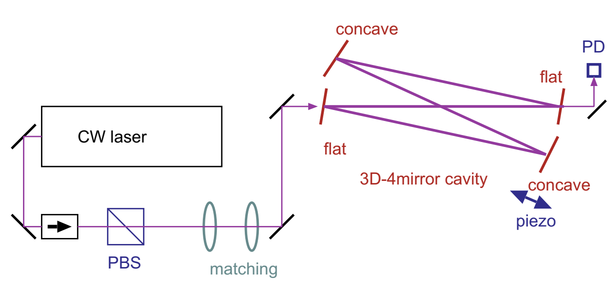

Fig. 2. Setup for the preliminary test.

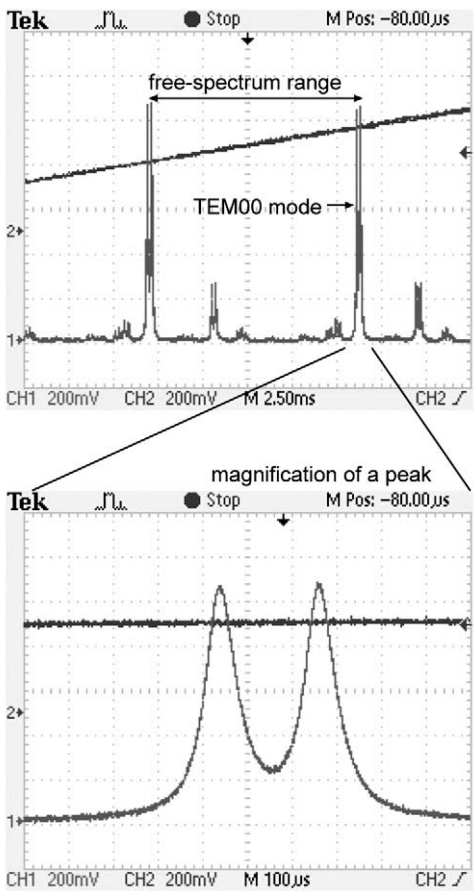

Fig. 3. Observation of the resonance signal. The cavity's transmission power was measured with a photo-diode while scanning the cavity length by the piezo-controlled mirror. The upper figure shows the whole period of a free spectrum range. The highest peaks correspond to the fundamental transverse mode. The bottom figure is a magnification of one of the fundamental mode peaks.

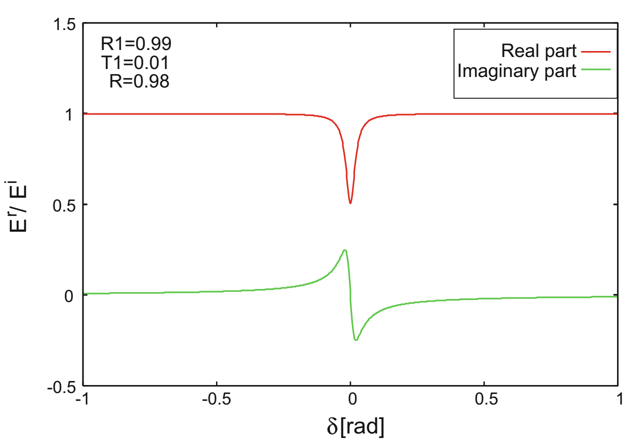

Fig. 4. Amplitude of the reflection wave when $R1 = 0.99, T1 = 0.01$ and $R = 0.98$. The real part and the imaginary part are plotted as a function of δ.

finesse as the present case of four-mirror cavity. δ is the phase difference from the resonance condition.

Fig. 4 shows a plot of this function when $R_1 = 0.99, T_1 = 0.01$ and $R = 0.98$.

3.1.2. Description of the polarization

Using Jones matrix, polarization of the wave can be described as follows [12]:

$$\boldsymbol{E} = \begin{pmatrix} E_p \\ E_s \end{pmatrix}. \tag{6}$$

Each component of the vector denotes p-polarization and s-polarization. The electric field is parallel and perpendicular to the table. If the wave is split with a PBS placed horizontally on the table, the two components can be measured separately.

The circular-polarization waves are described as follows:

$$\widehat{\boldsymbol{E_R}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \widehat{\boldsymbol{E_L}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \tag{7}$$

where $\widehat{\boldsymbol{E_R}}$ and $\widehat{\boldsymbol{E_L}}$ are the right- and left-polarization of the unit amplitude. Since the eigen states of a non-planar cavity corresponds to circular polarizations, the injection wave can be conveniently described as a superposition of $\widehat{\boldsymbol{E_R}}$ and $\widehat{\boldsymbol{E_L}}$.

A linear polarization that is rotated in 45° relative to the table surface, $\widehat{\boldsymbol{E_{45}}}$, can be described as follows:

$$\widehat{\boldsymbol{E_{45}}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1+i}{2}\widehat{\boldsymbol{E_R}} + \frac{1-i}{2}\widehat{\boldsymbol{E_L}}. \tag{8}$$

3.2. Scheme of the proposed system

As explained in Section 2.2, due to the additional geometric phase, the resonance conditions of the right- and left- polarization waves shift in the opposite sign. The reflection wave of each component can be written as follows, with ϕ_{geo} as the additional geometric phase:

$$\boldsymbol{E^r_R} = E^r_R\widehat{\boldsymbol{E_R}} = E^i_R F(\delta - \phi_{geo})\widehat{\boldsymbol{E_R}}, \tag{9}$$

$$\boldsymbol{E^r_L} = E^r_L\widehat{\boldsymbol{E_L}} = E^i_L F(\delta + \phi_{geo})\widehat{\boldsymbol{E_L}}. \tag{10}$$

When Eq. (8) is the injected wave, its right and left polarization components independently interact with the cavity. The resultant reflection wave of the superposition can be described as follows:

$$\begin{aligned} \boldsymbol{E^r} &= \boldsymbol{E^r_R} + \boldsymbol{E^r_L} = \frac{1+i}{2}F(\delta - \phi_{geo})\widehat{\boldsymbol{E_R}} + \frac{1-i}{2}F(\delta + \phi_{geo})\widehat{\boldsymbol{E_L}} \\ &= \begin{pmatrix} \frac{1+i}{2\sqrt{2}}F(\delta - \phi_{geo}) + \frac{1-i}{2\sqrt{2}}F(\delta + \phi_{geo}) \\ \frac{-i(1+i)}{2\sqrt{2}}F(\delta - \phi_{geo}) + \frac{i(1-i)}{2\sqrt{2}}F(\delta + \phi_{geo}) \end{pmatrix} = \begin{pmatrix} E^r_p \\ E^r_s \end{pmatrix}. \end{aligned} \tag{11}$$

$|E^r_s|^2 - |E^r_p|^2$ is the differential signal for locking the optical cavity proposed in this paper.

Fig. 5 shows an example calculation when $R_1 = 0.99$, $T_1 = 0.01, R = 0.98$, and $\phi_{geo} = -0.0575$ rad. This corresponds to

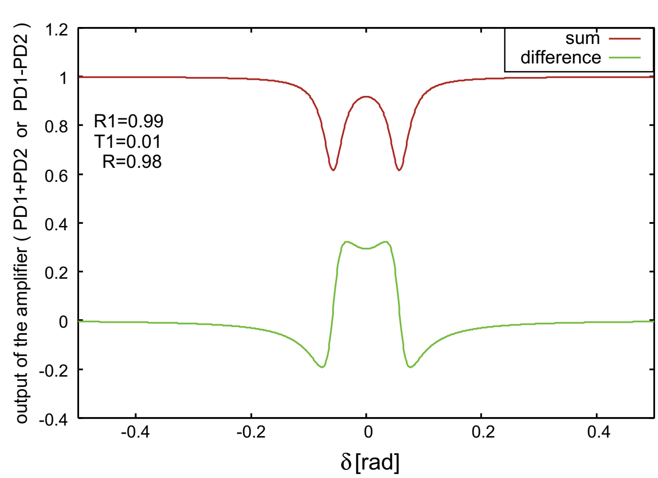

Fig. 5. An example calculation of the differential signal for our test cavity, with plots for $|E_s|^2 - |E_p|^2$ (difference) and $|E_s|^2 + |E_p|^2$ (sum).

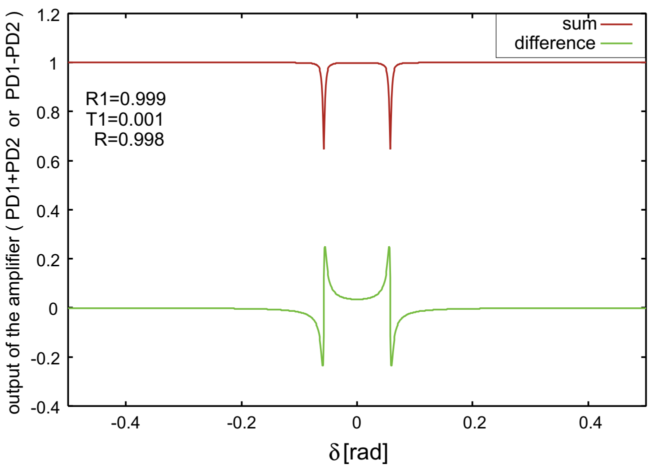

Fig. 6. An example calculation of the differential signal for a higher finesse, with plots for $|E_s|^2 - |E_p|^2$ (difference) and $|E_s|^2 + |E_p|^2$ (sum).

the configuration of our test cavity. A calculation for a case of higher finesse is shown in Fig. 6. The parameters are $R_1 = 0.999, T_1 = 0.001, R = 0.998$, and $\phi_{geo} = -0.0575$ rad. The signal crosses zero at the peaks of the resonances. By choosing the sign of the signal slope, it is possible to lock the system at one of the circular-polarization resonance.

3.3. Experiment

To confirm the calculation, we conducted an experiment using the setup shown in Fig. 7. A linear polarization wave was injected to the cavity. The reflection wave from the entrance mirror was guided to the detection system consisting of a PBS and two photo-diodes monitoring each output of the splitter. PD1 monitored the intensity of p-polarization component ($|E_p|^2$) as the basis of the PBS, while PD2 monitored the intensity of s-polarization component ($|E_s|^2$). The signals from the photo-diodes were fed to the differential amplifier and output as $|E_s|^2 - |E_p|^2$. The $\lambda/2$ retarder at the input of the detection system defined the relative plane of the incoming beam's polarization and that of the detection system. The angle of the $\lambda/2$ retarder was adjusted to balance the outputs of the two photo-diodes when the cavity was far from the resonances. This situation corresponds that the incoming beam at the input of the PBS is described as Eq. (8). Cavity resonance was

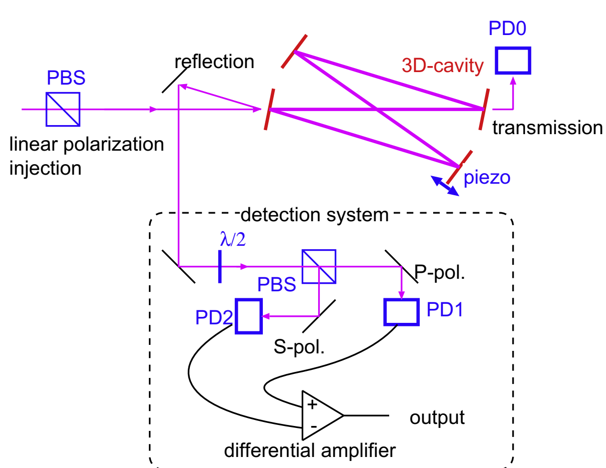

Fig. 7. Setup of the differential signal measurement.

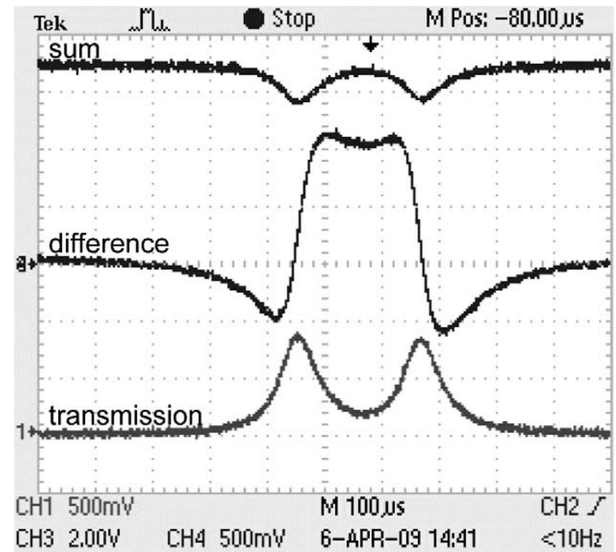

Fig. 8. Signal observed while scanning the cavity length. The middle line is the output of the differential amplifier and the bottom line is the output of the PD0.

monitored using a photo-diode, PD0, to measure the cavity's transmission.

We measured the output of the differential amplifier while scanning the cavity length using the piezo-controlled mirror. The signal observed near the cavity's point of resonance is shown in Fig. 8. The bottom line is the signal of the photo-diode that measured the transmission; it shows the point that the cavity's resonance. The middle line is the output of the differential amplifier. The signal shape agrees well with the result calculated in Fig. 5. The signal crossed zero at the point of resonance and showed a different sign in the vicinity of each circular-polarization peak; it provides a good differential signal for locking the cavity to one of the peaks of resonance.

4. Conclusions

We describe a new method for obtaining a differential signal that can be used to lock an optical cavity at a resonance peak. This method is a variation of the HC method, but does not require additional material to be inserted in the cavity. Combined with the unique polarization property of non-planar cavities, the phase shift at the peak of resonance appears as a polarization variation of the reflected wave. Theoretical calculations and the results of an experiment are shown. The non-planar four-mirror optical cavity can achieve the small spot that is required in the field of accelerator physics for an optical cavity to be used in a laser-Compton X-ray source.

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