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## The relation between the boundary diffraction wave theory and physical optics

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## ABSTRACT

The physical optics surface integral is asymptotically reduced to a line integral along the contour of the diffracting edge. It is shown that the resultant integral can be separated into two sub-integrals which represent the reflected and transmitted diffracted fields. The integrands are transformed into the same forms with the potential function of the boundary diffraction wave theory.

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## 1. Introduction

Diffraction is an important aspect of light besides reflection and refraction. This feature has been observed and investigated for nearly 350 years. In spite of the rigorous solutions and asymptotic methods that deal with diffraction, there have been efforts that try to enlighten the true nature of the phenomena [1,2]. An interference scheme is observed theoretically and experimentally, but there does not appear to be a consensus on the structure of the interfering fields. According to the decomposition of the Fresnel function, there seems to be three different possibilities for the fields that interfere on the observation plane. This subject has important applications in electromagnetics [3], optics [4] and also quantum mechanics [5].

The first qualitative explanation of diffraction was suggested by Young [6,7], who thought that the scattered field by an edge discontinuity was composed of two sub-fields. The first field is the geometrical optics (GO) wave that passes through the aperture without obstruction. The second field is the edge diffracted wave which originates from the edge discontinuity. The total field is the interference of these two sub-fields. Fresnel's quantitative theory, which was the mathematical application of the Huygen's principle, dominated Young's ideas [8] in the community of science. It was the work of Rubinowicz [9] that supplied the theoretical basis to the proposal of Young. He rigorously managed to reduce the dif-

fraction integral of Kirchhoff into a line integral along the edge contour of the scatterer. Although Maggi [10] had performed a similar reduction, his resultant line integral was not in a form that can provide a physical interpretation. Since Rubinowicz's method was valid for spherical and plane waves, the following endeavors were focused on the generalization of the boundary diffraction wave (BDW) theory. Miyamoto and Wolf [11,12] managed to show that a potential function can always be found for more general rays. The final development of the BDW theory was completed by Rubinowicz [13]. The method was widely used by the community of optics especially for the diffraction of Gaussian beams by apertures and half-planes [14–16]. The theory of BDW was applied to the problem of diffraction by a half-plane, the rigorous solution [17] of which was well known in the literature, by Ganci [18,19]. This study showed that the method of BDW was only leading to approximate results that were not equal to the rigorous field expressions. The potential functions which were leading to the exact diffracted waves was recently developed by Umul [20] based on the modified theory of physical optics (MTPO) [21].

The method of physical optics (PO) is an important tool in the analysis of high frequency electromagnetic scattering [22]. Besides its advantages, the defect of PO is the incorrect diffracted waves that are evaluated asymptotically from the PO integrals. The physical theory of diffraction (PTD) [23,24] was developed by Ufimtsev in order to obtain integrals that will yield to the exact field expressions. With this aim he introduced the fringe currents which are evaluated from the difference of the PO and rigorous fields that are found by the solution of the canonical diffraction problems.

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But PTD needed the exact solutions of some fundamental problems in order to construct the fringe currents. Umul [21] developed a consistent method which was leading to the exact diffracted waves for conducting geometries. This method, which is named as MTPO, removes the defect of PO by defining three axioms which does not require the knowledge of the solutions of canonical problems.

It is the aim of this paper is to investigate the connection between the BDW theory and PO. The fundamental problem of diffraction of plane waves by a conducting half-plane will be taken into account. The PO scattering integral will be constructed by using the diffraction theory of Kirchhoff for Dirichlet and Neumann boundary conditions. The surface integral will be reduced to a line integral by using the method of asymptotic reduction [25,26]. The resultant integrand will be transformed into the potential function of BDW by using trigonometric identities. Such an investigation is important in order to put forward the reason of the defects of BDW and PO theories by showing that the two methods have the same nature. There are also papers which use the methods of PO [27] and BDW [28] in literature and these studies can be improved by considering the mathematical and physical insights of these theories. Another originality of this paper is the evaluation of the potential function for the reflected diffracted waves which does not exist in the literature to our knowledge.

A time factor of  $\exp(j\omega t)$  will be considered and suppressed throughout the paper.

## 2. Theory

The diffraction theory of Kirchhoff [4] relies on the surface integration of the fields in an aperture. The integral can be given by

$$u_s = \frac{1}{4\pi} \int \int_S (u \nabla G - G \nabla u) \cdot \vec{n} \, dS \quad (1)$$

where  $u$  is the total field on the surface.  $G$  expresses the Green's function.  $\vec{n}$  is the unit normal vector of the surface. Eq. (1) can be rewritten as

$$u_s = \frac{1}{4\pi} \int \int_S \left( u \frac{\partial G}{\partial n} - G \frac{\partial u}{\partial n} \right) dS \quad (2)$$

by multiplying the normal vector with the gradients. Eq. (2) will be considered for two boundary conditions. These are the Dirichlet (soft surface)

$$u = 0 \quad (3)$$

and Neumann (hard surface)

$$\frac{\partial u}{\partial n} = 0 \quad (4)$$

conditions. The scattered field can be expressed as

$$u_{s1} = -\frac{1}{4\pi} \int \int_S G \frac{\partial u}{\partial n} \, dS \quad (5)$$

and

$$u_{s2} = \frac{1}{4\pi} \int \int_S u \frac{\partial G}{\partial n} \, dS \quad (6)$$

for soft and hard surfaces, respectively. The surface integrals, in Eqs. (5) and (6), will be reduced to line integrals along the contour of the diffracting edge by using the method of asymptotic reduction. The scattering surface will be considered to be a half-plane, placed at  $S = \{(x, y, x); x \in (0, \infty), y = 0, z \in (-\infty, \infty)\}$ . A plane wave of

$$u_i = u_0 \exp[jk\rho \cos(\phi - \phi_0)] \quad (7)$$

is illuminating the surface. The geometry of the problem is given in Fig. 1.  $\beta$  and  $\eta$  are the modified angles of scattering [21].  $C$  is the edge contour of the half-plane.  $z$  and  $z'$  are the projection of the observa-

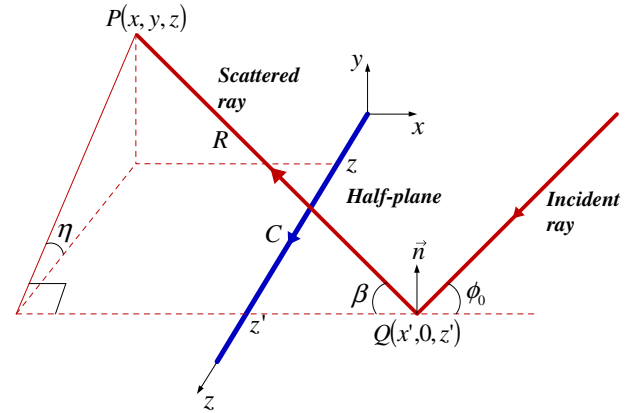


Fig. 1. Geometry of the half-plane.

tion and integration points on the edge, respectively.  $(\rho, \phi)$ , in Eq. (7), are the polar coordinates.  $\phi_0$  is the angle of incidence.  $Q$  and  $P$  are the integration and observation points.

### 2.1. Soft surface

The integral, in Eq. (5) is taken into account. The PO [24] approximation can be defined as

$$\frac{\partial u}{\partial n} \approx 2 \frac{\partial u_i}{\partial n} \quad (8)$$

The unit normal vector is equal to  $\vec{e}_y$ . Eq. (8) reads

$$\frac{\partial u}{\partial n} \approx 2jk u_0 \sin \phi_0 \exp(jkx' \cos \phi_0) \quad (9)$$

according to the PO approximation.  $x'$  is used instead of  $x$  since the integral is written on the surface of the half-plane ( $y' = 0$ ). The PO integral can be written as

$$u_{s1} = -\frac{jk u_0 \sin \phi_0}{2\pi} \int_{x'=0}^{\infty} \int_C e^{jkx' \cos \phi_0} \frac{e^{-jkR}}{R} \, dx' \, dl \quad (10)$$

for  $C$  represents the edge contour of  $z' \in (-\infty, \infty)$ . The  $x'$  part of the surface integral, in Eq. (10), can be reduced to a line integral by using the technique of asymptotic reduction [20]. The edge point contribution of an integral gives

$$\int_{\alpha_e}^{\infty} f(\alpha) e^{jk g(\alpha)} \, d\alpha \approx \frac{1}{jk} \frac{f(\alpha_e)}{g'(\alpha_e)} e^{jk g(\alpha_e)} \quad (11)$$

for sufficiently large value of  $k$ .  $f(\alpha)$  and  $g(\alpha)$  are the amplitude and phase functions of the integral, respectively.  $\alpha_e$  is the edge point of the integral. The phase function of the integral, in Eq. (10), is

$$g(x', z') = x' \cos \phi_0 - R \quad (12)$$

where  $R$  is equal to

$$R = \sqrt{(x - x')^2 + y^2 + (z - z')^2} \quad (13)$$

The first derivative of the phase function gives

$$\frac{\partial g}{\partial x'} = \cos \phi_0 + \frac{x - x'}{R} \quad (14)$$

which is equal to

$$\frac{\partial g}{\partial x'} = \cos \phi_0 - \cos \beta \quad (15)$$

according to the geometry, in Fig. 1. As a result one obtains

$$u_{d1} = -\frac{u_0}{2\pi} \int_C \frac{\sin \phi_0}{\cos \phi_0 - \cos \beta_e} \frac{e^{-jkR_e}}{R_e} \, dl \quad (16)$$

by applying Eq. (11) to Eq. (10).  $u_{d1}$  is the diffracted wave on the edge contour of  $C$  whereas  $u_{s1}$  is the scattered wave, which contains both of the diffracted and GO waves as  $u_{d1} + u_{G01}$ .  $\beta_e$  is the value of  $\beta$  at  $x' = 0$ .  $R_e$  is equal to

$$R_e = \sqrt{\rho^2 + (z - z')^2} \quad (17)$$

for  $\rho$  is  $\sqrt{x^2 + y^2}$ .  $R_e$  is the value of  $R$  on the edge contour of  $C$ . We will consider the term of

$$I = \frac{\sin \phi_0}{\cos \phi_0 - \cos \beta_e} \quad (18)$$

and show that it directly gives the potential function of the BDW theory as a sum of the transmitted and incident diffracted waves. Eq. (18) can be rewritten as

$$I = \frac{\sin \phi_0 \sin \left(\frac{\beta_e - \phi_0}{2}\right) \sin \left(\frac{\beta_e + \phi_0}{2}\right)}{2 \sin^2 \left(\frac{\beta_e - \phi_0}{2}\right) \sin^2 \left(\frac{\beta_e + \phi_0}{2}\right)} \quad (19)$$

which also yields the equation of

$$I = 2 \frac{\sin \phi_0 \sin \left(\frac{\beta_e - \phi_0}{2}\right) \sin \left(\frac{\beta_e + \phi_0}{2}\right)}{[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]}. \quad (20)$$

Eq. (20) gives

$$I = - \frac{\sin \phi_0 (\cos \beta_e - \cos \phi_0)}{[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]}. \quad (21)$$

Eq. (21) can be arranged as

$$I = - \frac{2 \sin \phi_0 \cos \beta_e - 2 \sin \phi_0 \cos \phi_0}{2[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]} \quad (22)$$

which leads to the equation of

$$I = - \frac{\sin(\phi_0 + \beta_e) - \sin(\beta_e - \phi_0) - \sin 2\phi_0}{2[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]}. \quad (23)$$

The term of  $\sin 2\phi_0$  can be expressed as

$$\sin 2\phi_0 = -\sin(\beta_e - \phi_0 - \beta_e - \phi_0). \quad (24)$$

Eq. (23) can be rewritten as

$$I = - \frac{\sin(\beta_e + \phi_0)[1 - \cos(\beta_e - \phi_0)] - \sin(\beta_e - \phi_0)[1 - \cos(\beta_e + \phi_0)]}{2[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]} \quad (25)$$

when Eq. (24) is taken into account. As a result one obtains

$$I = \frac{1}{2} \left[ \frac{\sin(\beta_e - \phi_0)}{1 - \cos(\beta_e - \phi_0)} - \frac{\sin(\beta_e + \phi_0)}{1 - \cos(\beta_e + \phi_0)} \right]. \quad (26)$$

The line integral of edge diffraction can be found as

$$u_{d1} = - \frac{u_0}{4\pi} \int_C \frac{\sin(\beta_e - \phi_0)}{1 - \cos(\beta_e - \phi_0)} \frac{e^{-jkR_e}}{R_e} dl + \frac{u_0}{4\pi} \int_C \frac{\sin(\beta_e + \phi_0)}{1 - \cos(\beta_e + \phi_0)} \frac{e^{-jkR_e}}{R_e} dl \quad (27)$$

when Eq. (26) is used in Eq. (16). The first integral represents the reflected diffracted waves whereas the second integral gives the

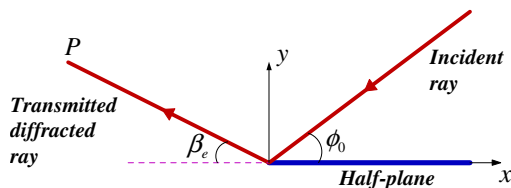


Fig. 2. Edge diffraction of the incident wave.

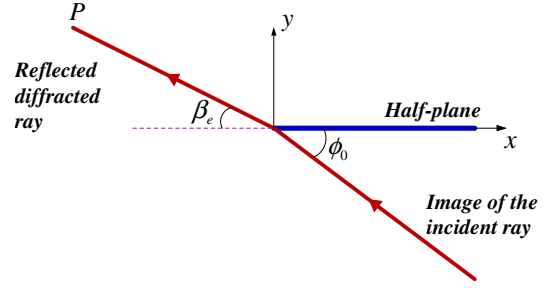


Fig. 3. Edge diffraction of the reflected wave.

transmitted diffracted fields. In order to show that Eq. (27) expresses the BDW theory, we will consider Figs. 2 and 3.

Fig. 2 shows the incident wave and the transmitted diffracted field. Fig. 3 represents a similar geometry for the reflected GO and reflected diffracted waves. The unit vectors of

$$\vec{s}_i = -\cos \phi_0 \vec{e}_x - \sin \phi_0 \vec{e}_y \quad (28)$$

and

$$\vec{s}_{td} = -\cos \beta_e \vec{e}_x + \sin \beta_e \vec{e}_y \quad (29)$$

can be defined for the incident and diffracted waves. The vector and scalar products of these vectors give

$$\vec{s}_i \times \vec{s}_{td} = -\sin(\beta_e + \phi_0) \vec{e}_z \quad (30)$$

and

$$\vec{s}_i \cdot \vec{s}_{td} = \cos(\beta_e + \phi_0), \quad (31)$$

respectively. Similar unit vectors can be defined as

$$\vec{s}_r = -\cos \phi_0 \vec{e}_x + \sin \phi_0 \vec{e}_y \quad (32)$$

and

$$\vec{s}_{rd} = -\cos \beta_e \vec{e}_x + \sin \beta_e \vec{e}_y \quad (33)$$

for the reflected GO and reflected diffracted waves. The vector and scalar products of the vectors, in Eqs. (32) and (33), give

$$\vec{s}_r \times \vec{s}_{rd} = -\sin(\beta_e - \phi_0) \vec{e}_z \quad (34)$$

and

$$\vec{s}_r \cdot \vec{s}_{rd} = \cos(\beta_e - \phi_0). \quad (35)$$

Eq. (27) can be rewritten as

$$u_{d1} = \frac{u_0}{4\pi} \int_C \frac{\vec{s}_r \times \vec{s}_{rd}}{1 - \vec{s}_r \cdot \vec{s}_{rd}} \frac{e^{-jkR_e}}{R_e} dl - \frac{u_0}{4\pi} \int_C \frac{\vec{s}_i \times \vec{s}_{td}}{1 - \vec{s}_i \cdot \vec{s}_{td}} \frac{e^{-jkR_e}}{R_e} dl \quad (36)$$

when Eqs.(30), (31), (34) and (35) are used in Eq. (27). It is apparent that the integrals are the same with the ones which are defined with the vector potential of the BDW theory [11–13]. This result also shows that the PO integral [29] contains the two edge diffracted fields (reflected diffracted and transmitted diffracted).

## 2.2. Hard surface

The integral, in Eq. (6), is valid for this case. The PO [24] approximation can be applied as

$$u \approx 2u_i \quad (37)$$

for a hard surface. The normal derivative of the Green's function gives

$$\frac{\partial G}{\partial n} \approx -jk \sin \beta \frac{e^{-jkR}}{R}. \quad (38)$$

The scattering integral can be written as

$$u_{s2} = -\frac{jku_0}{2\pi} \int_{x'=0}^{\infty} \int_C \sin \beta e^{jkx' \cos \phi_0} \frac{e^{-jkR}}{R} dx' dl \quad (39)$$

by considering Eqs. (6), (37) and (38). The line integral of diffraction can be obtained as

$$u_{d1} = -\frac{u_0}{2\pi} \int_C \frac{\sin \beta_e}{\cos \phi_0 - \cos \beta_e} \frac{e^{-jkR_e}}{R_e} dl \quad (40)$$

when Eq. (11) is taken into account. We will work on the term of

$$M = \frac{\sin \beta_e}{\cos \phi_0 - \cos \beta_e}. \quad (41)$$

$M$  can be rewritten as

$$M = \frac{\sin \beta_e \sin \left(\frac{\beta_e - \phi_0}{2}\right) \sin \left(\frac{\beta_e + \phi_0}{2}\right)}{2 \sin^2 \left(\frac{\beta_e - \phi_0}{2}\right) \sin^2 \left(\frac{\beta_e + \phi_0}{2}\right)} \quad (42)$$

which can be reduced to

$$M = -\frac{\sin \beta_e (\cos \beta_e - \cos \phi_0)}{[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]}. \quad (43)$$

Eq. (43) can be arranged as

$$M = -\frac{\sin 2\beta_e - \sin(\beta_e + \phi_0) - \sin(\beta_e - \phi_0)}{2[1 - \cos(\beta_e - \phi_0)][1 - \cos(\beta_e + \phi_0)]} \quad (44)$$

which yields the equation of

$$M = \frac{1}{2} \left[ \frac{\sin(\beta_e - \phi_0)}{1 - \cos(\beta_e - \phi_0)} + \frac{\sin(\beta_e + \phi_0)}{1 - \cos(\beta_e + \phi_0)} \right]. \quad (45)$$

The line integral of diffracted fields can be written as

$$u_{d2} = -\frac{u_0}{4\pi} \int_C \frac{\sin(\beta_e - \phi_0)}{1 - \cos(\beta_e - \phi_0)} \frac{e^{-jkR_e}}{R_e} dl - \frac{u_0}{4\pi} \int_C \frac{\sin(\beta_e + \phi_0)}{1 - \cos(\beta_e + \phi_0)} \frac{e^{-jkR_e}}{R_e} dl \quad (46)$$

which can also be expressed by

$$u_{d2} = \frac{u_0}{4\pi} \int_C \frac{\vec{s}_r \times \vec{s}_{rd}}{1 - \vec{s}_r \cdot \vec{s}_{rd}} \frac{e^{-jkR_e}}{R_e} dl + \frac{u_0}{4\pi} \int_C \frac{\vec{s}_i \times \vec{s}_{td}}{1 - \vec{s}_i \cdot \vec{s}_{td}} \frac{e^{-jkR_e}}{R_e} dl \quad (47)$$

in terms of the BDW theory. This result is also in harmony with the one, found for the soft surface. This analysis also gives mathematical insight about the error of the BDW theory. These points will be discussed in the conclusion.

### 3. Evaluation of the diffraction integrals

In this section, we will evaluate the integrals, given by Eqs. (27) and (46), asymptotically. Since the  $z'$  part of the integrals change in the interval of  $z' \in (-\infty, \infty)$ , the integrals can be evaluated directly by using the method of the stationary phase for  $k \gg 1$ . A general formula can be given by

$$\int_{-\infty}^{\infty} f(\alpha) e^{jk g(\alpha)} d\alpha \approx e^{j \text{sign}[g''(\alpha_s)] \frac{\pi}{4}} \sqrt{2\pi} \frac{f(\alpha_s)}{\sqrt{|g''(\alpha_s)|}} e^{jk g(\alpha_s)} \quad (48)$$

for the method of stationary phase.  $\alpha_s$  is the stationary phase point which is found by equating the first derivative of the phase function to zero. The phase function of the related integrals is

$$g(z') = \sqrt{\rho^2 + (z - z')^2}. \quad (49)$$

The stationary phase point can be found to be  $z_s = z$ .  $\beta_e$  is equal to  $\pi - \phi$  at the stationary phase point. As a result the diffracted fields can be evaluated as

$$u_{d1} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \left[ \frac{\sin(\phi - \phi_0)}{1 + \cos(\phi - \phi_0)} - \frac{\sin(\phi + \phi_0)}{1 + \cos(\phi + \phi_0)} \right] \quad (50)$$

and

$$u_{d2} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \left[ \frac{\sin(\phi - \phi_0)}{1 + \cos(\phi - \phi_0)} + \frac{\sin(\phi + \phi_0)}{1 + \cos(\phi + \phi_0)} \right] \quad (51)$$

for the soft and hard surfaces, respectively. It is apparent that the diffracted fields approach to infinity at the transition regions. There are two transition regions for this problem. The first one is the reflection boundary which can be found from the equation of  $1 + \cos(\phi + \phi_0) = 0$ . This equation gives the pole of the reflected diffracted field, which has the expression of

$$u_{rd} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{\sin(\phi + \phi_0)}{1 + \cos(\phi + \phi_0)}. \quad (52)$$

The geometrical place of the reflection boundary is at  $\phi = \pi - \phi_0$ . The reflected GO field is discontinuous at this point and the edge diffracted wave compensates this discontinuity.

The second transition region is the shadow boundary where the incident GO field has a discontinuity. The placement of the shadow boundary can be evaluated from the equation of  $1 + \cos(\phi - \phi_0) = 0$ . This equation gives the pole of the transmitted diffracted wave which has the representation of

$$u_{td} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{\sin(\phi - \phi_0)}{1 + \cos(\phi - \phi_0)}. \quad (53)$$

The infinities of the diffracted waves can be compensated by using the uniform theories of diffraction [30,31]. In this paper we will prefer the method, given in Ref. 31, because of its simplicity. The uniform diffracted fields can be defined by

$$u_{rd} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{f_+ \sin(\phi + \phi_0)}{1 + \cos(\phi + \phi_0)} \quad (54)$$

and

$$u_{td} = \frac{u_0 e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{f_- \sin(\phi - \phi_0)}{1 + \cos(\phi - \phi_0)} \quad (55)$$

where  $f_{\mp}$  is equal to

$$f_{\mp} = p_{\pm} \left( 1 - e^{-\sqrt{2\pi k \rho} |\cos \frac{\phi \pm \phi_0}{2}|} \right). \quad (56)$$

$p_{\pm}$  can be defined by

$$p_{\pm} = \exp\{j(\pi/4) \exp[-(|\phi - (\pi \pm \phi_0)|)]\}. \quad (57)$$

Although the field expressions, in Eqs. (54) and (55), are uniform, they do not represent the exact diffracted waves as in PO.

### 4. Numerical analysis

In this section, the diffracted waves, given by Eqs. (54) and (55), will be plotted numerically in order to investigate the behavior of the fields. The comparison of the BDW fields with the exact waves that are found from MTPO can be found in Ref. [20]. The angle of incidence is taken as  $60^\circ$ .  $\rho$  is equal to  $6\lambda$  for  $\lambda$  is the wave-length.

Fig. 4 plots the variation of the reflected diffracted wave versus the observation angle. There are two important points to be mentioned on the graphic. The first one is the place of the reflection boundary. It gives the correct value of  $120^\circ$  and the diffracted field is equal to 0.5 at this point. This is an expected value since the diffracted wave compensates the GO wave at the reflection boundary. The second important point is at  $\phi = 300^\circ$ . The value of the diffracted wave is equal to zero since the image of the incident wave hits the edge at this point. Fig. 5 presents similar values for the incident diffracted field. The shadow boundary is placed at  $\phi = 240^\circ$  and the field is equal to zero at  $\phi = 60^\circ$  where the incident wave hits the edge.

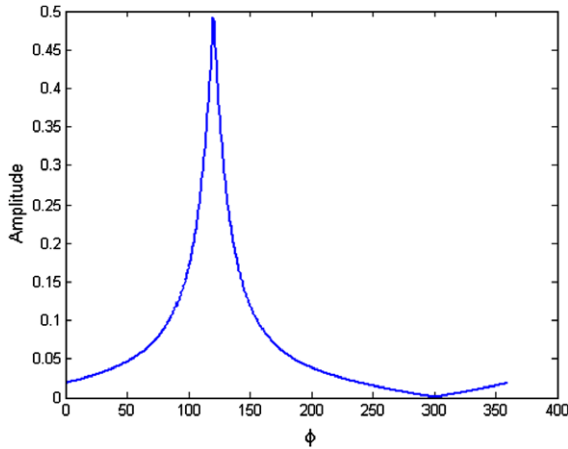


Fig. 4. Reflected diffracted field.

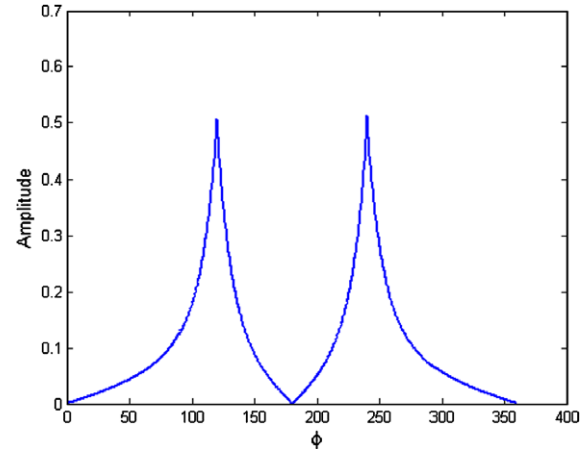


Fig. 7. Total diffracted field (hard surface).

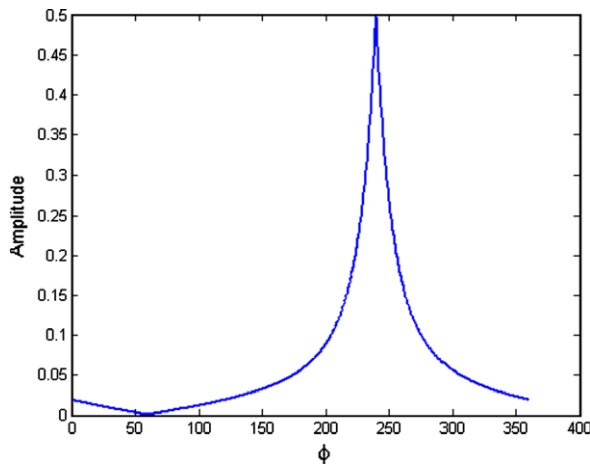


Fig. 5. Transmitted diffracted field.

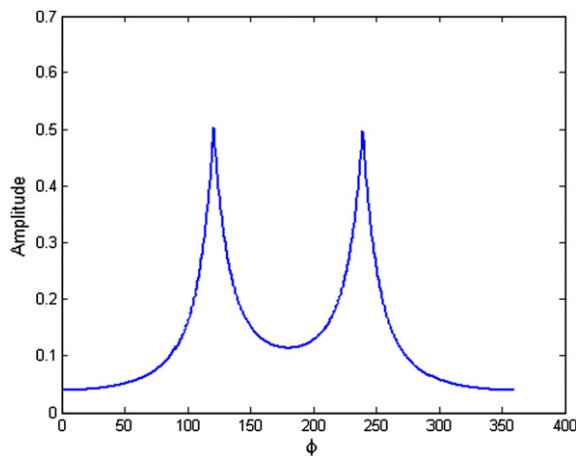


Fig. 6. Total diffracted field (soft surface).

Figs. 6 and 7 show the variation of the total diffracted fields with respect to the observation angle for soft and hard surfaces. It can be observed that the total diffracted waves do not satisfy the boundary conditions on the surfaces of the half-plane. These plots are well known in the literature [32].

## 5. Conclusion

In this paper we obtained the line integrals of the BDW theory by using the asymptotic reduction of the PO surface integrals. This analysis puts forward two important points. The first point is the equivalence of the PO and BDW methods. The second one is that the asymptotic reduction of surface integrals gives the same result with the Rubinowicz's method [9,21]. As mentioned earlier, the methods of PO and BDW yields incorrect diffraction coefficients, but the analysis, performed in this paper, is unique gives insights for the in depth investigation of the MTPO method, which gives the exact diffracted fields for conducting bodies.

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