Collective excitation of quantum wires and effect of spin-orbit coupling in the presence of a magnetic field along the wire

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The band structure of a quantum wire with the Rashba spin-orbit coupling develops a pseudogap in the presence of a magnetic field along the wire. In such a system spin mixing at the Fermi wave vectors $-k_F$ and k_F can be different. We have investigated, using bosonization techniques, the collective mode of this system, and found that the velocity of this collective excitation depends sensitively on the strength of the Rashba spin-orbit interaction and magnetic field. Our result suggests that the strength of the spin-orbit interaction can be determined from the measurement of the velocity.

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I. INTRODUCTION

Recently active research is taking place on how to manipulate spin properties of single electrons, and several semiconductor spin devices based on spin-orbit coupling have been proposed.[1,2] Among these we focus on a spin filter[3] proposed by Středa and Šeba.[4] They proposed a spin filter combining strong Rashba spin-orbit interaction (SOI) and the magnetic field *parallel* to a quantum wire (see Fig. 1). This system has an interesting one-dimensional band structure (see Fig. 2): a pseudogap is present at zero wave number and the orientation of electron spin depends on the wave vector.[4] For the lower band the electron with sufficiently negative k is mostly polarized in the $+z$ direction while that of sufficiently positive k is mostly polarized in the $-z$ direction. When the Fermi energy lies in the pseudogap substantial spin-mixing exists for moderate value of the Fermi energy. The transmission/reflection coefficients of such a wire in the presence of a step potential has been calculated in the presence of electron-electron interaction using poor man's renormalization group approach.[5] Optical and superconductor junction properties have been also investigated in the presence of the Rashba term.[5]

The dispersion of the collective mode of quantum wires in the presence of a magnetic field perpendicular to the wire has been investigated for many years.[6] Recently the interplay of Rashba SOI and electron-electron interaction in quantum wires have been studied by several groups.[7,8] However, none of these studies have dealt with the case where the applied magnetic field is parallel to quantum wire in the presence of spin orbit interaction. In this paper we investigate how the collective electronic properties may be manipulated by spin-orbit coupling. In II-VI semiconductors the Rashba term is expected to be larger than the the Dresselhaus coupling. In III-V semiconductors, such as GaAs, the opposite is true.[1] However, in these quantum wires the Dresselhaus term can be rather small under certain conditions as we argue below. The band structure of such quantum wires in the presence of a parallel magnetic field is as displayed in Fig. 2. The nature of the collective mode is unclear when the spin mixing at the Fermi wavevectors $-k_F$ and k_F are different. We have obtained, employing bosonization methods,[9,10] the exact dispersion relation of the collective mode of the lower band when the Fermi energy lies in the pseudogap. The dispersion relation of this mode is

$$\omega = [v_\theta(q) v_\phi(q)]^{1/2} q \equiv v_0 q. \tag{1}$$

$v_\theta(q)$ and $v_\phi(q)$ are defined as follows:

$$v_\theta(q) = v_F\left(1 + \frac{V_q}{\pi v_F} - \frac{g V_{2k_F}}{2\pi v_F}\right),$$

$$v_\phi(q) = v_F\left(1 + \frac{g V_{2k_F}}{2\pi v_F}\right), \tag{2}$$

where v_F is the Fermi velocity and the renormalization factor of the strength of backscattering is

$$g = \frac{\epsilon_Z^2}{\epsilon_Z^2 + (\eta_R k_F)^2}. \tag{3}$$

θ and ϕ are the phase fields which are basically linear combinations of density operators $\rho_{R/L}$ and they are defined in

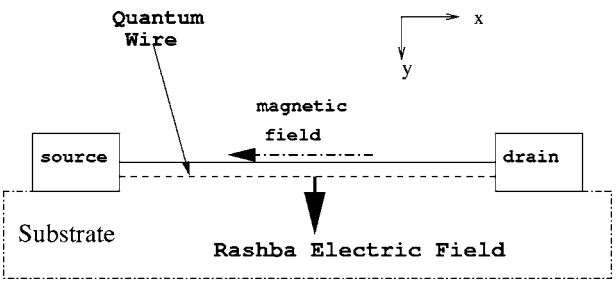

FIG. 1. The geometry of a quantum wire with a magnetic field along the wire.

Eq. (33). V_q is the interaction matrix element, ϵ_Z is the magnitude of Zeeman coupling, and η_R is a parameter characterizing Rashba SOI (see below). From the expression of g [Eq. (3)] we see that the velocity of this collective excitation depends sensitively on the Rashba SOI and magnetic field. This result differs from the that of an ordinary Luttinger liquid in that the back scattering term V_{2k_F} is renormalized by a factor g. The physical origin of this factor reflects the different spin mixing of single particle states near the Fermi wave vectors, which are coupled by backscattering.

The presence of the renormalization factor g may be exploited to determine the constant η_R. There is no simple way to calculate η_R because it depends both on the electric field inside the semiconductor heterostructure and on the detailed boundary conditions at the interface. Instead these spin-orbit coupling constants were measured by electric, optical, and photoelectrical means.[11–14] We suggest that the measurement of the velocity of the collective excitation v_0 may provide another way to determine the value of η_R. This measurement can be carried out using tunneling between two parallel wires in the presence of an additional magnetic field $\mathbf{B}_t=\nabla\times\mathbf{A}_t$ along the y axis.[15,16] This method allows one to determine the spectrum of elementary excitations[17,18] momenta much larger than $2k_F$.[19]

This paper is organized as follows. In Sec. II we introduce our model and review the results obtained by Středa and Šeba for the noninteracting case. In Sec. III we incorporate the electron-electron interaction and obtain an effective Hamiltonian for the system. In Sec. IV the dispersion of collective excitation is computed based on the effective action obtained in Sec. III. Section V we discuss how our result for the velocity differs from the results of ordinary Luttinger liquids. An experiment is proposed to measure η_R.

II. MODEL FOR SINGLE PARTICLE HAMILTONIAN

In our model confinement potentials are present along the y and z axes and quasi-one-dimensional motion of electrons is possible along the x axis. The widths of the wave function along both the y and z axes are assumed to be negligible. The lowest subband energies along the y and z axes are denoted by E_y and E_z. A magnetic field parallel to the quantum wire along the x axis is present $\mathbf{B}=-B\hat{\mathbf{x}}$. The corresponding vector potential can be chosen to be $\mathbf{A}=-By\hat{\mathbf{z}}$, $B>0$. In our model Rashba electric field is applied along the y axis (see Fig. 1), and is given by $\mathbf{E}=+E_0\hat{\mathbf{y}}$ ($E_0>0$). The Rashba spin-orbit interaction[20,21] then takes the form

$$\mathcal{H}_R=\eta_R(k_x\sigma_z-k_z\sigma_x), \tag{4}$$

where $\eta_R=|e|\hbar^2E_0/4m_e^2c^2>0$. The strength of Rashba SOI can be controlled by changing electric field.[22,23] Note that in quantum wires with electron propagating along the x axis k_y and k_z must be replaced by dynamical momentum operators. The expectation value of k_y, $k_z+eA_z/\hbar c$ with respect to the lowest subband state wave function of transverse degrees of freedom (y,z) vanish by symmetry considerations[3]

$$\mathcal{H}_R=\eta_Rk_x\sigma_z. \tag{5}$$

The bulk Hamiltonian of Dresselhaus SOI is given by[1]

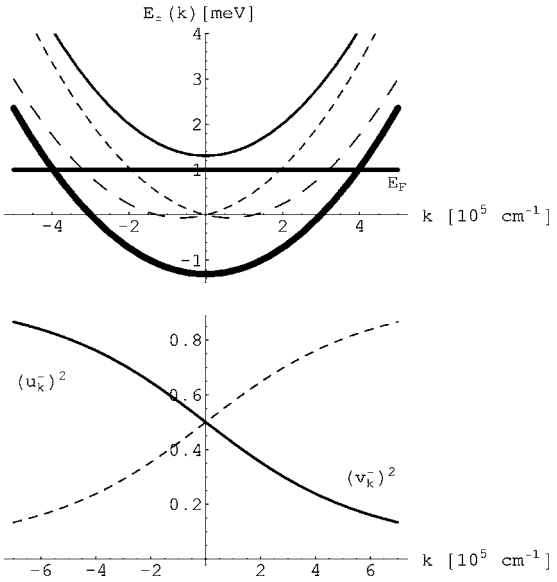

FIG. 2. Upper figure: Solid lines represent the lowest energy subband structure of the quantum wire in the absence of the Dresselhaus term. (Dashed lines are for zero magnetic field.) Note that the Fermi energy lies in the pseudogap. When a finite value of magnetic field is present bands do anticross (in the figure $B=3$ T). The input parameters are $\eta_R=2\times10^{-9}$ eV cm, $m^*=0.024m_e$. In this case the numerical value of g is approximately 0.7. Lower figure: The spin-up (solid line) and -down (dashed line) components $(u_k^-)^2$ and $(v_k^-)^2$ for the lower $E_-(k)$ band. The input parameters are identical with the above figure. Note that $(v_k^-)^2=1-(u_k^-)^2$.

$$\mathcal{H}_{\text{bulk},D}=\gamma_c[\sigma_xk_x(k_y^2-k_z^2)+\sigma_yk_y(k_z^2-k_x^2)+\sigma_zk_z(k_x^2-k_y^2)]. \tag{6}$$

To obtain the effective Hamiltonian of quantum wire we have to take the average of the above bulk Hamiltonian with respect to the ground state wave function of transverse (y,z) degrees of freedom. In our geometry the Rashba electric field is applied in the y direction, and the lateral confining potential enforcing quasi one-dimensional motion is applied in the z direction. Clearly $\langle k_z\rangle=0$ since the subband wave function along the z axis has even parity. The subband wave function along the y axis is a real function and therefore the expectation value $\langle k_y\rangle=0$, too. But we have to note that $\langle y\rangle\neq0$ since the inversion symmetry is lacking in the y direction. The effective Hamiltonian for quantum wire is then

$$\mathcal{H}_D=\gamma_c\sigma_xk_x(\langle k_y^2\rangle-\langle k_z^2\rangle)=\eta_D\sigma_xk_x, \tag{7}$$

where $\eta_D=\gamma_c(\langle k_y^2\rangle-\langle k_z^2\rangle)$.

Now the one-particle Hamiltonian becomes

$$\mathcal{H}_1=E_y+E_z+\frac{\hbar^2k^2}{2m^*}+\eta_Rk\sigma_z+\eta_Dk\sigma_x-E_Z\sigma_x. \tag{8}$$

The Dresselhaus term can be absorbed into the Zeeman term $E_Z=g_0\mu_BB/2$ ($g_0\approx15$ for InAs) in the following way:

$$\epsilon_Z \equiv E_Z - \eta_D k. \tag{9}$$

For the sake of completeness we include the Dresselhaus term in the calculation of the band structure. Later we will ignore it in the bosonization procedure. By the diagonalization of the Hamiltonian (8) the energy eigenvalues and the corresponding normalized eigenvectors are obtained as follows: For the lower band the eigenvalue is (E_y, E_z put to zero)

$$E_-(k) = \frac{\hbar^2 k^2}{2m^*} - \sqrt{\epsilon_Z^2 + \eta_R^2 k^2} \tag{10}$$

and the eigenvector is

$$\xi_- = \begin{pmatrix} u_k^- \\ v_k^- \end{pmatrix}, \tag{11}$$

where

$$u_k^- = \frac{\epsilon_Z}{\sqrt{(\eta_R k + D)^2 + \epsilon_Z^2}}, \tag{12}$$

$$v_k^- = \frac{\eta_R k + D}{\sqrt{(\eta_R k + D)^2 + \epsilon_Z^2}}. \tag{13}$$

$$\tag{14}$$

Here

$$D \equiv \sqrt{(\eta_R k)^2 + \epsilon_Z^2}. \tag{15}$$

u_k^- and v_k^- represents the amplitudes for the spin to point in the $+z$ and $-z$ directions, respectively. For the upper band the results are given in Ref. 24.

Quantum wires can be tailor made so that the quantities $\langle k_y^2 \rangle$ and $\langle k_z^2 \rangle$ are *almost equal*. If we assume the harmonic confining potential $m^* \omega_0^2 z^2/2$ along the z axis we have $\langle k_z^2 \rangle = m^* \omega_0/2\hbar$. For the y direction the constant Rashba electric field is acting so that the potential is linearly rising. In this case[25] $\langle k_y^2 \rangle \sim 0.8(2m^*|e|E_0/\hbar^2)^{2/3}$. The condition $\langle k_y^2 \rangle = \langle k_z^2 \rangle$ is satisfied when the value of the electric field is given by $eE_0 z_0 = 0.49(\hbar^2/2m^*)(1/z_0^2)$, where $z_0 = \sqrt{\hbar/m^* \omega_0}$. For this particular value of the electric field the Dresselhaus term is negligible compared to the Zeeman energy and the Rashba coupling. Note that $E_-(k)$ becomes an even function of k in this case. Hereafter we assume this. If the Rashba coupling becomes sufficiently strong such that

$$\eta_R^2 \geqslant \epsilon_Z \hbar^2/m^* \tag{16}$$

then the energy spectrum develops a double minium at

$$k = \pm \frac{1}{\eta_R} \left[\left(\frac{m^* \eta_R^2}{\hbar^2} \right)^2 - \epsilon_Z^2 \right]^{1/2}. \tag{17}$$

The energy at the minimum is given by

$$E_{\min} = -\frac{m^* \eta_R^2}{2\hbar^2} - \frac{\hbar^2 \epsilon_Z^2}{2m^* \eta_R^2}. \tag{18}$$

In such a case $E_-(0) > E_{\min}$. In our work we assume that $E_-(0) - E_{\min} = -\epsilon_Z - E_{\min}$ is less than the Fermi energy so that there are only two Fermi wave vectors.

III. MODEL FOR MANY-BODY HAMILTONIAN

Let a_k and b_k be the quasiparticle operators corresponding to $E_-(k)$ and $E_+(k)$, respectively. They can be explicitly expressed in terms of electron operators as follows:

$$b_k^\dagger = c_{k\uparrow}^\dagger u_k^+ + c_{k\downarrow}^\dagger v_k^+, \quad a_k^\dagger = c_{k\uparrow}^\dagger u_k^- + c_{k\downarrow}^\dagger v_k^-,$$

$$c_{k\uparrow}^\dagger = b_k^\dagger u_k^+ + a_k^\dagger u_k^-, \quad c_{k\downarrow}^\dagger = b_k^\dagger v_k^+ + a_k^\dagger v_k^-. \tag{19}$$

When electrons are filled such that the Fermi energy is located in the energy gap between a and b bands at $k=0$, we can safely neglect the b-type quasiparticles in the low-energy regime. Then the Eq. (19) can be simplified.

$$a_k^\dagger = c_{k\uparrow}^\dagger u_k^- + c_{k\downarrow}^\dagger v_k^-,$$

$$c_{k\uparrow}^\dagger \sim a_k^\dagger u_k^-, \quad c_{k\downarrow}^\dagger \sim a_k^\dagger v_k^-. \tag{20}$$

A general electron-electron interaction in a quantum wire is given by

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{k_1,k_2,q,\sigma,\sigma'} V_q c_{k_1\sigma}^\dagger c_{k_2+q\sigma'}^\dagger c_{k_2+q\sigma'} c_{k_1-q\sigma}, \tag{21}$$

where V_q is the interaction matrix element. Note that this interaction is spin conserving. For the long-range Coulomb interaction the interaction matrix element is

$$V_q = \frac{2e^2}{\epsilon} K_0(|q|w) \to \frac{2e^2}{\epsilon} \ln \frac{1}{|q|w} \quad \text{for } |q|w \ll 1. \tag{22}$$

K_0 is the modified Bessel function and w is the cutoff length scale which is the order of the width of the quantum wire. ϵ is the bulk dielectric constant. For the short-range interaction such as screened Coulomb interaction the matrix element V_q can be taken to be independent of the momentum transfer q. Projecting the Hamiltonian (21) to the a band with the use of Eq. (20), we obtain

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{k_1,k_2,q} \langle k_1, k_2 | \hat{V} | k_1 - q, k_2 + q \rangle a_{k_1}^\dagger a_{k_2}^\dagger a_{k_2+q} a_{k_1-p}, \tag{23}$$

where

$$\langle k_1, k_2 | \hat{V} | k_1 - q, k_2 + q \rangle = V_p [\xi_-^\dagger(k_1) \xi_-(k_1 - q)] \times [\xi_-^\dagger(k_2) \xi_-(k_2 + q)] \tag{24}$$

is the projected interaction matrix element in the low-energy Hilbert space. The explicit expression of eigenvector ξ_- is given by Eqs. (11) and (12).

At low energy, only the electron states near $-k_F$ and k_F Fermi points need to be considered. Following the usual procedures of g-ology and bosonization method[9] we can express the interaction Hamiltonian (23) within g-ology scheme. Forward scattering g_2 and g_4 process. Backscattering g_1 process We note further that for fermions of a single species (such as the a quasiparticle here) g_1 process is identical with g_2 process.[9] In this paper a commensurate filling is not consid-

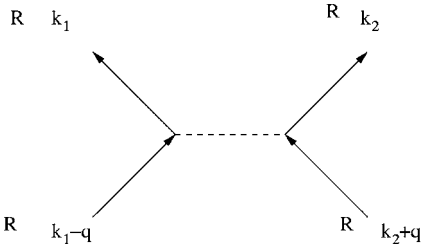

FIG. 3. A Feynman diagram of the g_4 process. All four momenta k_1, k_2, k_2+q, k_1-q are located near the right Fermi point. The dotted line indicates the matrix element V_q. See text for details.

ered, so that the Umklapp processes (g_3) can be neglected. From now on we will call electrons with $k<0$ ($k>0$) left (right) movers.

g_4 process. For instance let us assume that all four momenta k_1, k_2, k_2+q, k_1-q are located near the right Fermi point in the following (see Fig. 3). Then one can write $k_i=+k_F+p_i$ with a condition $|p_i|<\Lambda<k_F$. Λ is the momentum cutoff scale, within which the linearization of the a-band dispersion is valid. It is convenient to introduce the right moving Dirac fermion operator ψ_R: $\psi_R(p_i)\equiv a_{k_i}$ for $k_i=+k_F+p_i$. We can make following approximation if we neglect subleading contributions proportional to $(k-k_F)$ which are irrelevant at low energy: $[\xi_-^\dagger(k_1)\xi_-(k_1-q)]\approx[\xi_-^\dagger(k_F)\xi_-(k_F)]=1$. Thus the effect of spin mixing reflected in the matrix elements ξ_σ does not play any role for g_4 process.

The contributions from the neighborhood of left Fermi point can be treated in the same way. The left moving Dirac fermion operator ψ_L can be introduced similarly. $\psi_L(p_i)\equiv a_{k_i}$ for $k_i=-k_F-p_i$. The low energy effective Hamiltonian describing g_4 process can be read off from the original Hamiltonian (23):

$$\mathcal{H}_{g_4}=\frac{1}{2N}\sum_{|q|<\Lambda}V_q[\rho_R(q)\rho_R(-q)+\rho_L(-q)\rho_L(q)], \quad (25)$$

where $\rho_{R/L}(q)=\Sigma_p\psi_{R/L}^\dagger(p+q)\psi_{R/L}(p)$ are the density operators of right- and left-moving Dirac fermions. N is the number of lattice sites of quantum wire. In the above expression the low-momentum asymptotics of V_q must be used.

g_2 processes. According to the same reason as g_4 interaction one can make the approximations in Fig. 4,

$$[\xi_-^\dagger(k_1)\xi_-(k_1-q)]\approx[\xi_-^\dagger(k_F)\xi_-(k_F)]=1,$$

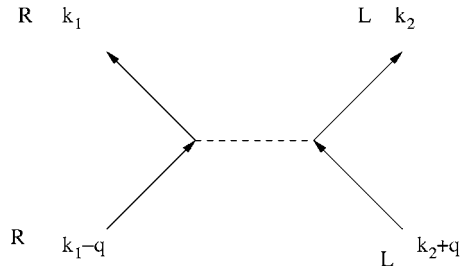

FIG. 4. A Feynman diagrams of the g_2 process. See text for details. There is another g_2 Feynman diagram in which $R\leftrightarrow L$.

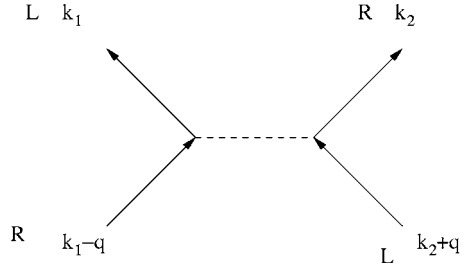

FIG. 5. A Feynman diagram of the $g_{1,\parallel}$ processes. There exists one more diagram where $R\leftrightarrow L$. See text.

$$[\xi_-^\dagger(k_2)\xi_-(k_2+q)]\approx[\xi_-^\dagger(-k_F)\xi_-(-k_F)]=1.$$

Again the spin-mixing effect represented by the matrix elements does not modify the interaction. Due to this one can easily read off the low-energy effective Hamiltonian describing g_2 process from the original Hamiltonian (23) using the definition of density operators $\rho_{R/L}(q)$:

$$\mathcal{H}_{g_2}=\frac{1}{N}\sum_{|q|<\Lambda}V_q\rho_R(q)\rho_L(-q). \quad (26)$$

$g_{1,\parallel}$ processes. In this case the matrix elements play crucial role as can be seen in (see Fig. 5)

$$[\xi_-^\dagger(k_1)\xi_-(k_1-q)][\xi_-^\dagger(k_2)\xi_-(k_2+q)]\approx[\xi_-^\dagger(-k_F)\xi_-(+k_F)][\xi_-^\dagger(+k_F)\xi_-(-k_F)]. \quad (27)$$

Evidently the dominant momentum transfer q must be $2k_F$. By changing the order of operators (thereby changing overall sign of interaction) and by summing over momenta one arrives at

$$H_{g_1}=-gV_{2k_F}\int dx\rho_R(x)\rho_L(x), \quad (28)$$

where

$$g=[\xi_-^\dagger(-k_F)\xi_-(+k_F)][\xi_-^\dagger(+k_F)\xi_-(-k_F)]=\frac{\epsilon_Z^2}{\epsilon_Z^2+(\eta_R k_F)^2}. \quad (29)$$

As can be seen from Eq. (29) the coupling constant g depend on the applied magnetic field and the Rashba SOI as well as Fermi momentum.

We observe that g_2 Hamiltonian (26) and g_1 Hamiltonian (28) can be combined completely. This is a special feature of fermions of *single* species. In the presence of other degrees of freedom such as spin a backscattering term ($g_{1,\perp}$) appears which is not of the Luttinger interaction form.

IV. BOSONIZATION AND COLLECTIVE EXCITATIONS

The total effective Hamiltonian incorporating interaction is given by

$$\mathcal{H}=\mathcal{H}^{(0)}+\mathcal{H}_{g_4}+\mathcal{H}_{g_2}+\mathcal{H}_{g_1}. \quad (30)$$

The linearized noninteracting Hamiltonian $\mathcal{H}_0$ is

$$\mathcal{H}^{(0)} = \sum_p [v_F p \psi_R^\dagger(p)\psi_R(p) - v_F p \psi_L^\dagger(p)\psi_L(p)]. \quad (31)$$

The Hamiltonian (30) can be bosonized straightforwardly:[9]

$$\mathcal{H} = \pi v_F \int dx[:\rho_R^2(x): + :\rho_L^2(x):] + \frac{1}{2N}\sum_q V_q[\rho_R(q)\rho_R(-q) + \rho_L(-q)\rho_L(q)] + \frac{1}{N}\sum_q [V_q - gV_{2k_F}]\rho_R(q)\rho_L(-q). \quad (32)$$

The colons denotes normal ordering of operators. It is convenient to introduce phase fields as follows:

$$\theta(x) = \frac{1}{2}[\phi_R(x) + \phi_L(x)],$$

$$\phi(x) = \frac{1}{2}[\phi_R(x) - \phi_L(x)], \quad (33)$$

where $\rho_{R/L}(x) = (1/2\pi)\partial_x \phi_{R/L}(x)$. In terms of phase fields

$$\begin{aligned}\mathcal{H} &= \frac{v_F}{2\pi}\int dx[(\partial_x\theta)^2 + (\partial_x\phi)^2] \\ &+ \frac{1}{N}\sum_p \frac{V_q q^2}{(2\pi)^2}[\theta(q)\theta(-q) + \phi(q)\phi(-q)] \\ &+ \left(\frac{1}{2\pi}\right)^2 \frac{1}{N}\sum_p (V_q - gV_{2k_F})q^2[\theta(q)\theta(-q) - \phi(q)\phi(-q)] \\ &= \frac{v_F}{2\pi}\int dx[(\partial_x\theta)^2 + (\partial_x\phi)^2] + \frac{1}{(2\pi)^2 N}\sum_q [2(V_q \\ &- gV_{2k_F}/2)q^2\theta(q)\theta(-q) + gV_{2k_F}q^2\phi(q)\phi(-q)]. \quad (34)\end{aligned}$$

The Euclidean action is given by

$$S[\theta,\phi] = \int d\tau\left[\int dx \frac{i}{\pi}\partial_x\phi\partial_\tau\theta + H\right]. \quad (35)$$

In matrix form the above can be written as

$$S = \frac{1}{2\pi}\int \frac{d\omega dq}{(2\pi)^2}[\theta(-q,-\omega)\ \ \phi(-q,-\omega)] \times \begin{bmatrix} v_\theta(q)q^2 & iq\omega \\ iq\omega & v_\phi(q)q^2 \end{bmatrix}\begin{bmatrix}\theta(q,\omega)\\ \phi(q,\omega)\end{bmatrix},$$

where

$$v_\theta = v_\theta(q) = v_F\left(1 + \frac{V_q}{\pi v_F} - g\frac{V_{2k_F}}{2\pi v_F}\right),$$

$$v_\phi = v_\phi(q) = v_F\left(1 + \frac{gV_{2k_F}}{2\pi v_F}\right). \quad (36)$$

θ and ϕ are the phase fields which are basically linear combination of density operators $\rho_{R/L}$ and they are defined in Eq. (33).

The dispersion relation of the collective excitation can be obtained from the kernel of action Eq. (36).

$$\det\begin{bmatrix} v_\theta(q)q^2 & iq\omega \\ iq\omega & v_\phi(q)q^2 \end{bmatrix} = 0. \quad (37)$$

After analytic continuation $i\omega \to \omega$ we find

$$\omega = [v_\theta(q)v_\phi(q)]^{1/2}q \equiv v_0 q. \quad (38)$$

v_0 is the velocity of collective excitation. From Eq. (36) one can write

$$v_0 = v_F\left[1 + \frac{V_q}{\pi v_F} + \frac{(V_q - gV_{2k_F}/2)(gV_{2k_F}/2)}{(\pi v_F)^2}\right]^{1/2}. \quad (39)$$

The quantity in the bracket of Eq. (39) represents the renormalization effect due to electron-electron interaction. The velocity of collective excitation can be controlled by band filling, Rashba SOI, and magnetic field through dependence on v_F and g. Let us estimate the magnitude of the correction terms. In $v_\theta(q)$ the backscattering term $g(V_{2k_F}/2\pi v_F)$ is a factor $g/2$ smaller than the forward term $V_q/\pi v_F$. In $v_\phi(q)$ the correction term $gV_{2k_F}/2\pi v_F \sim 0.1g$ for the width of the quantum wire $w \sim 100$ Å and $2k_F \approx 1\times 10^6$ cm^{-1}. We also note that for the screened short range Coulomb interaction the interaction matrix element V_q is almost independent of momentum transfer q, and the backscattering term plays an equally important role as forward the scattering.

V. DISCUSSIONS AND SUMMARY

It is instructive to compare our result with the velocities of phase fields of collective excitation of ordinary Luttinger liquids. For spinless fermions it is given by

$$v_\theta(q) = v_F\left(1 + \frac{V_q}{\pi v_F} - \frac{V_{2k_F}}{2\pi v_F}\right),$$

$$v_\phi(q) = v_F\left(1 + \frac{V_{2k_F}}{2\pi v_F}\right). \quad (40)$$

In Eq. (2) this corresponds to $g=1$, which implies absence of spin-orbit coupling and one type of spin, either up or down. For the Luttinger liquids of spinful fermions the velocity of charge mode is given by

$$v_{\theta_\rho}(q) = v_F\left(1 + \frac{2V_q}{\pi v_F} - \frac{V_{2k_F}}{2\pi v_F}\right),$$

$$v_{\phi_\rho}(q) = v_F\left(1 + \frac{V_{2k_F}}{2\pi v_F}\right). \quad (41)$$

θ_ρ and ϕ_ρ are the phase fields in the charge sector. The spinful velocity is recovered with the replacement $V_q \to 2V_q$ and $g=1$ in Eq. (2). The velocity of the spin mode is

$$v_{\theta_s}(q) = v_F\left(1 - \frac{V_{2k_F}}{2\pi v_F}\right),$$

$$v_{\phi_s}(q) = v_F\left(1 + \frac{V_{2k_F}}{2\pi v_F}\right). \quad (42)$$

θ_s and ϕ_s are the phase fields in the spin sector. This corresponds to $V_q=0$ and $g=1$ in Eq. (2).

The dispersion relation of the collective mode may be measured by adding another quantum wire parallel to the original wire in the presence of a second magnetic field $\vec{B}_t$ along the y axis. When the first wire is located at $z=0$ and the second wire at $z=z_0$ the single particle energy dispersion of the second wire is $E(k)=\hbar^2(k-k_0)^2/2m$, where $k_0=eB_tz_0/\hbar c$, m is the electron mass in the second wire, and the Landau gauge $\mathbf{A}_t=(zB_t,0,0)$ is used. Wave-number selectivity due to momentum-resolved tunneling between them, $E_-(k)=E(k)$, allows a mapping of the dispersion.[15,16,19] Even in the presence of electron interactions this technique allows direct measurement of the collective excitation spectrum.[17,18]

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