Tunable terahertz band planar Bragg reflectors

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A tunable planar narrow-band Bragg reflector based on coupling of the two propagating modes and a cutoff mode is considered. Coupled-wave analysis together with direct numerical simulations demonstrate operation of the proposed scheme up to the terahertz frequency band. Compatibility with the transportation of an intense electron beam encourages the use of a novel Bragg reflector in powerful long-pulse free electron lasers. © *2009 American Institute of Physics*.
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Reflectors (filters) based on Bragg coupling of counterpropagating waves on the periodic structures are widely used both in quantum[1,2] and classical[3] electronics. In the millimeter wavelength band Bragg structures formed by hollow corrugated waveguides enables selectivity of the system to be combined with effective electron beam transportation. However, their scaling to shorter wavelength bands is limited by the fact that at large values of the transverse oversize parameter the overlapping of different Bragg zones, which corresponds to the coupling of numerous pairs of propagating modes, occurs. As a result, the radiation generated by an electron beam would represent an uncontrollable mixture of the waveguide modes.[4] Moreover, the absolute values of the reflection coefficients decrease with increase in the oversize parameter.

These problems can be partially solved by using the coupling between the propagating and the cutoff modes in an advanced Bragg structure. In the case of a planar geometry (Fig. 1) such a reflector is formed by two parallel plates with shallow periodic corrugation of the inner walls given by

$$b(z) = b_1 \cos(\bar{h}z), \qquad (1)$$

where $\bar{h}=2\pi/d$, d is period of the structure, and $2b_1$ is the corrugation depth. The electromagnetic field inside the structure can be presented as a sum of two counterpropagating TEM waves, which are defined by the vector potential

$$\vec{A} = \vec{y}^0 \, \mathrm{Re}[[\hat{A}_+(z)e^{-ihz} + \hat{A}_-(z)e^{ihz}]e^{i\omega t}], \qquad (2)$$

$h=\omega/c$ is the wavenumber and the quasicutoff TM_n wave

$$\vec{A} = \vec{z}^0 \, \mathrm{Re}\left[\hat{B}(z)\cos\left(\frac{n\pi y}{b_0}\right)e^{i\omega t}\right]. \qquad (3)$$

We assume that $\omega \approx \omega_c$, where $\omega_c=n\pi c/b_0$ is the cutoff frequency, b_0 is the mean distance between plates, and n is an integer. Coupling between the propagating and cutoff waves is efficient under the Bragg resonance condition

$$h \approx \bar{h}, \qquad (4)$$

which is satisfied when the mean distance between the plates $b_0 \approx nd/2$. It should be noted that for an advanced Bragg reflector the period of structure d is approximately two times larger than in the case of a conventional Bragg reflector of the same frequency based on direct coupling of forward and backward propagating waves.[1–3]

The process of reflection of the waves Eq. (2) via the excitation of the cutoff mode Eq. (3) can be described by the equations

$$\frac{dA_+}{dZ} + i\Omega A_+ = i\alpha B, \quad \frac{dA_-}{dZ} - i\Omega A_- = -i\alpha B, \qquad (5a)$$

$$\frac{1}{2}\frac{d^2B}{dZ^2} + (\Delta - i\sigma)B + \Omega B = \alpha(A_+ + A_-), \qquad (5b)$$

with the boundary conditions

$$A_+|_{Z=0} = A_0, \quad A_-|_{Z=L} = 0, \qquad (6a)$$

$$\left.\left(\frac{dB}{dZ} + i\sqrt{2(\Omega+\Delta-i\sigma)}B\right)\right|_{Z=0} = 0,$$
$$\left.\left(\frac{dB}{dZ} - i\sqrt{2(\Omega+\Delta-i\sigma)}B\right)\right|_{Z=L} = 0, \qquad (6b)$$

where A_0 is the amplitude of the incident wave. Here $Z=\bar{h}z$, $\Omega=(\omega-\bar{\omega})/\bar{\omega}$ is the detuning between the Bragg frequency $\bar{\omega}=\bar{h}c$ and that of the incident wave, $\Delta=(\bar{\omega}-\omega_c)/\bar{\omega}$ is the mismatch between the cutoff frequency ω_c and the Bragg frequency, $\alpha=b_1/\sqrt{2}b_0$ is the coupling coefficient, $L=hl$, l is the length of the structure, $\sigma=\delta/b_0$ is the Ohmic losses parameter for the cutoff mode, and δ is the skin depth. In Eqs. (5) and (6) the amplitudes $A_\pm=\hat{A}_\pm/\sqrt{N_A}$, $B=\hat{B}/\sqrt{N_B}$ are normalized over the wave norms: $N_A=ch^2b_0/2\pi$, N_B

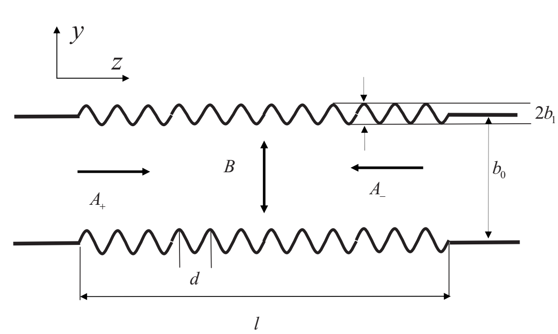

FIG. 1. Scheme of terahertz band Bragg reflector of planar geometry.

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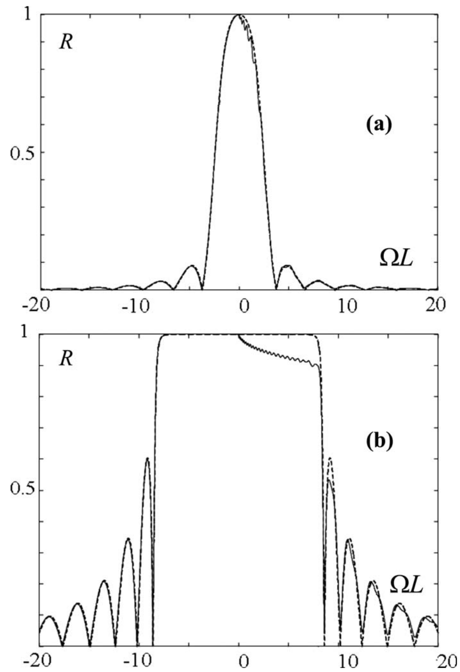

FIG. 2. Reflection coefficient vs frequency detuning calculated from simplified formula (9) (solid lines) and from full Eqs. (5) and (6) (dashed lines) at $L=520$ ($l=1.5$ cm). (a) $\alpha=0.0025$ and (b) $\alpha=0.01$.

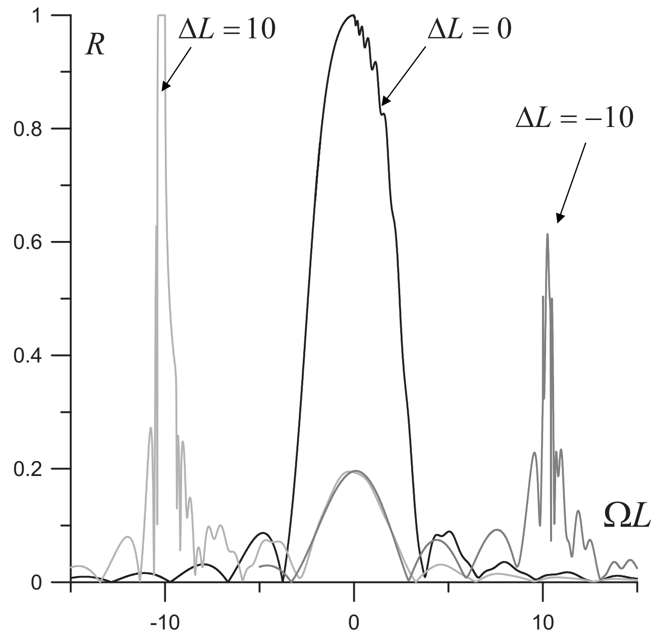

FIG. 3. The shift in the reflection zone when varying the mismatch parameter Δ, which depends on distance between plates b_0 at $L=520$ and $\alpha=0.0025$.

$=ch^2b_0/4\pi$. It should be noted that under the assumption that the reflections for the partial cutoff mode B from the edges of the corrugation are negligibly small, similarly to[5] the radiation boundary conditions Eq. (6b) can be used.

Neglecting the diffraction of the cutoff mode one can obtain the amplitude profiles of the reflected and transmitted waves from Eqs. (5) and (6)

$$A_- = A_0 \frac{2i(\Omega^2 - K^2)\sin K(L-Z)}{(\Omega+K)^2 e^{iKL} - (\Omega-K)^2 e^{-iKL}}, \tag{7a}$$

$$A_+ = A_0 \frac{(\Omega+K)^2 e^{iK(L-Z)} - (\Omega-K)^2 e^{-iK(L-Z)}}{(\Omega+K)^2 e^{iKL} - (\Omega-K)^2 e^{-iKL}}, \tag{7b}$$

where $K=(\Omega^2-2\alpha^2\Omega/(\Omega+\Delta-i\sigma))^{1/2}$. For the cutoff mode we have

$$B = \frac{\alpha(A_+ + A_-)}{\Omega+\Delta-i\sigma}. \tag{8}$$

The reflection coefficient is

$$R = \frac{A_-(Z=0)}{A_+(Z=0)} = \frac{2i(\Omega^2-K^2)\sin KL}{(\Omega+K)^2 e^{iKL} - (\Omega-K)^2 e^{-iKL}}. \tag{9}$$

At $\Delta=0$ maximum of the reflection coefficient

$$R_{\max} = \frac{\alpha^2 L}{\alpha^2 L + \sigma}, \tag{10}$$

corresponds to the exact Bragg resonance $\Omega=0$. The longitudinal profiles of the partial wave amplitudes in this case are given by

$$A_+ = A_0 \frac{L-Z}{L}, \quad A_- = A_0 \frac{Z-L}{L}, \quad B = \frac{iA_0}{\alpha L}. \tag{11}$$

The width of the reflection band in the absence of Ohmic losses can be estimated as $\Delta\Omega \approx 2\sqrt{\pi^2/L^2+2\alpha^2}$. Figure 2 shows the frequency dependence of the reflection coefficient R at $\Delta=0$, which demonstrate that the advanced Bragg structure can be used as an effective narrow-band reflector. Unlike the case of traditional Bragg structures[1–3] the decrease in waves coupling coefficient α in this scheme sharpens the reflection peak while the maximum value of reflection coefficient does not depend on α being close to unity. As follows from Eq. (10) only nonzero Ohmic losses lead to a decrease in $R_{\max}$. In the same figures the doted line shows the frequency dependence calculated from full Eqs. (5) and (6) which include diffraction of the cutoff mode B. Obviously simplified formula (9) gives a reliable expression for the reflection coefficient.

Another advantage is the tunability of the reflector by varying the distance between the plates b_0. The shift in the reflection zone with variation in the mismatch parameter Δ is shown in Fig. 3. Actually, the reflection zone is shifted together with the cutoff frequency (maximum of reflection coefficient corresponds to the cutoff frequency). For positive values of mismatch the maximum reflectivity is close to unity over a fairly broad frequency band.

Thus, the coupled-wave approach demonstrates that a narrow-band reflector is effective at large oversize values and can be realized by the coupling between the propagating and cutoff modes. This conclusion is confirmed by the results of the direct simulation of advanced Bragg reflectors in the terahertz frequency band by the use of a 3D electromagnetic numerical code. The parameters of the structure were: period $d=0.03$ cm, distance between plates $b_0=0.6$ cm, and length $l=1.5$ cm. At $n=40$ the expected resonance frequency is $f=1$ THz. In Fig. 4 the frequency dependencies of the reflection and transmission coefficients are shown for the incident TEM wave. One can see that for an oversized factor $b_0/\lambda\sim 20$ several reflection bands are present. These bands correspond to the excitation of cutoff modes with a different number of the field variations n. Nevertheless, the reflection near

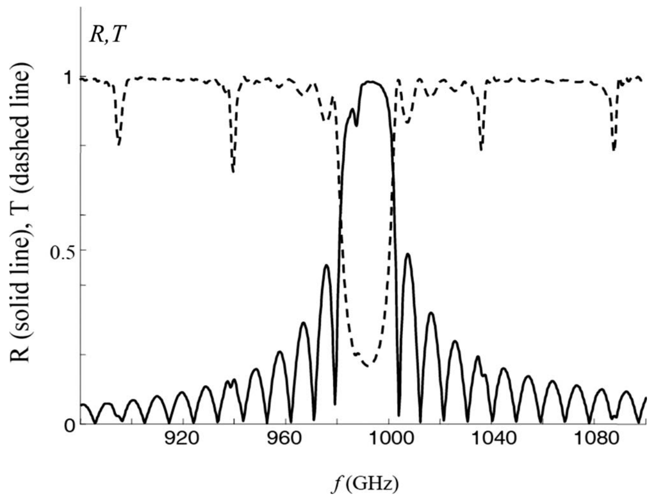

FIG. 4. Reflection (solid line) and transmission (dashed line) coefficients vs frequency found in 3D simulations at $b_0=0.6$ cm, $d=0.03$ cm, $b_1=0.003$ cm, and $l=1.5$ cm.

the frequency of 1 THz corresponding to the excitation of the TM_{40} wave dominates.

Compatibility with intense electron beam transportation encourages the use of a novel Bragg reflector in terahertz band free electron lasers (FELs).[6–8] Note that for effective single-mode operation, it is sufficient to provide conditions when the distance between the reflection zones related with excitation of the cutoff modes with different transverse indices n exceeds the FEL amplification band

$$c\pi/b_0 > \omega/N, \tag{12}$$

which is defined by the number $N=l_w/d_w$ of the wiggler periods d_w over the interaction space l_w. Taking into account the FEL operation wavelength: $\lambda \approx d_w/2\gamma^2$ we get a restriction for the cavity width

$$b_0 < l_w \gamma^{-2}, \tag{13}$$

where γ is the relativistic mass-factor. For example, for $\lambda=0.03$ cm, $\gamma=10$, $d_w=3$ cm, and $l_w=100$ cm the distance between plates admissible for mode selection is $b_0 \leq 1$ cm.

It should be noted in conclusion that it is reasonable to use advanced Bragg reflector in a two-mirror resonator scheme as an upstream reflector. To avoid large Ohmic losses associated with excitation of the cutoff mode it is sufficient to use a conventional Bragg reflector as a downstream mirror with a fairly small reflectivity.[9] We should also emphasize that the advanced Bragg reflector can be considered as an example of a more general principle of making narrow-band reflectors which exploit the coupling between propagating waves and the modes trapped in the resonator.[10] Another important version of such reflectors is a 2D Bragg structure of coaxial geometry,[11,12] which allows separation of reflection zones for the modes with different azimuthal indices in the large oversized waveguide.

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[1] H. Kogelnik and C. V. Shank, J. Appl. Phys. **43**, 2327 (1972).
[2] A. Yariv, *Quantum Electronics* (Wiley, New York, 1975).
[3] V. L. Bratman, G. G. Denisov, N. S. Ginzburg, and M. I. Petelin, IEEE J. Quantum Electron. **19**, 282 (1983).
[4] N. S. Ginzburg, A. A. Kaminsky, A. K. Kaminsky, N. Yu. Peskov, S. N. Sedykh, A. P. Sergeev, and A. S. Sergeev, Phys. Rev. Lett. **84**, 3574 (2000).
[5] V. L. Bratman, M. A. Moiseev, and M. I. Petelin Izv. Vyssh. Uchebn. Zaved., Radiofiz. **16**, 622 (1973).
[6] L. R. Elias, G. Ramian, J. Hu, and A. Amir, Phys. Rev. Lett. **57**, 424 (1986).
[7] A. Abramovich, M. Canter, A. Gover, J. S. Sokolowski, Y. M. Yakover, Y. Pinhasi, I. Schnitzer, and J. Shiloh, Phys. Rev. Lett. **82**, 5257 (1999).
[8] Y. Socol, A. Gover, A. Eliran, M. Volshonok, Y. Pinhasi, B. Kapilevich, A. Yahalom, Y. Lurie, M. Kanter, M. Einat, and B. Litvak Phys. Rev. ST Accel. Beams **8**, 080701 (2005).
[9] N. S. Ginzburg, A. M. Malkin, N. Yu. Peskov, A. S. Sergeev, and V. Yu. Zaslavsky Phys. Rev. ST Accel. Beams **12**, 060702 (2009).
[10] M. I. Petelin and I. V. Turchin, J. Commun. Technol. Electron. **46**, 1331 (2001).
[11] N. S. Ginzburg, N. Yu. Peskov, A. S. Sergeev, I. V. Konoplev, A. W. Cross, A. D. R. Phelps, G. R. M. Robb, K. Ronald, W. He, and C. G. Whyte, J. Appl. Phys. **92**, 1619 (2002).
[12] A. W. Cross, N. S. Ginzburg, W. He, I. V. Konoplev, N. Yu. Peskov, A. D. R. Phelps, K. Ronald, A. S. Sergeev, and C. G. Whyte, Appl. Phys. Lett. **80**, 1517 (2002).