Time-to-frequency converter for measuring picosecond optical pulses

M. T. Kauffman, W. C. Banyai, A. A. Godil,^{a)} and D. M. Bloom *E. L. Ginzton Laboratory, Stanford University, Stanford, California* 94305-4085

(Received 21 June 1993; accepted for publication 10 November 1993)

We present a new technique for measuring the intensity I(t) of optical pulses using a temporal optical system. A diffraction grating pair followed by a microwave-driven, optical phase modulator configured as a time lens is used to uniquely map the pulse shape from the time domain to the frequency domain, allowing measurement of the pulse shape with a spectrometer. We discuss the theory of operation and present experimental results illustrating 3 ps time resolution.

Direct measurement of optical pulses using either photodiodes or streak cameras involves high-speed electronics and is currently limited to approximately 1 ps time resolution. Most techniques for measuring shorter pulse shapes require the use of nonlinear phenomena, but they do alleviate the need for high-speed electronics. For example, second-order intensity and interferometric autocorrelations¹ can be used together to deduce the pulse shape from a train of sampled pulses, while frequency resolved optical gating,² utilizing third-order Kerr nonlinearity, can be used to deduce the pulse shape on a single-shot basis. The advantage of these two techniques is that they yield useful phase information, however, both require adequate pulse energy to excite the respective nonlinearities. In this letter, we present a new technique for the direct measurement of short optical pulses that does not rely on high speed electronics or optical nonlinearities. This technique is best described as time-to-frequency conversion: pulse amplitude is uniquely mapped from the time domain to the frequency domain such that the pulse's temporal shape can be directly measured with a spectrometer. It is important to note that this technique will work with chirped pulses; in general, time-bandwidth limited pulses are not required for this technique to work. To perform the timeto-frequency conversion, we take advantage of the unique properties of temporal optical systems and recently developed technology that allows implementation. With our current system, we have achieved 3 ps resolution, which is in agreement with our predicted resolution.

The time-to-frequency conversion process is best understood by noting the analogy between a temporal optical system manipulating pulses of light and a spatial optical system manipulating beams of light. An equivalent temporal optical system is found for a spatial optical system by exchanging spatial variables with time variables, and spatial frequencies with spectral frequencies. The effects of diffraction and spatial lenses on a beam of light are equivalent to the effects of dispersion and time lenses on a pulse of light. Because a spatial lens causes a spatially varying phase shift, a time lens must cause a time-varying phase shift. This space-time analogy was noticed by Treacy,³ formalized by Kolner,⁴ and demonstrated experimentally by Godil⁵ and Kauffman.⁶

The temporal optical system performing the time-tofrequency conversion is shown in Fig. 1. The pulse to be measured travels through one focal time of dispersion, supplied by a grating pair, then is phase modulated by the time lens. This is equivalent to a spatial optical system with the object placed at the front focal plane of the lens. We know from Fourier optics that the field distributions at the front focal plane and output plane of the spatial lens are related by a Fourier transform,⁷ and we will show below that the same relation holds for a time lens. Also, the output field distribution and its spectral frequency distribution are related by a Fourier transform. Therefore, the output spectrum and input field are related by two successive Fourier transforms, and must have the same functional dependence. As shown in Fig. 1, the output spectrum has the same shape as the input pulse. Therefore, measuring the shape of the output power spectrum gives us the temporal shape of the input pulse.

Quantitatively, the temporal system operation is determined by following the input field $\mathbf{E}_0(t)$, an optical carrier at frequency ω_0 modulated by the pulse envelope $\mathbf{u}_0(t)$,

$$\mathbf{E}_0(t) = \mathbf{u}_0(t) e^{j\omega_0 t}.$$
 (1)

Treacy [Eq. (14)] showed that if the propagation constant $\beta(\omega)$ of the dispersive medium can be approximated by a three term Taylor series

$$\beta(\omega) = \beta|_{\omega_0} + \beta'|_{\omega_0}(\omega - \omega_0) + \frac{1}{2}\beta''|_{\omega_0}(\omega - \omega_0)^2,$$
(2)

then the effect of the dispersion distorts the input pulse envelope $\mathbf{u}_0(t)$ into $\mathbf{u}_1(t')$:

$$\mathbf{u}_{1}(t') = \int dt \ \mathbf{u}_{0}(t) \exp\{j[(t-t')^{2}/2\beta''L]\},\tag{3}$$



FIG. 1. Experimental schematic of the time-to-frequency converter with the ideal time lens approximated by a sinusoidally driven, electro-optic phase modulator.

^{a)}Currently at Kubota Lab, Sony Corp., Tokyo, Japan.

where L is the distance traveled in the dispersive medium. The pulse $\mathbf{u}_1(t')$ is then phase modulated by the time lens. Kolner showed that the ideal time lens has a quadratic phase versus time:

$$\exp\left(j\frac{\omega_0 t'^2}{2f_T}\right),\tag{4}$$

where f_T is the focal time of the lens. For a lens approximated with the quadratic portion of a sinusoidal phase modulator driven at frequency ω_m , the focal time is given by $f_T = \omega_0 / A \omega_m^2$, where A is the peak phase modulation of the modulator. After passing through the dispersive grating pair and the optical phase modulator, the output pulse $\mathbf{u}_2(t')$ is related to the input pulse $\mathbf{u}_0(t)$ by

$$\mathbf{u}_{2}(t') = \mathbf{u}_{1}(t') \exp\left(j\frac{\omega_{0}t'^{2}}{2f_{T}}\right)$$

$$= \exp\left[j\frac{t'^{2}}{2}\left(\frac{\omega_{0}}{f_{T}} + \frac{1}{\beta''L}\right)\right]\int dt \,\mathbf{u}_{0}(t)$$

$$\times \exp\left(j\frac{t^{2}}{2\beta''L}\right) \exp\left(-j\frac{tt'}{\beta''L}\right).$$
(5)

If the amount of dispersion is adjusted such that the object pulse $\mathbf{u}_0(t)$ is at the front focal plane of the time lens, that is, $\omega_0/f_T = -1/\beta''L$, then the quadratic phase term outside the integral disappears. Under this condition, the form of Eq. (5) shows that the output pulse $\mathbf{u}_2(t')$ is a Fourier transform of the input pulse $\mathbf{u}_0(t)$ multiplied by a quadratic phase factor

$$\mathbf{u}_{2}(t') = \int dt \ \mathbf{u}_{0}(t) \exp\left(j\frac{t^{2}}{2\beta''L}\right) \exp\left(-j\frac{tt'}{\beta''L}\right). \tag{6}$$

The Fourier transform of the output pulse is

$$\mathbf{U}_{2}(\omega) = \mathbf{u}_{0}(t) \exp\left(-j \frac{\omega_{0} t^{2}}{2 f_{T}}\right)\Big|_{t=\omega f_{T}/\omega_{0}}.$$
(7)

The power spectrum measured by the optical spectrum analyzer is the pulse spectrum centered at the carrier frequency ω_0 ,

$$|\mathbf{U}_{2}(\omega-\omega_{0})|^{2} = |\mathbf{u}_{0}(t)|_{t=(\omega-\omega_{0})f_{T}/\omega_{0}}^{2}.$$
(8)

Therefore, the input pulse intensity is found by measuring the power spectrum of the output pulse and scaling the frequency axis to a time axis using

$$t = \frac{(\omega - \omega_0)f_T}{\omega_0} = -\frac{(\lambda - \lambda_0)f_T}{\lambda} .$$
(9)

The time lens used to demonstrate this measurement technique is a LiNbO₃ electro-optic phase modulator built by Godil. It produces A = 51 rad of peak phase modulation for 16 W of microwave power at $\omega_m = 2\pi 5.2$ GHz. The experiments were carried out using a mode-locked, Nd:YAG laser operating at 1.06 μ m. A fiber-grating pulse compressor was used to produce the short pulses. The input dispersion is provided by a four pass dispersive delay line constructed with a 1700 lines/mm grating.

An optical pulse measurement is shown in Fig. 2. The output spectrum is measured and wavelength converted to



FIG. 2. Pulse shape measured by time-to-frequency conversion.

time using the scale factor -30.3 ps/nm given by Eq. (9). The pulse has a main lobe FWHM=11.8 ps with a significant, asymmetric tail. This asymmetric tail cannot be seen by intensity autocorrelation, which, by definition, must be symmetric. The directly measured autocorrelation of this pulse has a FWHM of 20.0 ps, while the computed autocorrelation FWHM is 18.7 ps.

The important performance measures of this system are the temporal resolution and temporal field of view. Just as a spatial lens has a diffraction limited spot size arising from its finite aperture, the time lens has a dispersion limited pulse width because of its finite time aperture. The time aperture is defined by the quadratic duration of the sinusoidal phase versus time of the time lens. The time aperture is approximately $1/\omega_m$, and the resolution⁵ 2.8/($A \omega_m$). For our lens, these numbers are 31 and 1.7 ps, respectively. When pulses shorter than this are measured, they will dispersively broaden and overfill the effective aperture. Also, pulses which are initially longer than the time aperture will overfill the effective aperture. Unlike a spatial lens which has a physical lens stop, a sinusoidal time lens has no stops, and light dispersing into the nonquadratic regions is not transformed correctly. That is, the nonquadratic phase error causes aberrations. Both of these limits have been explored experimentally, and the results are presented below.

To find the time resolution we used our system to measure a pulse whose measured autocorrelation FWHM is 1.9 ps. Figure 3 shows the output spectrum of the pulse. The spectral width is 1.8 nm at the input and a spectrometerlimited 0.08 nm at the output. After converting the wave-



FIG. 3. Measured system time resolution found by performing time-tofrequency conversion on a pulse whose autocorrelation width is 1.9 ps.



FIG. 4. Effect of aberrations on long pulses. (a) Photodiode measured input pulse. (b) Pulse shape measured with new technique shows aberrations due to overfilling the time-lens aperture.

length axis to time, the FWHM of the measured pulse is 3.0 ps.

The temporal field of view is given by the effective aperture of the sinusoidal time lens, t_a , which is 31 ps for our lens. This effect of exceeding this limit is demonstrated by measuring a long pulse whose width is greater than the effective time aperture. Figure 4(a) shows the photodiode measured 63 ps FWHM time domain of pulse before time-to-frequency conversion. The output spectrum is shown in Fig. 4(b), with the wavelength also converted to time. The time-

lens measured pulse shows obvious distortions arising from overfilling the effective aperture of the time lens. Unlike the short pulse limit, the long pulse limit is not a fundamental restriction to time-lens operation. The addition of a lens stop in the form of an optical intensity modulator synchronized to the time lens should allow aberration-free operation over larger fields of view.

In conclusion, we have demonstrated a fundamentally new technique for measuring the shape of optical pulses. The technique uses a temporal optical system for time-tofrequency conversion so that an optical spectrum analyzer can be used to measure the temporal intensity of the optical pulse. Since this technique measures the actual pulse shape, it provides more information about the pulse than an autocorrelation. We have shown that it works with non-timebandwidth limited pulses, and also investigated the temporal resolution and temporal field of view. Using an electro-optic phase modulator operating at 5.2 GHz as a time lens, 3.0 ps system time resolution was achieved. Time lenses with more phase modulation should reduce this resolution to the subpicosecond regime. We have also demonstrated the limited field of view of a sinusoidally driven time lens, but note that this can be overcome with the use of an appropriate lens stop.

This work was supported by the Air Force Office of Scientific Research under Contract No. F49620-92-J-0099 and by SDI through the Office of Naval Research under Contract No. N00014-93-1-0443.

- ¹J.-C. M. Diels, J. J. Fontaine, I. C. McMichael, and F. Simoni, Appl. Opt. **24**, 1270 (1985).
- ²R. Trebino and D. J. Kane, J. Opt. Soc. Am. A 10, 1101 (1993).
- ³E. B. Treacy, IEEE J. Quantum Electron. 5, 454 (1969).
- ⁴B. H. Kolner and M. Nazarathy, Opt. Lett. 14, 630 (1989).
- ⁵A. A. Godil, B. A. Auld, and D. M. Bloom, Appl. Phys. Lett. **62**, 1047 (1993).
- ⁶M. T. Kauffman, A. A. Godil, B. A. Auld, W. C. Banyai, and D. M. Bloom, Electron. Lett. **29**, 268 (1993).
- ⁷ Introduction to Fourier Optics, edited by J. W. Goodman (McGraw-Hill, New York, 1968).