Two-Dimensional Metal-Insulator Transition as a Percolation Transition in a High-Mobility Electron System

S. Das Sarma,[1,2] M. P. Lilly,[2] E. H. Hwang,[1] L. N. Pfeiffer,[3] K. W. West,[3] and J. L. Reno[2]

[1]*Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

[2]*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

[3]*Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA*

(Received 25 June 2004; published 5 April 2005)

By carefully analyzing the low temperature density dependence of 2D conductivity in undoped high-mobility n-GaAs heterostructures, we conclude that the 2D metal-insulator transition in this 2D electron system is a density inhomogeneity driven percolation transition due to the breakdown of screening in the random charged impurity disorder background. In particular, our measured conductivity exponent of ~ 1.4 approaches the 2D percolation exponent value of $4/3$ at low temperatures and our experimental data are inconsistent with there being a zero-temperature quantum critical point in our system.

DOI: 10.1103/PhysRevLett.94.136401 PACS numbers: 71.30.+h, 73.40.–c, 73.50.Bk

Ever since the pioneering observation by Kravchenko *et al.* [1] of an apparent two-dimensional (2D) metal-insulator transition (MIT) in Si-metal-oxide-semiconductor field-effect transistor (MOSFET) inversion layers, the nature of the transition has remained a controversial enigma. The 2D MIT has been claimed by some, most notably by Kravchenko and collaborators [1,2], to be an interaction driven and carrier-density tuned $T = 0$ quantum phase transition whereas others have argued [3] that it is a disorder-driven crossover phenomenon between weakly and strongly localized 2D electron states. Density dependent scanning studies of the 2D chemical potential [4], surface acoustic wave measurements on low-density 2D electrons [5], and compressibility measurements [6,7], as analyzed by direct numerical simulations [8] of the energetics of the 2D disordered system, indicate very strong density inhomogeneities in the 2D system around the critical density (n_c) for the putative 2D MIT. Also, the critical density n_c is highly system dependent and varies very strongly with the impurity disorder in the 2D system. In relatively highly disordered Si MOSFETs the typical $n_c \approx 10^{11}\text{cm}^{-2}$ and in very high-quality 2D n-GaAs heterostructures the typical $n_c \sim 10^{9}\text{cm}^{-2}$. The strong dependence of n_c on disorder, the long-range nature of the charged impurity disorder potential in 2D semiconductor structures, the observed density inhomogeneities around n_c, all taken together, suggest a percolation-type transition underlying the 2D MIT. Indeed such a percolation transition for 2D systems was discussed by Efros 17 years ago [9], and has recently been suggested in the context of the 2D MIT phenomena by several authors [8,10–13]. There are a few experimental works in the literature [4,12,14] indicating the 2D MIT to be a percolation transition, but the regime of parameter space (i.e., temperature, density, mobility) studied in this context has been quite limited in contrast to the large number of claims propounding the more popular viewpoint that 2D MIT must be a quantum phase transition. In this Letter we provide experimental evidence in support of the 2D MIT being a percolation transition, at least for our experimental system. We report for the first time in the literature percolation scaling in 2D transport properties ranging over a decade in density and conductivity in a single high-mobility 2D system. We emphasize that ours is the first experimental report of a percolation transition in a 2D *electron* system; the earlier percolation studies [11,12] focused on 2D hole systems. In addition, our 2D electron systems are much higher quality (mobility $\sim 10^7$ cm^2/V s; $n_c \sim 10^9$ cm^{-2}), i.e., less disordered, than the corresponding 2D hole systems (mobility $\sim 10^5$ cm^2/V s; $n_c \sim 10^{10}$ cm^{-2}) studied in Refs. [12,14].

The basic picture of the percolation transition for 2D MIT is simple and highly physically motivated. As the carrier density (n) is lowered in a 2D system, screening becomes progressively weaker and strongly nonlinear. A small decrease in n leads to a large decrease in screening, and eventually to a highly inhomogeneous 2D system as the electron gas becomes unable to screen the disorder potential. This gives rise to a random ''hill-and-valley'' potential landscape with the 2D carriers repelled from the potential hills and accumulating at the potential valleys in contrast to the high carrier density homogeneous situation. Once these ''depletion zones'', associated with the disorder potential hills are numerous enough to prevent percolating conducting paths to span the 2D system, an effective 2D MIT transition takes place with the system being an effective metal (insulator) for $n > (<) n_c$ where n_c is the critical percolation density. This percolation picture is particularly germane to 2D semiconductor systems because the disorder potential here arises from the presence of random charged impurities in the system making electronic screening a key ingredient in the effective disorder. Such a density-driven percolation transition is the hallmark of the long-range Coulomb disorder potential since the Coulomb disorder can be tuned by changing the carrier density through the nonlinear screening mechanism. The key qualitative features of the 2D MIT phenomenon,

namely, the correlation between n_c and the sample quality as well as the density inhomogeneity around n_c, are entirely consistent with a percolation transition in the screened Coulomb disorder potential.

Motivated by these percolation considerations, in this Letter we analyze the power law behavior of $\sigma(n)$ in the context of density dependent screening and a percolation transition. Most of the 2D MIT literature has emphasized the strong temperature dependence in the effective metallic phase, which is now well understood [15] as a manifestation of the strong temperature dependence of the effective Coulombic disorder seen by the low-density 2D carriers through the temperature dependence of screening. While this theory is consistent with higher density data, we present a detailed Boltzmann theory calculation [15] using realistic parameters appropriate for our system to show that the abrupt decrease in conductivity at low density cannot be accounted for using either impurity scattering or localization in a uniform 2D system. If, however, the percolation model for an inhomogenous 2D system is considered, fitting the density dependence of the conductivity data to the percolation scaling form is in agreement with percolation predictions.

The two samples used in this study are undoped GaAs/AlGaAs heterojunctions with field-induced carriers. From the surface, the epilayers consist of 60 nm $n+$ GaAs followed by a 600 nm $Al_{0.3}Ga_{0.7}As$ barrier, and then 1000 nm of GaAs. The 2D electrons are induced with a density that is proportional to the voltage on the $n+$ GaAs layer that serves as a top gate. Standard four terminal lock-in measurements are made to measure the resistance over a wide range of density. Conductivity of these square samples is determined using van der Pauw techniques. In sample A the density spans a range of 0.15 to 7.5×10^{10} cm^{-2} with a very high maximum mobility of 8.5×10^6 cm^2/V s at high density and sample B spans a density range of 0.5 to 13×10^{10} cm^{-2} with a lower mobility of 3.3×10^6 cm^2/V s at high density.

The experimentally measured conductivity $\sigma(n)$ is shown in Fig. 1 along with the calculated [15] $\sigma(n)$ curves. We first discuss a few salient qualitative features of our results. At high densities ($n \gtrsim 10^{10}$ cm^{-2}), the conductivity depends on the density approximately as $\sigma \sim n^{\alpha}$ with $\alpha \approx 1.6$. We emphasize that this is not a strict power law since $\alpha \equiv \alpha(n)$ depends weakly on the density. The high density behavior is completely consistent with our Boltzmann theory based calculations assuming the conductivity being limited by linearly screened charged impurity scattering. This is in agreement with recent work on dilute high-mobility 2D electron [16,17] and hole [18] systems where transport results are understood using Fermi liquid physics. As n decreases, σ starts decreasing faster with decreasing density and the experimental conductivity exponent $\alpha(n)$ becomes strongly density dependent with its value increasing substantially . At the lowest density for sample A, $\alpha \approx 5$, we find that screening in a *homogeneous* electron gas fails qualitatively in explaining the $\sigma(n)$ behavior at low densities whereas it gives quantitatively accurate results at high densities. As has been found from direct numerical simulations [8,9], homogeneous screening of charged impurities breaks down at low carrier densities with the 2D system developing strong inhomogeneities leading to a percolation transition at $n = n_c$. For $n < n_c$, the system is an insulator containing isolated puddles of electrons with no ''metallic'' conducting path spanning through the whole system.

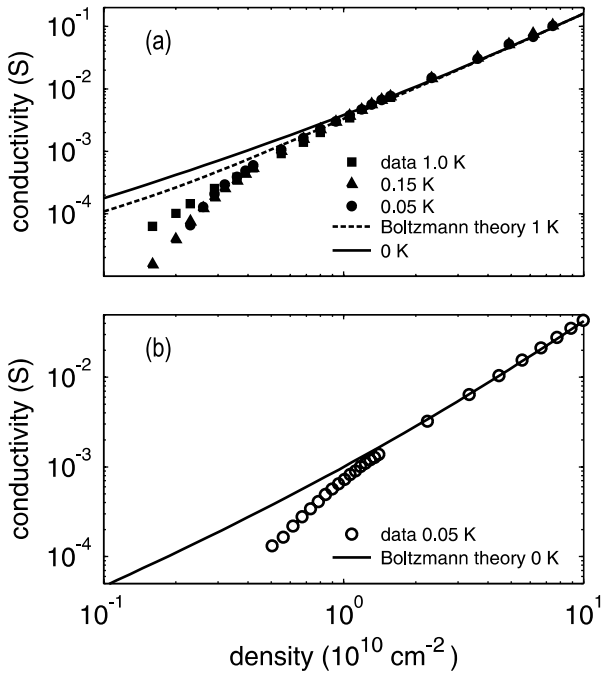

FIG. 1. Experimentally measured (symbols) and Boltzman scattering calculated (lines) values of conductivity for sample A (a) and sample B (b).

The percolation scenario also naturally explains the qualitative aspects of the observed strong temperature dependence of the effective metallic phase in low-disorder high-quality samples for $n \gtrsim n_c$ in the 2D MIT phenomenon. In particular, low disorder ensures a low value of n_c, which then automatically leads to a strongly temperature dependent screening correction to the conductivity as both T/T_F and $q_{TF}/2k_F$ are effectively large [15]. In samples with large values of n_c (low-quality samples) the temperature dependent screening effects are weak in the effective metallic phase as both $q_{TF}/2k_F$ and T/T_F are small [15].

In Fig. 2 we fit our measured conductivity $\sigma(n)$ to the expected [9,19] percolation ''critical'' behavior

$$\sigma(n) = A(n - n_c)^{\delta}, \tag{1}$$

where δ is the conductivity percolation exponent characterizing the vanishing of the conductivity. Fitting the conductivity to Eq. (1) is performed for a range of the low-density data to determine values of δ, n_c, and A. For sample A the data is $1 \times 10^9 < n < 4 \times 10^9$ cm^{-2}, although the results do not depend sensitively on the exact range of data used in the fit. For sample B, the ''goodness'' of the fit

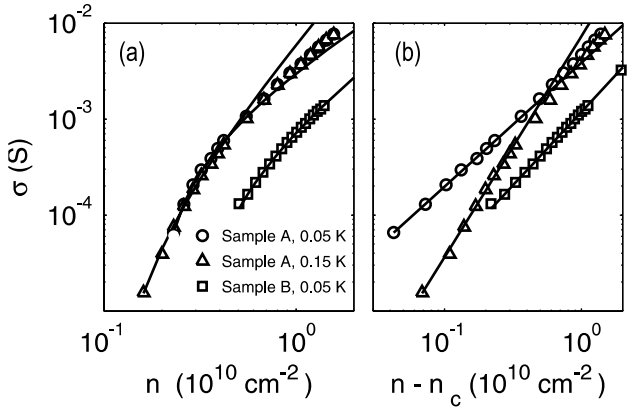

FIG. 2. Experimental conductivity (symbols) fit to Eq. (1) (lines) for both samples A and B as indicated in the legend. The same results are displayed as a function of n (a) and $n - n_c$ (b) to better show the low-density region.

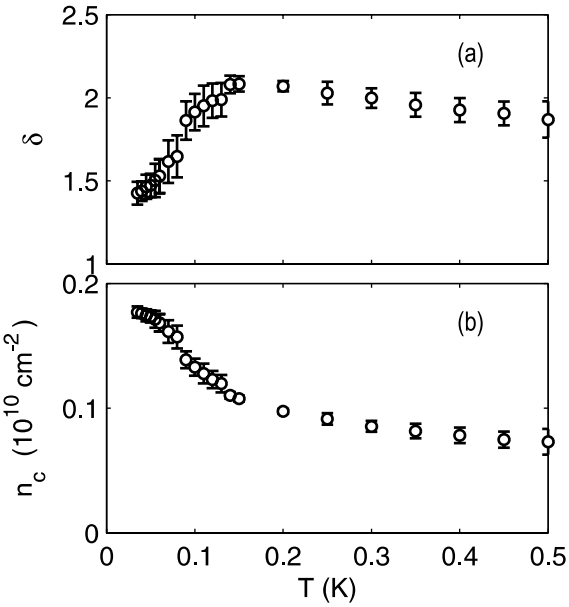

FIG. 3. Parameters δ and n_c for sample A for a range of temperature. Error bars are the 95% confidence interval as determined from the nonlinear fitting shown in Fig. 2. Sample B shows a much weaker temperature dependence of δ and n_c.

is more strongly dependent on the range of data used for fitting. For the data shown, the lowest density point is excluded from the fit, and the fitting range is $5 \times 10^9 < n < 8 \times 10^9$ cm^{-2}. The values of the relevant parameters are $\delta = 1.4 \pm 0.1$ and $n_c = 0.18 \pm 0.01 \times 10^{10}$ cm^{-2} at $T = 50$ mK for sample A and 1.5 ± 0.1 and $n_c = 0.28 \pm 0.02 \times 10^{10}$ cm^{-2} for sample B. These values of δ are close to the known 2D percolation exponent of 4/3 [19].

In Fig. 3 we show the dependence of the critical exponent $\delta(T)$ and the critical density $n_c(T)$ of sample A. At low T, $\delta \approx 1.4$ (1.5 for sample B). The decrease of δ with T for $T > 0.2$ K is caused by the increasing importance of phonon scattering. The increase of δ (for $T < 0.2$ K) and decrease of n_c with increasing T can be well understood by adding a noncritical temperature dependent contribution $f(T)$, where $f(T \to 0) = 0$ and $f(T) \neq 0$ for $T \neq 0$, to the right-hand side of Eq. (1) and realizing that at any finite T the system, by definition, is an ''effective'' metal since the conductivity is nonzero at all finite temperatures. This automatically implies that the effective $\delta(T)$ increases with T and the effective $n_c(T)$ decreases with T, since δ and n_c are strictly defined only at $T = 0$ and at all finite T the system tends to behave as an effective metallic phase. In fact, for sufficiently high values of T (≥ 1 K) one cannot distinguish at all between the metallic and the insulating phase by looking at the conductivity. The fact that critical behavior manifests over about a decade of density and conductivity as well as our finding of a conductivity exponent consistent with the 2D percolation transition, suggests that the 2D MIT in these low-disorder n-GaAs systems is a percolation transition.

It is important to point out that the critical density n_c we obtain by the optimal fitting of experimentally measured $\sigma(n)$ to the conductivity scaling formula of Eq. (1) is consistently lower than the corresponding estimate (n_c') based on the sign of $d\sigma/dT$ as is often done in the literature to distinguish an effective metal from the insulator. This is understandable based on the ''quantum-classical crossover'' scenario discussed in Ref. [15]—at low carrier densities the effective metallic phase exhibits an insulating $d\sigma/dT$ (>0) sign down to rather low temperature, and therefore any determination of the critical density n_c' based purely on the sign of $d\sigma/dT$ will produce $n_c' > n_c$. Thus, the 2D MIT transition density cannot be ascertained by looking for the point $d\sigma/dT|_{n=n_c'} = 0$—the sign of $d\sigma/dT$ is *not* a good indicator for the critical density. Our analyses imply that the transition itself is better understood by concentrating on $\sigma(n)$ at fixed (and very low) values of T whence the 2D MIT can be studied approaching entirely from the metallic regime with the sign of $d\sigma/dT$ playing no role in the analysis. The effect of electron-electron interaction is mainly to ''homogenize'' the density of the electron liquid and therefore decrease the effective value of the n_c [8].

Before concluding we emphasize that one obvious and immediate consequence of the 2D MIT being a disorder-driven percolation transition in our system is that it is manifestly *not* a quantum critical phenomenon as has sometimes been suggested in the literature. We have, in fact, attempted to fit our density and temperature dependent conductivity $\sigma(n, T)$ to the usual quantum critical scaling form [1,2], $\sigma(n, T) \sim F(T/T_0)$, where F is the universal scaling function with $T_0 \equiv T_0(n) \sim |n - n_c|^\gamma$, where the exponent $\gamma \equiv \nu z$ is a product of the correlation exponent ν and the dynamical exponent z. We have not been able to obtain any kind of scaling collapse of our $\sigma(n, T)$ data as a function of T/T_0. The fact that our fitted percolation exponent δ and critical density n_c both depend on temperature already indicates an absence of quantum critical scaling in the data. A more subtle point is that the

usual quantum critical ''fan'' diagram [20] indicates that the scaling regime should expand with increasing temperature (at low enough temperatures) since the quantum critical regime spreads out in density (around n_c) at higher values of T. The expected quantum critical fan behavior is in strong disagreement with our observed behavior of $\sigma(n, T)$ where we find that the best scaling of $\sigma(n) \sim (n - n_c)^{\delta}$ occurs at our lowest measurement temperatures, and the scaling regime definitively and systematically shrinks as temperature increases in sharp contrast to the quantum scaling fan behavior. It cannot, of course, be ruled out that the quantum critical regime ends at temperatures way below our lowest measurement temperature. We believe that our data and the percolation analyses presented in this Letter rules out a quantum critical phenomenon underlying 2D MIT.

In conclusion, we provide compelling evidence involving both experimental transport data and detailed theoretical analyses in support of the 2D MIT phenomenon in n-GaAs being a 2D percolation transition. As screening weakens at low carrier densities, the system breaks up into an inhomogeneous collection of disconnected local ''droplets'' of electrons localized in the potential valleys of the long-range Coulombic disorder potential as was discussed by Efros some years ago [9]. Our measured conductivity exponent $\delta \approx 1.4$–1.5, approaches the expected 2D percolation exponent of $4/3$, and we see impressive scaling behavior in our data consistent with a percolation transition. The percolation transition occurs at a critical density n_c considerably lower than the corresponding critical density n_c' one would infer for 2D MIT based just on the temperature dependence of $\sigma(n, T)$. Our transport data validates the existence of a percolation transition in 2D MIT but cannot provide a qualitative description of the underlying percolation process. We note that a percolation scenario has earlier been invoked [11,12] in describing 2D MIT data in p-GaAs hole samples, but the scaling behavior in these studies was limited to a very narrow density and conductivity range for any firm conclusion to be drawn. More experimental work, involving simultaneous measurements of compressibility, capacitance, transport, local chemical potential, and local conductivity, is required to better understand the details of the percolation mechanism. Our work also brings up important questions regarding the universal nature of 2D MIT itself; for example, we urge experimental analyses of $\sigma(n)$ at fixed low temperatures in *other* 2D systems to check whether this percolation scenario is universal or just applies to our ultrahigh mobility 2D n-GaAs system. In this context it is perhaps quite interesting to point out that even the extensively studied 3D MIT in doped Si systems has been speculated [21] to be a possible inhomogeneity driven percolation transition (similar to what we find for 2D MIT in our work), and such a percolation transition may very well be a generic mechanism for metal-insulator transitions in semiconductors where screened charged impurity potential is the dominant disorder mechanism.

We acknowledge outstanding technical assistance from Denise Tibbetts and Roberto Dunn. This work (S. D. S. and E. H.) is supported by the US-ONR, US-ARO, and NSF at the University of Maryland. This work was performed, in part, at the Center for Integrated Nanotechnologies, a DOE nanoscale science research center operated jointly by Los Alamos and Sandia National Laboratories. Sandia National Laboratories is a multiprogram laboratory operated by Sandia Corporation, a Lockheed-Martin Company, for the U. S. Department of Energy under Contract No. DE-AC04-94AL85000.

[1] S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio, Phys. Rev. B **50**, 8039 (1994); S. V. Kravchenko, *et al.*, Phys. Rev. B **51**, 7038 (1995).
[2] E. Abrahams *et al.*, Rev. Mod. Phys. **73**, 251 (2001).
[3] B. L. Altshuler *et al.*, Physica E (Amsterdam) **9**, 209 (2001).
[4] S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. **84**, 3133 (2000); Science **292**, 1354 (2001).
[5] J. P. Eisenstein (private communication); L. A. Tracy, *et al.*, APS March Meeting, Montreal, 2004.
[6] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **50**, 1760 (1994).
[7] S. C. Dultz and H. W. Jiang, Phys. Rev. Lett. **84**, 4689 (2000).
[8] J. Shi and X. C. Xie, Phys. Rev. Lett. **88**, 086401 (2002).
[9] A. L. Efros, Solid State Commun. **65**, 1281 (1988); **70**, 253 (1989); see also J. A. Nixon and J. H. Davies, Phys. Rev. B **41**, R7929 (1990).
[10] S. He and X. C. Xie, Phys. Rev. Lett. **80**, 3324 (1998).
[11] Y. Meir, Phys. Rev. Lett. **83**, 3506 (1999).
[12] R. Leturcq, D. L'Hôte, R. Tourbot, C. J. Mellor, and M. Henini, Phys. Rev. Lett. **90**, 076402 (2003).
[13] M. M. Fogler, Phys. Rev. B **69**, 121409 (2004).
[14] Ref. [11] presents analysis of experimental data from Y. Hanein *et al.*, Phys. Rev. B **58**, R7520 (1998); Phys. Rev. B **58**, R13338 (1998); and Y. Yaish *et al.*, Phys. Rev. Lett. **84**, 4954 (2000).
[15] S. Das Sarma and E. H. Hwang, Phys. Rev. Lett. **83**, 164 (1999); Phys. Rev. B **68**, 195315 (2003); **69**, 195305 (2004).
[16] M. P. Lilly *et al.*, Phys. Rev. Lett. **90**, 056806 (2003).
[17] J. Zhu, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **90**, 056805 (2003).
[18] See, for example, X. P. A. Gao, A. P. Mills, A. P. Ramirez, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **88**, 166803 (2002); X. P. A. Gao *et al.*, Phys. Rev. Lett. **93**, 256402 (2004); H. Noh *et al.*, Phys. Rev. B **68**, 165308 (2003).
[19] D. Stauffer and A. Aharaony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992); B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, New York, 1984).
[20] S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, Rev. Mod. Phys. **69**, 315 (1997).
[21] S. Bogdanovich, M. P. Sarachik, and R. N. Bhatt, Phys. Rev. B **60**, 2292 (1999).