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Peculiarities of Tamm states formed in degenerate photonic band gaps

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ABSTRACT

The structure of the Tamm state localized at the interface between anisotropic magnetophotonic crystal (anisotropic MPC) and a photonic crystal (PC) made of isotropic dielectrics is studied. It is shown that if the frequency of this state appears within the degenerate band gap then its structure qualitatively differs from the structure of a well-known Tamm state localized at the interface between two one-dimensional PC made of isotropic materials. Since inside the degenerate BG the real part of the Bloch wavenumber differs from the Brillouin value, two Bloch waves with different signs in the real part of the wavenumber and the same sign in the imaginary part have different input impedance values. Moreover, contrary to the case of a PC made of isotropic materials the impedance of each Bloch wave is a tensor. As a consequence to construct a surface state at least three evanescent Bloch waves are required. The conditions that determine the Tamm state frequency also change.

1. Introduction

Within the last decades the electrodynamics of photonic crystals has experienced rapid development. Magnetophotonic structures merit special attention [1]. The introduction of magnetic inclusions into photonic crystals enrich the electrodynamics by new physical phenomena and enlarge the possible applications of photonic crystals.

Due to the existing analogy between PC and crystalline solids many phenomena, well-known in solid state physics, are now observed in photonic crystals (PC). In particular, the surface states of electrons predicted by Tamm in 1934 have been intensively investigated both at optical and microwave frequencies in PC [2–12]. The optical Tamm state can form at the interface of two different PCs or between PC and a medium with negative permittivity or permeability. The frequency $k_0 = \omega/c$ at which the Tamm state lies in the intersection of the band gaps (BGs) of the first and second PCs and is determined by the equality of the PCs input admittances of the evanescent Bloch waves composed this Tamm state:

$$Y_1(k_0) = Y_2(k_0)$$

where the input admittance $Y(k_0)$ (which is electrodynamics analog of logarithmic derivative of psi-function) of a Bloch wave is equal to the ratio of tangential components of electric and magnetic fields H_t/E_t. Thus, this equation is equivalent to the usual Maxwell boundary conditions.

The Tamm state consists of two evanescent Bloch waves decreasing away from the interface [2]. In this communication we study the structure of the Tamm state, which appears in the degenerate BG [11,12]. The degenerate BG may form in magnetophotonic crystals whose primitive cell consists of anisotropic and magnetooptical layers [11–15].

2. Formation of the degenerate BG

Recently it has been shown that the combination of anisotropic and magnetooptic materials may result in the formation of band gaps of a new type [11–17]—the so called degenerate band gap.

To consider the formation of degenerate BG more carefully let us consider a 1D PC made up of anisotropic and isotropic magnetooptic layers [11] of thickness d. At zero magnetization, the MO layers are isotropic and we can choose two independent eigensolutions such that their polarizations coincide with the polarizations of the ordinary and extraordinary waves in the uniaxial crystal layers. In this case the boundary conditions connect the waves of identical polarization and two independent (complementary) Bloch waves have polarizations of ordinary and extraordinary waves and different Bloch wavenumbers $k_{Or}^{(Bl)}$ and $k_{Ex}^{(Bl)}$, respectively. In accordance with the Bragg condition ($2k_{Or,Ex}^{(Bl)}d = \pi n$) at the boundaries of the Brillouin zones each of the solutions has its own band gap (see Fig. 1) [11] (we will refer to such gap as Brillouin band gap). Since the ordinary and extraordinary waves are perfectly independent the possible intersections of dispersion curves have no affect.

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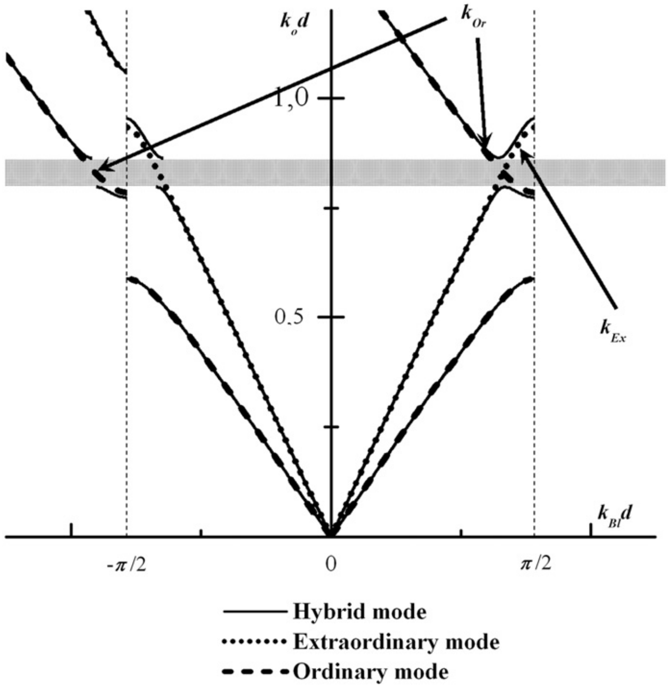

Fig. 1. Reduced (right side of the figure) and extended (left side of the figure) band patterns displaying degenerate band gaps. The solid curves correspond to the magnetic-field case. The dotted curve presents the dispersion of the extraordinary mode. The dashed curve presents the dispersion of the ordinary mode. A gray region corresponds to frequencies of degenerate band gap. The PC's primitive cell consists of a layer of anisotropic material with permittivity ε_{xx}=2.0, ε_{yy}=8.0 and of a MO layer with permittivity ε_{diag}=3.0, ε_{off_diag}=ig=0.5i, the thickness of each layer equals to d.

Magnetization of the PC in a direction perpendicular to the layer interfaces changes the character of the eigensolutions in the previously isotropic and now gyrotropic (magneto-optical) layers. In the gyrotropic medium the eigensolutions are left- or right-circularly polarized waves. Now, the boundary conditions connect waves of different polarizations and, say, left-circular polarized wave excites in the uniaxial crystal layers both ordinary and extraordinary waves and vice a versa linearly polarized wave excites both circularly polarized waves. Thus, waves of different polarization are mixed at the boundaries. The Bloch waves loose their well-defined polarization and become of waves of a hybrid type.

A possible intersection of dispersion curves of ordinary and extraordinary Bloch waves in the reduced zone pattern corresponds to the generalized Bragg condition $k_{Or}^{(Bl)}d+k_{Ex}^{(Bl)}d=\pi$ in the expanded zone pattern. Since the magnetooptical effects are very weak, at first approximation, we can consider the hybrid Bloch waves as slightly disturbed ordinary and extraordinary Bloch waves. Indeed, far from the point of intersection one observes a small disturbance in the Bloch wave numbers and the polarization of the main harmonic is nearly linear. At the point of intersection we observe resonant reflection of the Bloch waves because of the generalized Bragg condition $k_{Or}^{(Bl)}d+k_{Ex}^{(Bl)}d=\pi$. Indeed, the scattering channel when we take into account only cross-polarized reflections is resonant. Thus, at the point of intersection a new BG forms (see Fig. 1).

It is worth noting that this BG forms simultaneously for both Bloch waves, in other words formation of such band gap is degenerate with respect to polarization and we will call such band gaps-degenerate [15]. It should be pointed out that only at the edge of the degenerate BG Bloch wavevectors are the same for different solutions. For frequencies inside degenerate BG Bloch wavevectors in the reduced zone pattern have the special form $k_{1,2}^{(Bl)}(k_0)=\pm(q_1(k_0)+iq_2(k_0))$ and $k_{3,4}^{(Bl)}(k_0)=\pm(q_1(k_0)-iq_2(k_0))$ [15].

3. Tamm state based on degenerate band gap

Now let us consider a Tamm state based on the degenerate band gap in a system consisting of an anisotropic MPC (first PC in Fig. 2) and a PC made up of isotropic components (second PC in Fig. 2). The unit cell of the first PC consists of a uniaxial crystal and a magnetooptical layer and yields a degenerate BG. The unit cell of the second PC consists of two isotropic layers.

Let us consider the frequencies around the intersection of the dispersion curves of ordinary and extraordinary waves of a non-magnetized first PC. It is assumed that such frequencies correspond to the band gap of the second PC. At zero magnetization the transmittance is suppressed by the Bragg reflectance in the second PC (dotted line at Fig. 3).

Application of a magnetic field results in the appearance of a degenerate band gap in the first PC [11] and, respectively, leads to the formation of Tamm states at the boundary of the two PCs. Such state reveals itself as a transparency peak in the transmittance spectra (see Fig. 3).

In the present case only one Tamm state appears as result of magnetization, contrary to the case of a Tamm state at the boundary between an isotropic magnetooptical PC and a PC made up of isotropic components. [2] In the latter case the Tamm state exists without magnetization and splits into two Tamm states upon magnetization (corresponding to right- and left-circularly polarized waves). The present Tamm state is not doubly degenerate with respect to polarization like the Tamm state at the boundary between two PCs made of isotropic layers. Such peculiarities of the present Tamm state is caused by the hybrid nature of the eigensolutions in a PC based on anisotropic and magnetooptical materials.

4. Anisotropy of admittance

The Bloch waves in a 1D PC made up of anisotropic materials with coincident directions of anisotropy axes are TE or TM polarized, having different but scalar impedance like it happens in

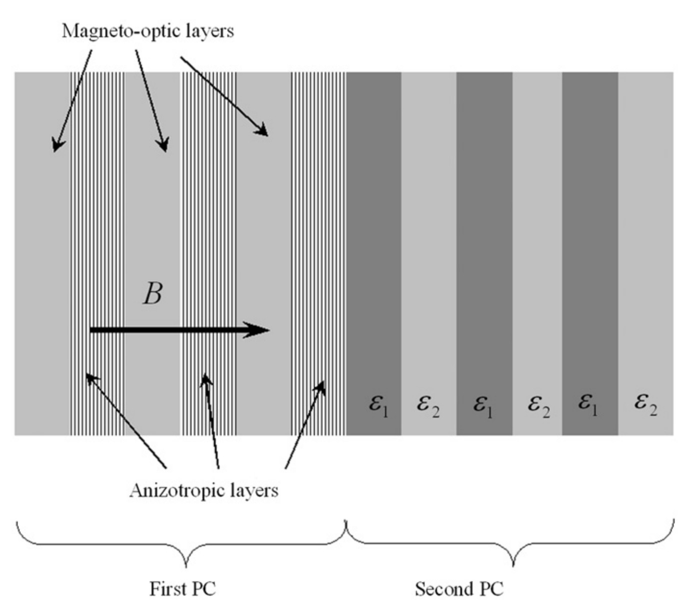

Fig. 2. System under consideration. The unit cell of the first PC consists of a uniaxial crystal (ε_{xx}=2.7, ε_{yy}=5.0) and a magnetooptical layer (ε_{diag}=3.0, ε_{off_diag}=$i\alpha$=0.02i and ε_{off_diag}=0 at zero magnetization). The unit cell of the second PC consists of two isotropic layers with permittivity ε_1=3.1, ε_2=3.4. The thickness of each layer equals d.

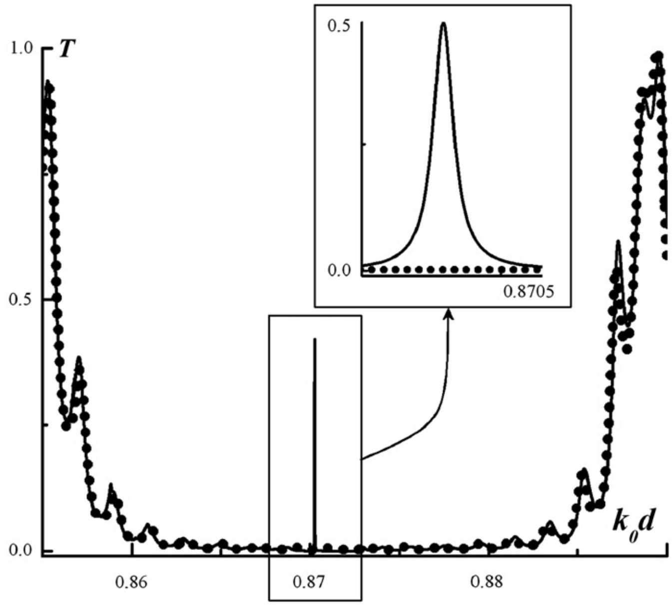

Fig. 3. Propagation coefficient of the system under consideration. The dotted line corresponds to zero magnetization; the solid line corresponds to the magnetized case.

a homogeneous medium. A non-zero angle between the axes or the presence of gyrotropy results in hybridization of the Bloch waves. These hybrid Bloch waves have neither linear nor circular (elliptical) polarization. As a consequence the problem cannot be reduced to two scalar problems with scalar (yet possibly different) impedance values. In the scalar problem the boundary conditions reduce to equality of impedance values on both sides of an interface.

In the case of a primitive cell consisting of layers with anisotropic and gyrotropic permittivities, the Bloch wave consists of four waves differing in polarization and traveling directions in each layer. Waves with different polarizations are characterized by different scalar impedance values. The Bloch wave, as a consequence, is characterized by an impedance value tensor. If the second PC is made of isotropic material we can construct a couple of Bloch waves having any type of polarization: linear, circular or elliptical. In any case there are two Bloch functions with complementary polarizations and the same impedance value. The boundary conditions still reduce to equality of input impedance values. It is obvious that to satisfy such a condition we have to employ both hybrid Bloch waves in order to obtain a symmetric tensor of impedance values.

Let us deduce the required equality following from the boundary conditions and serving as dispersion equation of the surface state. As was mentioned above, two complementary Bloch waves in the anisotropic MPC have different Bloch wave numbers: $k_1(k_0)=q_1(k_0)+iq_2(k_0)$, $k_2(k_0)=-q_1(k_0)+iq_2(k_0)$ and different periodic factors:

$$\vec{E}_\alpha=\begin{pmatrix} f_{\alpha x}(z)\\ f_{\alpha y}(z)\end{pmatrix}e^{ik_1 z},\quad \vec{H}_\alpha=\begin{pmatrix} Y_{\alpha y}f_{\alpha y}(z)\\ -Y_{\alpha x}f_{\alpha x}(z)\end{pmatrix}e^{ik_1 z}$$

where

$$Y_{\alpha\beta}(z)=n_\alpha+\frac{f'_{\alpha\beta}}{f_{\alpha\beta}ik_0}$$

is a tensor of the local impedance values, α=1,2—is a label of the Bloch wave and β=x,y—is a label of the component. To satisfy the boundary conditions we have to identify the fields in the adjoining layers belonging to the different PC. Below, for distinctness, we consider the case when the layer with anisotropic permittivity adjoins the PC made of isotropic materials. The hybrid Bloch waves in this layer may be presented as a sum of two ordinary and two extraordinary waves traveling in opposite directions:

$$\vec{E}_\alpha=\begin{pmatrix} E_{\alpha x}\\ E_{\alpha y}\end{pmatrix}=A_\alpha\begin{pmatrix}1\\0\end{pmatrix}e^{ik_o z}+B_\alpha\begin{pmatrix}1\\0\end{pmatrix}e^{-ik_o z}+C_\alpha\begin{pmatrix}0\\1\end{pmatrix}e^{ik_e z}+D_\alpha\begin{pmatrix}0\\1\end{pmatrix}e^{-ik_e z}$$

and

$$k_0\vec{H}_\alpha=k_0\begin{pmatrix} H_{\alpha x}\\ H_{\alpha y}\end{pmatrix}=k_oA_\alpha\begin{pmatrix}0\\1\end{pmatrix}e^{ik_o z}-k_oB_\alpha\begin{pmatrix}0\\1\end{pmatrix}e^{-ik_o z}+k_eC_\alpha\begin{pmatrix}1\\0\end{pmatrix}e^{ik_e z}-k_eD_\alpha\begin{pmatrix}1\\0\end{pmatrix}e^{-ik_e z}$$

where α=1,2 is a label of the Bloch wave A_i, B_i, C_i, D_i are the phazors of plane waves, which are eigensolutions of the Maxwell equations in the layer, k_o and k_e are the wave numbers of ordinary and extraordinary waves. It is easy to see that in this representation the admittance tensor is diagonal with $Y_{\alpha x}=Y_o=H_x/E_y$ and $Y_{\alpha y}=Y_e=-H_y/E_x$. In the isotropic medium the corresponding admittance values are identical: $Y_x=H_x/E_y=k/k_0=-H_y/E_x=Y_y=Y_i$. Hence we cannot simultaneously satisfy all boundary conditions confining ourselves to a single Bloch wave in MPC. We have to consider a linear combination of two complementary Bloch waves in the anisotropic MPC:

$$\vec{E}=a\begin{pmatrix} f_{1x}(z)\\ f_{1y}(z)\end{pmatrix}e^{ik_1 z}+b\begin{pmatrix} f_{2x}(z)\\ f_{2y}(z)\end{pmatrix}e^{ik_2 z}\quad\text{and}$$

$$\vec{H}=a\begin{pmatrix} Y_{1y}f_{1y}(z)\\ -Y_{1x}f_{1x}(z)\end{pmatrix}e^{ik_1 z}+b\begin{pmatrix} Y_{2y}f_{2y}(z)\\ -Y_{2x}f_{2x}(z)\end{pmatrix}e^{ik_2 z}$$

Taking into account that in the PC layer adjoining MPC the Bloch wave has a form

$$\vec{E}=\begin{pmatrix} E_x\\ E_y\end{pmatrix}=\begin{pmatrix} cf(z)\\ df(z)\end{pmatrix}e^{ikz}\quad\text{and}\quad \vec{H}=\begin{pmatrix} H_x\\ H_y\end{pmatrix}=\begin{pmatrix} d\left(n+\frac{f'}{fik_0}\right)f\\ -c\left(n+\frac{f'}{fik_0}\right)f\end{pmatrix}e^{ikz}$$

the boundary conditions can be written down as

$$\begin{cases} a\begin{pmatrix} f_{1x}\\ f_{1y}\end{pmatrix}+b\begin{pmatrix} f_{2x}\\ f_{2y}\end{pmatrix}=\begin{pmatrix} cf\\ df\end{pmatrix}\\ a\begin{pmatrix} Y_{1y}f_{1y}\\ -Y_{1x}f_{1x}\end{pmatrix}e^{ik_1 z}+b\begin{pmatrix} Y_{2y}f_{2y}\\ -Y_{2x}f_{2x}\end{pmatrix}=\begin{pmatrix} dYf\\ -cYf\end{pmatrix}\end{cases}$$

This linear system (with respect to a,b,c,d) has a non-zero solution if

$$\begin{vmatrix} (Y_{1y}-Y)f_{1y} & (Y_{2y}-Y)f_{2y}\\ (Y_{1x}-Y)f_{1x} & (Y_{2x}-Y)f_{2x}\end{vmatrix}=0$$

Thus, we obtain the equation that determines the frequency of the Tamm state in the degenerate BG.

5. Conclusion

It has been shown that at the interface between a MPC, the primitive cell of which is made up of a layer with anisotropic permittivity and a layer with gyrotropic permittivity magnetophotonic crystal, and a PC, the primitive cell of which is made up of layers with isotropic permittivity, there may appear a surface Tamm state. The frequency of this state lies in the degenerate BG of the first PC and in Brillouin BG of the second one. Contrary to the quantum case and the case when both BG are of the Brillouin type, this Tamm state consists of three evanescent Bloch waves. The necessity of the third Bloch wave is a consequence of the different bases in the PCs.

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