Diffusion thermopower at even-denominator fractions

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We compute the electron diffusion thermopower at compressible quantum Hall states corresponding to even-denominator fractions in the framework of the composite-fermion approach. It is shown that the deviation from the linear low-temperature behavior of the thermopower is dominated by the logarithmic temperature corrections to the conductivity and not to the thermoelectric coefficient, although such terms are present in both quantities. The enhanced magnitude of this effect compared to the zero-field case may allow its observation with the existing experimental techniques. [S0163-1829(96)52644-5]

The thermoelectric effect in metals and semiconductors gives valuable information about the underlying electron-transport processes. A field of a high current activity is the thermoelectric effect in quasi-two-dimensional (2D) heterostructure inversion layers. These systems are characterized by a nearly 2D metallic conduction with very low Fermi energies $E_F \sim 100$ K compared to ordinary metals, which results in a large electron diffusion thermopower. This quantity is experimentally accessible at low enough temperatures in contrast to most of the other parameters related to the thermodynamic properties of 2D electronic systems, such as the electronic specific heat, which are hardly measurable because of the dominant lattice contribution.

In the presence of an electric field $\vec{E}$ and a temperature gradient $\vec{\nabla} T$ the electric current can be written in terms of the conductivity σ_{ij} and the thermoelectric coefficients η_{ij} tensors,

$$J_i = \sigma_{ij} E_j + \eta_{ij} \nabla_j T. \tag{1}$$

The normally measurable quantity is the thermopower tensor $S_{ij} = -\sigma_{ik}^{-1} \eta_{kj}$, which relates $\vec{E}$ and $\vec{\nabla} T$ provided $\vec{J}=0$.

In the presence of a quantizing magnetic field the GaAs/Al_xGa_{1-x}As heterostructures demonstrate the variety of phenomena known as the quantum Hall effect. The corresponding behavior of the electron diffusion thermopower S_{ij} as a function of the filling factor $\nu = 2\pi n_e / B$ is quite complicated. At $\nu = N + \frac{1}{2}$ the theory of noninteracting electrons predicts universal peaks of the diagonal thermopower (Seebeck coefficient) $S_{xx} = \ln 2/e(N+1/2)$ given by the entropy per particle for the half-filled $N+1$st Landau level[1] (here and hereafter we assume the carrier charge e to be of either sign). The thermopower of the integer quantum Hall states ($\nu = N$) vanishes at $T=0$, as the entropy of any number of completely filled Landau levels is zero, and demonstrates the thermally activated behavior for small T.

In the presence of impurities the thermopower tensor develops off-diagonal components increasing in magnitude with the strength of disorder for partially occupied Landau levels, which are now broadened into bands. The S_{xx} maxima at half-integer filling factors, on the contrary, are predicted to reduce by a factor dependent only on the ratio between T and the Landau band width.[2] It implies, in particular, that the S_{xx} maxima increase approximately linearly with the magnetic field B as ν decreases.

The experimental data obtained at $\nu > 4$ agree reasonably well with the above theoretical predictions. However, at small ν, where the system is believed to be in the fractional quantum Hall (FQHE) regime dominated by the electron interactions, both longitudinal and transverse components of the thermopower behave qualitatively differently.

In the insulating phases in the vicinity of $\nu = 1/3$ and 2/7 the diagonal thermopower diverges at $T \to 0$ suggesting the spectrum gap.[3] One might also expect that the zero-entropy argument can be applied to incompressible quantum Hall states at odd-denominator fractions which demonstrate a vanishing diagonal thermopower. At present there are no firm analytical results available for the FQHE states.

In the present paper we analyze the behavior of $S_{xx}(T)$ at primary even-denominator fractions ($\nu \sim 1/\Phi$, where $\Phi = 2,4$, etc.) which correspond to compressible states with no gap. The theoretical framework for our calculations is provided by the theory developed by Halperin, Lee, and Read[4] which explains the metal-like features observed at these fractions[5] as the formation of the Fermi surface of a new sort of fermionic quasiparticles named composite fermions (CF's).

At $T > 0.1$ K the measured thermopower is dominated by phonon drag.[6–8] The phonon drag contribution to S_{xx} scales with temperature approximately as $T^{3.5\pm 0.5}$ and acquires equal values at ν with the same even denominator, such as 1/2 and 3/2 or 1/4 and 3/4, which agrees reasonably well with the recent theoretical estimate based on the picture of the 2D CF coupled to phonons in the substrate.[8]

At low enough T the phonon drag term dies off and one expects the diffusion contribution to take over. To date, the low-temperature measurements which revealed the approximately linear behavior of $S_{xx}(T)$ were only reported on 2D hole systems.[6,7]

The features exhibited by S_{xx} at filling fractions $\nu = (2N+1)/(4N+2\pm 2)$ in the range from 1/3 to 2/3 are strikingly similar to those at high half-integer ν values.[7] This observation lends support to the CF picture where on the level of the mean-field description those fractions correspond to half-filled effective Landau levels of CF ($\nu^* = N + \frac{1}{2}$) in the residual field $B^* = B - 2\pi\Phi n_e$ at $\Phi = 2$.[4]

The data at the primary fractions $\nu=1/2$ and 3/2 were interpreted in Ref. 6 by means of the standard Mott formula for noninteracting fermions at zero field,

$$\eta_{xx}=-\frac{\pi^2 T}{3e}\left(\frac{d\sigma(E)}{dE}\right)\bigg|_{E=E_F} \tag{2}$$

where the energy-dependent Drude conductivity $\sigma(E)=(2\pi e^2/h)N(E)D(E)$ is a product of the density of states $N(E)$ and the diffusion coefficient $D(E)$ determined by the transport time $\tau_{tr}(E)\sim E^p$, which results in the classical Drude thermopower,

$$S_{xx}^{(0)}=\frac{\pi^2 T}{3e}\left(\frac{d\ln\sigma(E)}{dE}\right)\bigg|_{E=E_F}=\frac{\pi^2(p+1)T}{3eE_F}. \tag{3}$$

In the zero-field case formula (3) was conventionally applied to ordinary electrons which undergo scattering from remote as well as background impurities and from the surface roughness.[9] Assuming the applicability of the Matthiessen's rule it was shown in Ref. 9 that all together these three mechanisms lead to a fairly complicated dependence of the exponent p on electron density n_e. At very low densities the scattering by remote ionized donors dominates and yields $p=3/2$. The results obtained in Ref. 9 predict that at higher $n_e\sim 1\times 10^{11}$ cm^{-2} the overall effect of the three mechanisms can be approximated by $p\approx 1$ while at $n_e\approx 7\times 10^{11}$ cm^{-2} the exponent p was found to change sign.

In the framework of the CF theory the only mechanism considered so far was the effect of ionized donors placed on a distance $\xi\sim 10^2$ nm apart from the 2D electron gas.[4] Under the mapping of electrons at $\nu\sim 1/\Phi$ to the CF the Coulomb impurities become sources of the spatially random static magnetic field (RMF) correlated as $\langle B_q^* B_{-q}^*\rangle=4\pi^2\Phi^2 n_i e^{-2\xi q}$, where n_i stands for the impurity concentration.

The RMF scattering appears to be essentially more efficient than the ordinary potential one and leads to a lower value of the exponent p. Namely, the result of the lowest Born approximation $\tau_{\rm tr}^B(E)=\xi\sqrt{2E}m^{3/2}/\pi\Phi^2 n_i$ (Ref. 4) suggests that at low enough n_e the exponent p might be close to 1/2. A more systematic treatment of the RMF problem beyond the lowest Born approximation[10] gives $\tau_{\rm tr}(E)=(2\xi\sqrt{m}/\sqrt{2E})\exp(\pi\Phi^2 n_i/Em)K_1(\pi\Phi^2 n_i/Em)$, from which one readily obtains an even smaller value $p\approx 0.13$.

In the above discussion of the CF thermopower we neglected the effects of the CF gauge interactions which develop beyond the mean-field approximation.[4] The available experimental data seem to suggest that the effect of these interactions on $S_{xx}(T)$ is relatively small in spite of the absence of any small parameter in the present CF theory. Namely, the low-temperature value of the ratio $S_{xx}(3/2)/S_{xx}(1/2)$ was found in Ref. 6 to be close to $E_F(1/2)/E_F(3/2)=\sqrt{3}$ in a good agreement with the mean-field picture of free CF with the effective mass $m^*\sim\nu^{-1/2}$ forming the metal-like state characterized by the Fermi momentum $k_F=[4\pi n_e(1-[\nu]/\nu)]^{1/2}$ on the $[\nu]+1$st partially occupied Landau level (however, the validity of this interpretation for $\nu>1$ was recently questioned in Ref. 11).

Given the complexity of the problem and the previous reports of non-Fermi-liquid–type features revealed by the resistivity measurements at $\nu=1/2$ and 3/2 (Ref. 12) the issue of interaction corrections to the classical Drude thermopower (3) deserves further theoretical analysis.

We note at this point that even in the zero-field case the effects of the electron-electron interactions on the 2D thermopower remain only poorly understood. Therefore as a prelude to our discussion we summarize the results obtained in this field so far.

Following the theoretical predictions of the logarithmic temperature corrections to the 2D electrical conductivity due to both the effects of weak localization and Coulomb interactions (see, for example, Ref. 13 and references therein) a similar effect on the thermopower was discussed in Ref. 14. It was argued in Ref. 14 that not only the conductivity σ_{xx} but also the thermoelectric coefficient η_{xx} receive $\ln T$ corrections. This prediction was refuted in a number of subsequent publications where it was shown that neither weak localization effects[15] nor interference between Coulomb interactions and impurity scattering[16] produce such corrections to the Peltier coefficient Π related to η by the Onsager relation $\Pi=\eta/T$. It was pointed out in Refs. 15 and 16 that the calculation of $\Pi_{ij}={\rm Im}(1/\omega)\int_0^\infty dt e^{it\omega}\langle[Q_i(t),J_j(0)]\rangle$ as a correlation function of the heat $\vec{Q}=(\vec{p}/m)\epsilon$ and the electric current $\vec{J}=e(\vec{p}/m)$ operators requires an application of the finite- temperature formalism or an accurate analytic continuation from imaginary frequencies which had not been done in Ref. 14. As a result, the zero-field diagonal thermopower was predicted to receive $\ln T$ corrections solely from σ_{xx}. To the best of our knowledge an experimental confirmation of such terms remains an open question.

Recently strong although sample-dependent $\ln T$ terms in the resistivity at $\nu=1/2$ and 3/2 were reported.[17] As we already mentioned, in the CF picture the impurity scattering is translated to the RMF problem, which belongs to the unitary ensemble characterized by broken time-reversal symmetry. Therefore the localization effects in the RMF are strongly suppressed which rules out $\ln T$ localization corrections.[4,18] On the other hand, it was shown in Ref. 19 that the interference between CF gauge interactions and impurity scattering indeed leads to $\ln T$ terms which are enhanced by the nonuniversal factor as compared to the well-known exchange correction $\Delta\sigma_{xx}=(e^2/\pi h)(\ln T\tau_{\rm tr})$ due to the Coulomb interaction[13] which is known to be independent of the magnetic field.[20]

In order to estimate quantum corrections to the classical CF thermopower (3) and to facilitate a forthcoming calculation of $\Delta\eta_{ij}^{\rm CF}$ we remind a reader of the basic notions of the CF theory, which was recently used to compute $\Delta\sigma_{ij}^{\rm CF}$.[19]

First we recall that the free CF Green function in the presence of disorder $G_{R(A)}(E,\vec{p})=1/E-p^2/2m\pm i/2\tau$ is determined by the (formally divergent) RMF scattering rate $1/\tau$. It also shows up in the particle-hole diffusion amplitude, which develops a pole (here $\epsilon=E-E_F$):

$$\Gamma(\epsilon,\omega,q)=\frac{1}{2\pi N(E)\tau^2}\frac{1}{i\omega-D(E)q^2}, \tag{4}$$

provided that $\epsilon(\epsilon+\omega)<0$.[18] The gauge invariant physical observables are, however, independent of τ.

It was pointed out in Ref. 19 that in the CF theory the diffusive regime (4) extends up to momentum transfers $q \sim B^{1/2}$ while the energy transfers remain small ($\omega < 1/\tau_{\rm tr}$). The diffusive behavior at large q can be readily seen in the original electron representation where it is due to electron hopping between adjacent Larmor orbits being $\sim B^{-1/2}$ distance apart.

In Ref. 19 we showed that the main negative temperature correction to the CF magnetoconductivity tensor $\sigma_{ij}^{\rm CF}$ comes from the transverse vector gauge coupling mediated by the propagator (here E is the energy of the CF emitting the frequency ω gauge boson):

$$\mathcal{D}_{\perp}(\omega,q) = \frac{1}{-iN(E)D(E)\omega + \chi_q q^2}, \tag{5}$$

where $\chi_q = [1/12\pi^2 N(E)](1/6 + 1/\Phi^2) + [1/(2\pi\Phi)^2]V_q$ is the CF diamagnetic susceptibility determined by the form of the pairwise electronic potential V_q.

In contrast to the zero-field case[20] this correction computed in the first order in $\mathcal{D}_{\perp}(\omega,q)$ depends on the effective magnetic field B^* according to the relations

$$\Delta\sigma_{xx}^{\rm CF}(B^*) = (1 - (\Omega_c^* \tau_{\rm tr})^2)\Delta\sigma_{xx}^{\rm CF},$$

$$\Delta\sigma_{xy}^{\rm CF}(B^*) = 2(\Omega_c^* \tau_{\rm tr})\Delta\sigma_{xx}^{\rm CF}. \tag{6}$$

Keeping the explicit dependence on E_F one can write $\Delta\sigma_{xx}^{\rm CF}$ in the short-range case ($V_q \approx V_0 = 2\pi e^2/\kappa$, where κ is a screening constant) as

$$\Delta\sigma_{xx}^{\rm CF} = \frac{2e^2}{\pi h}(\ln T\tau_{\rm tr})\ln[N(E_F)D(E_F)] \tag{7}$$

whereas in the case of the unscreened Coulomb potential ($V_q = 2\pi e^2/q$) the double-logarithmic terms occur:

$$\Delta\sigma_{xx}^{\rm CF} = \frac{2e^2}{\pi h}(\ln T\tau_{\rm tr})[\ln[N(E_F)D(E_F)] + \tfrac{1}{4}\ln T\tau_{\rm tr}], \tag{8}$$

which reduce the correction (8) with respect to (7) by a factor of 2 in the range of temperatures $E_F[1/(E_F\tau_{\rm tr})^3] < T < 1/\tau_{\rm tr}$ [at lower T the divergency in Eq. (8) is cut off]. Because of the extra logarithm of $N(E_F)D(E_F)$ which equals $k_F l/4\pi$ at $B^* = 0$ the corrections (7) and (8) are stronger in samples of higher density and/or mobility.

It was also argued in Ref. 19 that the higher-order corrections do not alter the above $\ln T$ behavior of $\Delta\sigma_{ij}^{\rm CF}$, which exhibits the diffusive character of the low-energy CF dynamics.

The physical magnetoresistivity tensor is related to the CF magnetoconductivity tensor as follows:[4]

$$\rho_{ij} = \sigma_{ij}^{-1} = (\sigma^{\rm CF})_{ij}^{-1} + \frac{2h}{e^2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{9}$$

while the physical tensor of thermoelectric coefficients η_{ij} is expected to be simply equal to $\eta_{ij}^{\rm CF}$.

On the basis of relation (9) we concluded in Ref. 19 that in contrast to the case of the Coulomb interacting 2D electron gas the CF Hall conductivity acquires nonzero $\ln T$ corrections which lead to the minima of the diagonal resistivity ρ_{xx} as a function of ν in the vicinity of the primary even-denominator fractions. The temperature dependence of ρ_{xx} at these minima exhibits the $\ln T$ behavior whereas the associated Hall resistivity ρ_{xy} shows no such term.

Formula (9) also allows one to understand the well-pronounced symmetric V-shaped maxima of $S_{xx}(B^*)$ observed at $\nu = 1/2$ and 3/2.[6,7] Namely, by using Eq. (9) and the classical Drude-like $\eta_{xy}(B^*) = [\Omega_c^*\tau_{\rm tr}/1 + (\Omega_c^*\tau_{\rm tr})^2]\eta_{xx}(0)$ with $\eta_{xx}(0)$ given by Eq. (2) one obtains $S_{xx}(B^*) - S_{xx}(0) = -[2\pi^2(p+1)/3][E_F T\tau_{\rm tr}^2/\nu^*]$ in the vicinity ($\Omega_c^*\tau_{\rm tr} < 1$) of the primary fractions.

Now we return to the question of gauge interaction corrections to $\eta_{ij}^{\rm CF}$ proceeding along the lines of the formalism developed in Ref. 16 for the zero-field case of Coulomb interacting electrons. A similar procedure yields the leading correction due to the transverse gauge interactions (f_ϵ is the Fermi distribution function):

$$\Delta\eta_{xx} = -\frac{e\tau^2}{2}\int d\omega d\epsilon \frac{\epsilon}{T}\frac{\partial f_\epsilon}{\partial\epsilon}(2f_{\epsilon+\omega} - 1)N^2(E)D^2(E)$$

$$\times \mathrm{Im}\int \frac{d\vec{q}}{(2\pi)^2}\mathcal{D}_{\perp}(\omega,q)\Gamma(\epsilon,\omega,q), \tag{10}$$

which comes from small energy ($\omega\tau < 1$) transfer processes.

It can be readily seen that expression (10) vanishes unless one expands the result of the $\vec{q}$ integration in odd powers of ϵ/E_F. This is an example of the general rule that the diffusion thermopower must vanish in the limit of a zero Fermi surface curvature when the particle-hole symmetry gets restored.

In the conventional case of Coulomb interacting electrons[16] the integral over $\vec{q}$ is independent of ϵ and the expression similar to Eq. (10) vanishes, so that the only contribution to the electron thermopower

$$\Delta S_{xx} = S_{xx}^{(0)}\left(-\frac{\Delta\sigma_{xx}}{\sigma_{xx}} + \frac{\Delta\eta_{xx}}{\eta_{xx}}\right), \tag{11}$$

where $S_{xx}^{(0)}$ is given by Eq. (3), comes from $\Delta\sigma_{xx}$ and behaves at $T < 1/\tau_{\rm tr}$ as

$$\Delta S_{xx}(B=0) \approx -\frac{\pi(p+1)T}{3eE_F^2\tau_{\rm tr}}(\ln T\tau_{\rm tr}) \tag{12}$$

The zero-field electron diffusion thermopower correction (12) is further decreased by the Hartree term[16] whose magnitude depends on the screening length κ^{-1} of the Coulomb potential.

In the case of CF governed by transverse gauge interactions the integrand in Eq. (10) has an extra ϵ dependence due to the factor $\ln[N(E)D(E)]$ [which is the same as in Eqs. (7) and (8)] arising from the integral over $\vec{q}$. As a result, we obtain [p is the same as in (3)]

$$\Delta\eta_{xx}^{\rm CF} = -\frac{eT(p+1)}{12E_F}\ln T\tau_{\rm tr}. \tag{13}$$

It is worthwhile to note that corrections (7), (8), and (13) satisfy the Mott formula (2). Another known example of an approximate validity of the Mott formula for the case of interacting fermions is provided by the electron-phonon interaction in the diffusive regime $ql<1$.[21]

Repeating the calculations at finite B^* we also obtain that the corrections to the components of the tensor $\Delta\eta_{ij}^{\mathrm{CF}}(B^*)$ obey the relations analogous to Eq. (6).

Despite the fact that η_{ij}^{CF} receives a nonzero contribution (13), the overall correction to the Drude thermopower (11) is dominated by $\Delta\sigma_{ij}^{\mathrm{CF}}$ given by Eqs. (7) and (8). In the case of the screened electron Coulomb potential which appears to be better consistent[19] with the $\ln T$ behavior of the conductivity at $\nu=1/2$ and 3/2 reported in Ref. 17 the correction to the Drude thermopower is

$$\Delta S_{xx}=-\frac{2\pi(p+1)T}{3eE_F^2\tau_{\mathrm{tr}}}(\ln T\tau_{\mathrm{tr}})\ln k_F l. \tag{14}$$

Correction (14) developing at $T<1/\tau_{\mathrm{tr}}$ is enhanced as compared to Eq. (12) due to the extra factor $\ln(k_F l)^2$ and the absence of the corresponding Hartree term because of the singular behavior of $\mathcal{D}_\perp(0,q)$ at $q\to 0$.[19]

Although the available experimental data[6,7] are not sufficient to extract the nonlinear $T\ln T$ contribution, the enhanced magnitude of the effect may, in principle, provide its observation at $T\sim 10-100$ mK with improved techniques.

Before concluding the discussion of the nonlinear terms in $S_{xx}(T)$ we note that in the conventional zero-field case the subject of an even greater controversy is a role of large momenta transfer $(ql>1)$ processes in the thermopower renormalization.

Such effects were previously discussed in both cases of the 3D electron-phonon[22] and electron-electron[23] interactions. It was argued in Refs. 22 and 23 that the main contribution to η_{xx} comes from processes involving virtual bosons (phonons or plasmons). In the problem at hand this kind of correction is associated with the real part of the transverse gauge propagator as opposed to the kinetic terms, which describe the effects of real gauge bosons and contain $\mathrm{Im}\mathcal{D}_\perp(\omega,q)$.

Naively, these terms would lead to a contribution $\Delta' S_{xx}\sim(T\Phi^2k_F/e^3mE_F)\ln E_F/T$ for the unscreened Coulomb CF problem $[\Delta' S_{xx}\sim(1/e)(\Phi^2T/E_F)^{2/3}$ for the screened case], which one could conceivably relate to the singular CF effective mass renormalization[4]) via $E_F\sim 1/m^*(T)$.

Although such terms are indeed expected in a general time reversal symmetric 2D gauge theory[24] they do not appear in the CF problem in accordance with the fact that the CF dynamics remains diffusive up to $q\sim k_F$.[19] A more elaborated analysis, which is required to settle this subtle issue, demonstrates that time-reversal symmetry breaking inherent in the CF theory[4] is crucially important. The details will be presented elsewhere.

To summarize, we discuss the disorder and interaction effects on the diffusion thermopower of composite fermions in the vicinity of the primary even-denominator fractions. We show that in contrast to the zero-field case of Coulomb interacting electrons the thermoelectric coefficient of composite fermions acquires the $\ln T$ interference correction resulting from small momentum transfer processes. However, the main $T\ln T$ correction to the diagonal thermopower arises mostly from the composite-fermion conductivity. The enhanced magnitude of this term compared to the zero-field case makes it in principle possible to observe such a term experimentally, which would provide a new significant test for the composite-fermion theory.

[1] S. M. Girvin and M. Jonson, J. Phys. C **15**, L1147 (1982).

[2] W. Zawadski and R. Lassnig, Surf. Sci. **142**, 225 (1984); M. Jonson and S. M. Girvin, Phys. Rev. B **29**, 1939 (1984); H. Oji, *ibid.* **29**, 3148 (1984).

[3] V. Bayot *et al.*, Europhys. Lett. **25**, 613 (1994).

[4] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[5] R. L. Willett *et al.*, Phys. Rev. Lett. **65**, 112 (1990); H. W. Jiang *et al.*, *ibid.* **65**, 633 (1990); R. L. Willett *et al.*, *ibid.* **71**, 3846 (1993).

[6] X. Ying *et al.*, Phys. Rev. B **50**, 4969 (1994).

[7] V. Bayot *et al.*, Phys. Rev. B **52**, R8621 (1995).

[8] B. Tieke *et al.*, Phys. Rev. Lett. **76**, 3630 (1996).

[9] W. Walukiewicz, H. E. Ruda, J. Lagowski, and H. C. Gatos, Phys. Rev. B **30**, 4571 (1984); V. C. Karavolas and P. N. Butcher, J. Phys. Condens. Matter **3**, 2597 (1991).

[10] D. V. Khveshchenko, Phys. Rev. Lett. **77**, 1817 (1996).

[11] N. R. Cooper, B. I. Halperin, and I. M. Ruzin (unpublished).

[12] R. R. Du *et al.*, Phys. Rev. Lett. **73**, 3274 (1994); H. C. Manoharan *et al.*, *ibid.* **73**, 3270 (1994); R. L. Willett *et al.*, *ibid.* **75**, 2988 (1995); P. T. Coleridge *et al.*, Phys. Rev. B **52**, R11 603 (1995).

[13] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[14] C. S. Ting, A. Houghton, and J. R. Senna, Phys. Rev. B **25**, 1439 (1982).

[15] V. V. Afonin, Y. M. Galperin, and V. L. Gurevich, Zh. Eksp. Teor. Fiz. **87**, 335 (1984) [Sov. Phys. JETP **60**, 194 (1984)]; C. Castellani, C. Di Castro, M. Grilli, and G. Strinati, Phys. Rev. B **37**, 6663 (1988).

[16] J. W. P. Hsu, A. Kapitulnik, and M. Y. Reizer, Phys. Rev. B **40**, 7513 (1989).

[17] L. P. Rokhinson *et al.*, Phys. Rev. B **52**, R11 588 (1995); W. Kang *et al.*, Phys. Rev. Lett. **75**, 4106 (1995); R. Haug *et al.* (unpublished).

[18] A. G. Aronov, A. D. Mirlin, and P. Wolfle, Phys. Rev. B **49**, 16 609 (1994); V. I. Falko, *ibid.* **50**, 17 406 (1994).

[19] D. V. Khveshchenko, Phys. Rev. Lett. **77**, 362 (1966); (unpublished).

[20] A. Houghton, J. R. Senna, and S. C. Ying, Phys. Rev. B **25**, 2196, 6468 (1982); S. M. Girvin, M. Jonson, and P. A. Lee, Phys. Rev. B **26**, 1651 (1982).

[21] M. Jonson and G. D. Mahan, Phys. Rev. B **42**, 9350 (1990).

[22] M. Y. Reizer and A. Sergeev, Zh. Eksp. Teor. Fiz. **93,** 2191 (1987) [Sov. Phys. JETP **66**, 1250 (1988)].

[23] M. Y. Reizer, Phys. Rev. B **44**, 12 701 (1991).

[24] D. V. Khveshchenko and M. Y. Reizer (unpublished).