15 April 2002

ELSEVIER

Optics Communications 205 (2002) 221–226

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Stable propagation of ultrashort optical pulses in modified higher-order nonlinear Schroedinger equation

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Received 25 September 2001; received in revised form 10 December 2001; accepted 14 February 2002

Abstract

Stable propagation of single ultrashort (subpicosecond or femtosecond) optical pulses and pulse trains is obtained in numerical solutions of a higher-order nonlinear Schroedinger equation with bandwidth limited amplification and nonlinear gain. It is shown that stable single pulse propagation can be achieved for a broad class of pulses with different initial shapes, and that pulse trains can propagate stably without interaction under certain conditions. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 42.81.Dp; 42.65.R; 42.65.W

Keywords: Optical communication; Optical pulse; Propagation; Interaction

Soliton propagation in a single mode optical fiber was first suggested by Hasegawa and Tappert [1] for high-bit-rate optical communication. Propagation of a picosecond optical pulse governed by the nonlinear Schroedinger (NLS) equation can be realized by balancing the anomalous group velocity dispersion (GVD) and the Kerr nonlinearity or self-phase modulation (SPM) [2]. In order to increase the bit rate of a single channel, the transmission of ultrashort (subpicosecond or femtosecond) optical pulses is necessary. For ultrashort soliton pulses (USP), the NLS equation has to be converted into a higher-order nonlinear Schroedinger (HNLS) equation [3] in which effects such as third-order dispersion (TOD), self-steepening and self-frequency shift are considered. In the absence of the self-frequency shift term, a number of solitons, both bright and dark, or solitary wave solutions have been found in recent years in the balance between GVD, SPM, TOD, and self-steepening effects [4–13]. However, in the presence of self-frequency effect, the exact solitary wave or soliton solution has not, to our knowledge, been found yet. So stable USP transmission is still an open topic in this case. In addition, in actual fiber transmission system, loss is inevitable and the pulse is often deteriorated by loss, so an amplifier has to be employed to compensate the

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PII: S0030-4018(02)01316-0

loss. When the gain bandwidth of the amplifier is comparable to the spectral width of the ultrashort optical pulse, the effects of frequency- and intensity-dependent gains must be considered. These effects may lead to eventual instability of soliton pulse. However, Kodama et al. [14] suggested that the introduction of nonlinear gain, or the combined operation of gain and saturable absorption is quite effective for stabilizing the soliton propagation. When all these effects are considered, the HNLS equation must be replaced by the modified HNLS (MHNLS) equation.

The mathematical model for MHNLS equation describing all of the effects mentioned above is as follows:

$$\begin{aligned}&\Psi_\varsigma + \frac{\mathrm{i}}{2}\beta_2\Psi_{\tau\tau} - \tfrac{1}{6}\beta_3\Psi_{\tau\tau\tau} - \mathrm{i}\gamma|\Psi|^2\Psi \\ &\quad + a_1\left(|\Psi|^2\Psi\right)_\tau + \mathrm{i}a_2\left(|\Psi|^2\right)_\tau\Psi \\ &\qquad = a_0\Psi + b_2\Psi_{\tau\tau} + b_n|\Psi|^2\Psi + a_5|\Psi|^4\Psi, \end{aligned} \tag{1}$$

where ς is the propagation distance, τ is the retarded time, Ψ is the complex envelope of the electric field. β_2 and β_3 represent the net GVD and the TOD, respectively, γ is Kerr nonlinear parameter, $a_1 = 2\gamma/\omega_0$ is the coefficient of the cubic derivative term and is responsible for self-steepening of the pulse edge and ω_0 is the carrier frequency, $a_2 = \gamma T_{\mathrm{R}}$ (T_{R} is the slope of Raman gain) that results from the time-retarded induced Raman process and is responsible for the soliton self-frequency shift, a_0 is the linear gain ($a_0 > 0$) or loss ($a_0 < 0$) coefficient at the carrier frequency ω_0, b_2 describes the effect of spectral limitation due to bandwidth limited amplification and/or spectral filtering which is inverse proportional to gain bandwidth, b_n accounts for nonlinear gain/absorption processes, a_5 represents a higher-order correction to the nonlinear amplification/absorption namely the saturation to the nonlinear gain or loss.

In the absence of linear frequency- and intensity-dependent gain/or absorption ($a_0 = b_2 = b_n = a_5 = 0$), Eq. (1) reduces to the standard HNLS equation mentioned in [15], which has been extensively investigated in recent years. But to our knowledge, until now no one has considered the system described by Eq. (1) although it is worthy of investigation in actual ultrashort pulse transmission line.

In this paper, we investigate the propagation of the optical pulse by employing numerical split-step Fourier method to solve Eq. (1). It is found that a stable hyperbolic secant pulse can exist under the balance of all these effects: GVD, SPM, TOD, self-steepening, self-frequency shift, linear loss, band-limited filter, nonlinear gain and gain saturation. In addition, the interaction of adjacent pulses is discussed. The results show that appropriate phase-difference may be helpful for suppressing the interaction between pulses to some extent. For multi-pulse propagation there is a threshold for the separation between two adjacent pulses, when the separation is larger than the threshold, two or more pulses can stably transmit without clear interaction in a given distance, but when the separation is smaller than the threshold, the pulse train will attract each other and converge into one pulse along its transmission instead of transmitting separately. Finally, we present numerically 1-bit (8-pulses) propagation without clear interaction up to a distance of 1500 m.

For the reason of simplicity, we normalize Eq. (1) by scaling with: $U = \Psi/\sqrt{P_0}$, $t = \tau/T_0$, $z = \varsigma/L_{\mathrm{D}}$, respectively, where P_0 is proportional to peak power, T_0 is proportional to pulse width and L_{D} is so-called GVD length given by $L_{\mathrm{D}} = (T_0^2/|\beta_2|)$. Then Eq. (1) can be rewritten by

$$\begin{aligned}&U_z - \mathrm{i}\left[\tfrac{1}{2}U_{tt} + |U|^2U\right] - c_3U_{ttt} \\ &\quad - c_s\left(|U|^2U\right)_t - \mathrm{i}c_v\left(|U|^2\right)_tU \\ &\qquad = \delta_0U + \delta_2U_{tt} + \delta_n|U|^2U + \delta_5|U|^4U, \end{aligned} \tag{2}$$

where $C_3 = \beta_3/6|\beta_2|T_0$, $c_s = -(2/\omega_0T_0)c_n$, $c_v = -T_R/T_0$, $\delta_0 = L_{\mathrm{D}}a_0$, $\delta_2 = (L_{\mathrm{D}}/T_0^2)b_2$, $\delta_n = L_DP_0b_n$, and $\delta_5 = P_0^2L_Da_5$. Here we have set $P_0 = 1/(\gamma L_{\mathrm{D}})$, without loss of generality. For a pulse of $T_0 = 100$ fs, if we choose the typical parameter values for conventional single mode fiber as follows [15, 16] $\beta_2 = -20\ \mathrm{ps}^2/\mathrm{km}$, $\beta_3 = -0.1\ \mathrm{ps}^3/\mathrm{km}$, $\gamma = 20$ $\mathrm{W}^{-1}\ \mathrm{km}^{-1}$ and $T_{\mathrm{R}} = 5$ fs, then we can determine $c_3 \approx -0.008$, $c_s \approx -0.025$, $c_v = -0.05$, $L_{\mathrm{D}} = 0.5$ m and $P_0 = 100$ W. In addition, the corresponding parameters of modified terms can be given by [17]: $\delta_0 = -0.05, \delta_2 = 0.3, \delta_n = 0.5, \delta_5 = -0.34$.

Let us now consider the stable pulse transmission of Eq. (2). The evolution of single light pulse up to a distance of $z = 500$ ($\varsigma = 250$ m) is shown in Fig. 1(a), where the initial pulse is selected as $\mathrm{sech}(t)$. From the figure we can see that a solitary pulse can propagate stably in the system. In fact, at the beginning of the evolution process, the given initial pulse experiences a self-adjusting process and having walked a short distance, a stable light pulse transmission is approached eventually. The self-adjusting process can be seen clearly in Fig. 1(b) which shows the temporal width versus the propagation distance of the pulse of Fig. 1(a) within the distance of $z = 100$.

It is noteworthy that although the effects of higher terms are considered, the shape of the pulse remains unchanged. We haven't found the asymmetrical broadening in the time domain caused by TOD and the asymmetrical spectral broadening of the pulse made by self-steepening and even the red-shift of the spectrum caused by the self-frequency shift [18]. So there may exist a balance of the co-operation of all these effects: GVD, SPM, TOD, self-steepening, self-frequency shift, linear loss, band-limited filter, nonlinear gain and gain saturation, which lead to a stable pulse propagation. In order to prove this inference, we investigate the evolution of the optical pulse along the fiber for MHNLS Eq. (2) without the modified terms ($\delta_i = 0$). In this case, the TOD value aforementioned will cause dispersion waves on the tail of the pulse in a very short distance (in our case $z = 10$). The result is the same as that indicated in many papers (for example, see [15]). However, when the modified terms are added, as indicated in Figs. 1(a) and (b), these phenomena cannot be observed any more. In fact, when the modified terms are added, even the TOD value is increased to 10 times the value mentioned above, the pulse can still propagate stably without any asymmetrical broadening in the time domain and any dispersion wave on its tail. Similarly, if the modified terms are omitted, the phenomenon of self-frequency shift is clearly observed in a short distance. But when the modified terms are considered, as the contour of the spectrum of Fig. 1(a) shown in Fig. 2, the so-called self-frequency-shift cannot be observed even in such a long distance. In fact, there is barely any change in the spectrum after the stable pulse is formed. By the way, although the Raman gain slope T_R is generally selected as 3 or 5 fs [15,19,20], we find that even if it increases to 30 fs, the self-frequency shift phenomenon can still be suppressed completely. Therefore, from these investigations we can infer that there may exist a balance among all the effects to keep the stable pulse propagation.

By observing the pulse's half maximum spectral width and temporal width versus transmission distance for the case of Fig. 1(a), we find that at the beginning of the process both of them are oscillating with comparatively large amplitudes, then they quickly stabilize to fixed values as shown in Fig. 1(b). By fitting the transmission data we find that the stable pulse is Sech-shaped. Moreover we calculate the product of pulse temporal width

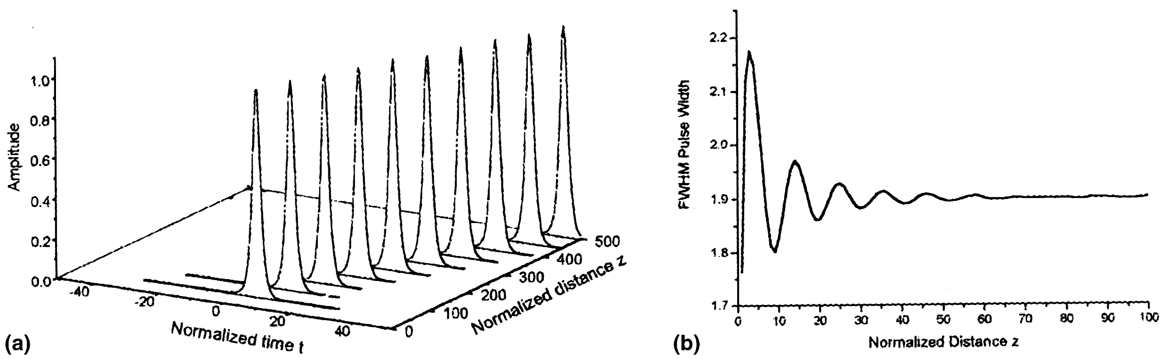

Fig. 1. (a) The stable evolution of a solitary pulse in MHNLSE up to a distance of 500. (b) Pulse width versus distance of (a).

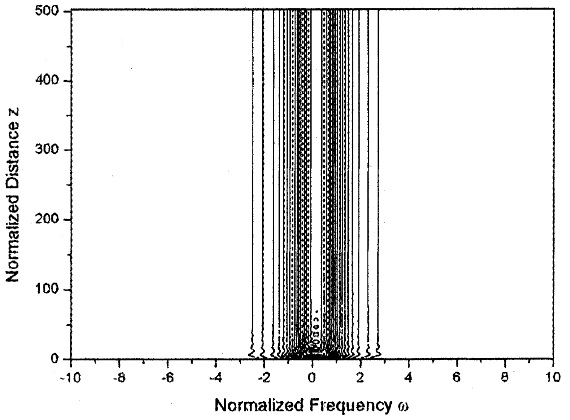

Fig. 2. Contour plot of the spectrum for the soliton-like pulse of Fig. 1(a).

(PW) and spectrum width (FW), which is around 0.34. This is a little larger than that of the exact light solitary wave (Sech-shaped), which equals to 0.32 approximately. This shows that the stable pulse might be expressed as a stable solitary solution with a little chirp. However the chirp is so small that it can't be seen from the real and the imaginary parts of the stable pulse. What made it important is that the stable solitary wave can be excited not only by Sech-shaped initial pulse but also by Gaussian-shaped. Detailed numerical studies reveal that when the Gaussian-shaped pulse is used as an initial pulse, there is threshold for the initial amplitude to form the stable pulse. For example, for the initial Gaussian-shaped input pulse $\Psi(t,0) = A_0 \exp(-t^2)$, A_0 must be larger than or equal to 1.08, otherwise, the initial pulse will decay to zero instead of forming a stable soliton pulse. In addition, the stable pulses obtained by different initial pulses are the same in shape. It should be noted that, although we only show the stable pulse transmission to a distance of $z = 500$ ($\varsigma = 250$ m) in Fig. 1(a), in fact, by investigating the evolution of the pulse in a much longer distance of $z = 10\,000$ ($\varsigma = 5000$ m) we find that the pulse can still propagate without any distortion. It implies that the undistorted stable pulse might propagate in an optical communication system for a practical distance.

It should be pointed out that the parameter values of the modified terms must be confined to some extent in order to realize stable propagation of the pulse. We have studied the evolution behavior of the pulse as one of these parameters changes but with the others keeping the forementioned values. The numerical results show that the undistorted stable pulse can be formed in certain regions. Such as when only δ_0 is varied, the range is $-0.08 \leqslant \delta_0 \leqslant 0$ with others the same as that given before. Similarly, when only δ_2, δ_n or δ_5 is varied, respectively, the range is correspondingly $0.1 \leqslant \delta_2 \leqslant 0.6$, $0.5 \leqslant \delta_n \leqslant 1.5$, or $-0.5 \leqslant \delta_5 \leqslant -0.1$, respectively. Out of the ranges, the pulse either decays to zero or breaks up. However, this doesn't imply that the regions for a stable pulse transmission are obtained completely. What would happen if several of these parameters were varied? It is still an open question.

Since the single solitary pulse can transmit stably in such a system, then what about the pulse train? Can they propagate without interaction or if the interaction exists, what will they be? We study them numerically by giving two parallel pulses as initial pulses that are expressed as:

$$\Psi(t,0) = \operatorname{sech}(t - T)\exp(\mathrm{i}\theta_1) + \operatorname{sech}(t + T)\exp(\mathrm{i}\theta_2).$$

The different results of dual-pulse transmission with different phase-differences but same separation for the initial input pulses are shown in Fig. 3 and Fig. 4, where $T = 3.5$ and $\theta = \theta_2 - \theta_1 = 0$ and π, respectively. Fig. 3 shows that in the absence of phase-difference ($\theta = 0$) the two input pulses attract each other and merge into one pulse quickly. But in the case of $\theta = \pi$, see Fig. 4, the two input pulses can propagate a longer distance before they merge into one. So for multi-pulses propagation, a π phase-difference may be helpful for suppressing the interaction between pulses to some extent. However, in both the two cases, the pulse-pair attract each other and eventually merge into a single pulse. This indicates from another aspect that there exists stable single solitary pulse for the system and that the initial condition of forming the solitary pulse is more broad. But as it is shown, when the two initial pulses are too close, they cannot propagate separately in the given distance without interaction. In contrast, the propagation

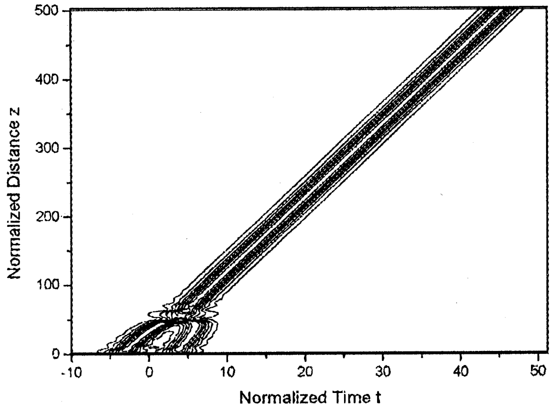

Fig. 3. Evolution contour of pulse-pair without phase-difference up to a distance of 500 for $T = 3.5$.

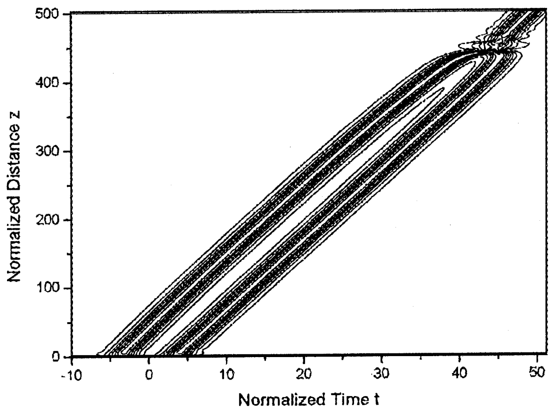

Fig. 4. Evolution contour of pulse-pair with phase-difference π up to a distance of 500 for $T = 3.5$.

of two pulses in the case of $T = 5.0$ is shown in Fig. 5, where the phase-difference $\theta = 0$. From it we can see clearly that there are no interaction between them. It means that, if the separation is large enough for the given propagation distance, it is possible to achieve stable pulse-pair transmission without interaction over a long distance even for zero phase-difference.

From the results obtained above, stable pulse-pair transmission can be achieved only if the separation between input pulses is large enough. But the broad-band information transmission requires a high repetition rate of soliton pulse, i.e., the separation should be short enough. So it is useful

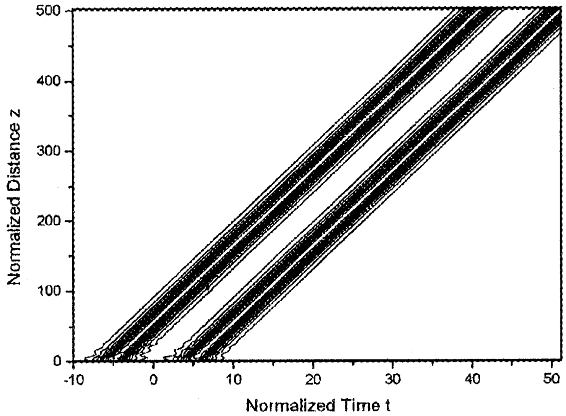

Fig. 5. Evolution contour of pulse-pair without phase-difference up to a distance of 500.

to find the optimized value that not only can insure no clear interactions among pulses but also can insure highest repetition rate for optical pulse propagation. By detailed numerical calculations, we find that for the distance of $z = 500$ ($\varsigma = 250$ m), if there is no phase-difference between adjacent pulses, the separation threshold between input pulses should be $T_{\mathrm{th}} = 4.4$, and if there is phase-difference π, the threshold should be $T_{\mathrm{th}} = 4.2$ which means that when $T < T_{\mathrm{th}}$, the two pulses always attract each other and merge into one in the distance, and when $T > T_{\mathrm{th}}$, the interaction cannot be seen in the given transmission distance. By the way, we found that the phase-difference π is more effective for weakening the interaction only when the initial separation is comparatively smaller. In addition, to confirm the stable transmission of multi-pulses, we investigated the propagation of three and four pulses and found that the results are similar to those results for pulse-pair.

Finally, the pulse train propagation of 1-bit (8-pulses) is shown in Fig. 6. Unlike the value of TOD selected in Fig. 5, here we choose it as $c_3 = -0.055$ and find that the velocity shift seems to be sensitive to the value of TOD. In addition, the phase-difference is given as $\theta = 0$. And the initial separation is selected as $T = 5.0$, which corresponds to the conventional case, namely the space between pulses is 10 times of the pulse width. From the figure we can see that the pulse train can transmit a distance up to $z = 500$ ($\varsigma = 250$ m)

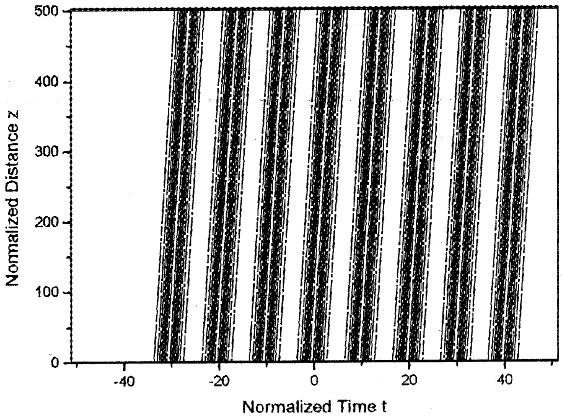

Fig. 6. Evolution contour of 8-pulse without phase-difference up to a distance of 500.

without clear interaction. Moreover, we cannot find the interaction of these pulses even for a longer distance up to $z = 3000$, which corresponds to 1500 m. It is expectable that the stable pulse train may transmit in an optical communication system for a practical distance.

In conclusion, we have studied the pulse transmission for a modified HNLS equation with bandwidth limited amplification and nonlinear gain by numerical simulation. It is found that a stable optical pulse with hyperbolic secant shape can exist under the cooperation of GVD, SPM, TOD, self-steepening, self-frequency shift, linear loss, band-limited filter, nonlinear gain and gain saturation. In addition, the interaction of adjacent pulses is also discussed. The results show that appropriate phase-difference may be helpful to suppress the interaction between pulses to some extent. For multi-pulse propagation there is a threshold for the separation between two adjacent pulses, when the separation is smaller than the threshold, the pulse train will attract each other and converge into one pulse along its transmission instead of transmitting separately. Finally, we present numerically 1-bit (8-pulses) propagation without clear interaction up to a distance of 1500 m. It may be significant for high-bit-rate optical communication in a practical distance.

Acknowledgements

The authors wish to thank gratefully the anonymous referee for insightful comments and valuable suggestions in English and thank gratefully Professor Su Dachun for the English revision of the manuscript. This research was supported by National Natural Science Foundation of China grant 10074041 and Provincial Natural Science Foundation of Shanxi grant 991004. Huiping Tian's present e-mail address is: tianhuip@mail.sxu.edu.cn.

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