Impurity center in a semiconductor quantum ring in the presence of crossed magnetic and electric fields

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An analytical approach to the problem of an impurity electron positioned in a quantum ring (QR) in the presence of crossed axially directed homogeneous magnetic and radially directed electric fields is developed. The quantum well wire and quantum disk regimes of the QR and weak and strong magnetic fields as well as low and high QR's are considered. The analytical dependences of the total and binding energies of the impurity electron on the strengths of the external fields, the radii, and height of the QR and the position of the impurity center within the QR are obtained. It is shown that if the QR confinement and/or magnetic field increase, the binding energy also increases. The binding energy reaches a maximum for the impurity center positioned at the midplane perpendicular to the symmetry axis of the QR. For the quantum disk regime the binding energy decreases while shifting the impurity from the internal surface towards the external one. The effects due to the confinement and magnetic field can be balanced by those produced by a radially directed electric field. For a relatively narrow QR the impurity influences the oscillations of the ground electron energy as a function of the magnetic field only marginally (magnetostatic Aharonov-Bohm effect). Estimates of the linear electron densities needed to bring in balance the blue energy shifts caused by the ring confinement and magnetic fields and the changes of the binding energy induced by the displacement of the impurity are made for parameters of a GaAs QR.

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I. INTRODUCTION

Electronic properties of low-dimensional semiconductor heterostructures such as quantum wells, superlattices, quantum wires, and quantum dots have become a very active field of research with respect to both theory and experiment. Advances with respect to growth as well as high-resolution electron-beam lithography techniques allow the fabrication of novel confined structures called quantum rings (QR's). This system is modeled by a cylindrical quantum dot containing an internal axially symmetric cavity. Similar to the properties common for the above-listed confined structures, the QR's possess unique features. Being subjected to an external magnetic field, QR's have been shown to bear persistent currents associated with the magnetostatic interference Aharonov-Bohm effect: oscillations of the electron energy as a function of the magnetic flux are observed.[1] The Aharonov-Bohm effect and persistent current become possible due to the ring topology.

First, persistent currents were studied in metallic and semiconductor mesoscopic rings in which the electrons move diffusively (noncoherently) throughout the ring. References to a selection of theoretical and experimental results may be found in recent papers by Bruno-Alfonso and Latge,[2] Song and Ulloa,[3] and Lavenere-Wanderlay *et al.*[4] However, recent progress in the fabrication of the self-assembled ringlike-shaped nanostructures allowed Lorke *et al.*[5] to observe phase coherence effects of the electrons in nanoscopic QR's subjected to an external magnetic field.

Investigation of electronic states in nanoscopic QR's in the presence of external fields is currently of major interest. The energy spectrum and optical absorption properties of QR's subjected to magnetic fields have been calculated numerically by Barticevic *et al.*[6] neglecting excitonic and impurity effects. Llorens *et al.*[7] performed numerical calculations of electronic states in QR's in the presence of an in-plane electric field. The energy levels and oscillator strengths were found as functions of the ratio of the internal and external radii of the QR and electric field strength. Barticevic, Fuster, and Pacheco[8] studied the effect of an in-plane electric field on the electronic energies and optical absorption properties in the QR. Effects of the eccentricity and an in-plane electric field on the electronic and optical properties of elliptical QR's were under consideration in Ref. 4. Magarill *et al.*[9] calculated the persistent current in elliptical QR's.

It is well known that the electronic, optical, and kinetic properties of low-dimensional structures are strongly affected by impurities and/or excitons. These effects become more pronounced in the presence of an external magnetic field. The reason for this is that the confinement and magnetic field both increase the binding energy of the impurity electrons and excitons (see, for example, Bruno-Alfonso and Latge[2] and references therein). The study of magnetoexcitons in QR's was originated by Chaplic[10] and continued in Refs. 11 and 12. Using a variational method, Bruno-Alfonso and Latge[2] comprehensively studied shallow donor states in QR's in the presence of a weak and moderate magnetic field up to 10 T. The dependences of the binding energy of the ground state on the position of the impurity center within the QR on the radii of the QR and on the strength of the magnetic field have been obtained.

The majority of the theoretical papers on the problem of the impurity electron in a QR exposed to external fields are based on numerical studies that rely on a variational method. Undoubtedly, numerical methods provide highly accurate results especially needed for a comparison with experiments. However, a desirable complementary approach is to perform analytical investigations on the above systems, which will be pursued in the present work. This allows us in particular to obtain in closed form the evolution of the impurity states as a function of the parameters of the QR and external field strengths, which is of major interest. The impact of the topology of the ring on the physical properties remains very transparent throughout an analytical study. The aim of the present paper is an analytical study of impurity effects provided by the ring topology and the external fields.

The paper is organized as follows. In Sec. II the general analytical approach is developed and the basic equations are derived. QR's of extremely different radii (quantum disk regime) in the presence of weak and strong magnetic fields are considered in Secs. III and IV, respectively. QR's with comparable radii (quantum well wire regime) are studied in Sec. V. Section VI contains a discussion of our results. Section VII provides the conclusions.

II. GENERAL APPROACH

We consider a QR formed by the revolution of a rectangle around the z axis. The plane of the rectangle is parallel to the z axis. The QR is bounded by infinite barriers at the planes $z=\pm d/2$ and cylindrical surfaces of internal radius $\rho=a$ and external radius $\rho=b$. The chosen model corresponds to hard-wall confinement potential. An alternative parabolic ring confinement potential determined by the radius of the ring $\overline{\rho}$ and by the effective frequency Ω was proposed by Chakraborty and Pietiläinen[13] and then very effectively applied to study of QR's.[3,6,11,12] For a comparison of several potential models for the ring confinement see Ref. 6. The position of the impurity center $\mathbf{r}_0$ is given by the cylindrical coordinates $a\leqslant\rho_0\leqslant b$, $-d/2\leqslant z_0\leqslant +d/2$, and $\varphi_0=0$. The uniform magnetic field B is assumed to be parallel to the z axis. A radially directed electric field is modeled by a field of the charged wire coinciding with the z axis and possessing a linear effective charge density λ. The other length scales relevant to our study are the Bohr radius $a_0=4\pi\varepsilon_0\varepsilon\hbar^2/\mu e^2$ and magnetic length $a_B=(\hbar/eB)^{1/2}$, where μ is the effective electron mass and ε is the dielectric constant. We take the conduction band to be parabolic, nondegenerate, and separated from the valence band by a wide energy gap.

In the effective mass approximation the equation describing the spinless impurity electron positioned at the point $r(\rho,\varphi,z)$ subject to a uniform magnetic field and the axially symmetric electric field produced by the charged wire has the form

$$\left\{-\frac{\hbar^2}{2\mu}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}+\frac{i}{a_B^2}\frac{\partial}{\partial\varphi}-\frac{\rho^2}{4a_B^4}\right]+\frac{e\lambda}{2\pi\varepsilon\varepsilon_0}\ln\frac{\rho}{a}-\frac{e^2}{4\pi\varepsilon\varepsilon_0|\mathbf{r}-\mathbf{r}_0|}\right\}\Psi(\rho,\varphi,z)=E\Psi(\rho,\varphi,z). \tag{2.1}$$

By solving this equation subject to the boundary conditions

$$\Psi(\rho,\varphi,z)=0 \quad \text{for } \rho=a, \quad \rho=b, \quad z=\pm d/2, \tag{2.2}$$

the total energy E and the wave function Ψ can be found in principle. In the following the effect of the lateral (within the x-y plane) confinement or magnetic field on the electron is taken to be much stronger than that of the Coulomb field of the impurity center. In Sec. III the strong lateral size confinement dominates the interaction due to the impurity potential and magnetic field. The strong lateral size confinement is provided by the small external radius of the QR, taken to be much less than the Bohr impurity radius and magnetic length. In Sec. IV the magnetic field confinement exceeds the lateral size confinement and effect of the impurity Coulomb field. The magnetic length is less than the Bohr impurity radius and the external radius of the QR. We consider in Sec. V the QR with the difference of the external and internal radii to be much less than the radii of the QR, the Bohr impurity radius, and the magnetic length. In this case the narrow QR causes a strong lateral size confinement. For these cases approximate solutions to Eq. (2.1) are given by the adiabatic separation of the (ρ, φ) and z degrees of freedom,

$$\Psi(\rho,\varphi,z)=\Theta_{\perp N,m}(\rho,\varphi)f^{(N,m)}(z), \tag{2.3}$$

where the function

$$\Theta_{\perp N,m}(\rho,\varphi)=\frac{\exp(im\varphi)}{\sqrt{2\pi}}R_{N,m}(\rho) \tag{2.4}$$

describes the lateral motion of the electron of energy $E_{\perp N,m}$ determined by the confinement and external fields and where $R_{N,m}(\rho)$ is the function of the Nth radial state ($N=1,2,3,...$) corresponding to the angular quantum number $m=0, \pm1, \pm2,...$. The function $f^{(N,m)}(z)$ corresponds to the longitudinal motion parallel to the z axis and satisfies the equation

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dz^2}f^{(N,m)}(z)+V_{N,m}(z)f^{(N,m)}(z)=W^{(N,m)}f^{(N,m)}(z), \tag{2.5}$$

with the boundary conditions

$$f^{(N,m)}\left(\pm\frac{d}{2}\right)=0 \tag{2.6}$$

and with

$$V_{N,m}(z)=-\frac{e^2}{4\pi\varepsilon_0\varepsilon}\times\int\frac{d\rho}{2\pi}\frac{|R_{N,m}(\rho)|^2}{[\rho^2-2\rho\rho_0\cos\varphi+\rho_0^2+(z-z_0)^2]^{1/2}}. \tag{2.7}$$

The total energy E then becomes

$$E=E_{\perp N,m}+W^{(N,m)}. \tag{2.8}$$

The binding energy E_b of the impurity in the QR is defined as usual by the difference between the energy of the free electron in the QR ($E_{\perp N,m}+\hbar^2\pi^2 l^2/2\mu d^2$, $l=1,2,...$) and the energy E, Eq. (2.8), of the impurity electron. We have

$$E_b=\frac{\hbar^2\pi^2 l^2}{2\mu d^2}-W^{(N,m)}. \tag{2.9}$$

For a low QR ($d<a_0$) the longitudinal energy becomes $W^{(N,m)}=\hbar^2\pi^2 l^2/2\mu d^2+\Delta W^{(N,m)}$, where $\Delta W^{(N,m)}$ is the correction to the size-quantized energy caused by the impurity potential. In this case Eq. (2.9) gives

$$E_b=-\Delta W^{(N,m)}. \tag{2.10}$$

Below we consider the regimes determined by the different relationships between the radii of the QR, the Bohr impurity radius, the magnetic length, and the height of the QR, d. The wave functions $R_{N,m}(\rho)$ describe the lateral motion of the electron in the presence of the magnetic field B (Sec. IV), in the quantum disk of radius b (Sec. III) or in the two-dimensional quantum well of width $b-a$ (Sec. V) depending on the different regimes. Using the above wave functions, we calculate the potential energies $V_{N,m}(z)$, Eq. (2.7), the energies of the longitudinal motion $W^{(N,m)}$ in Eq. (2.5), and then the binding energies E_b, Eqs. (2.9) and (2.10), for the high ($d>a_0$) and low ($d<a_0$) QR.

III. QUANTUM DISK REGIME: WEAK ELECTRIC AND MAGNETIC FIELDS

We consider a QR having an internal radius a much less than the external radius b [quantum disk (QD) regime], which in turn is much less than the impurity Bohr radius a_0. Under these conditions $a\ll b\ll a_0$ and for weak magnetic ($b\ll a_B$) as well as electric fields the confinement caused by the external radius b provides the dominant contribution to the lateral energy $E_{\perp N,m}$ in Eq. (2.8):

$$E_{\perp N,m}=E^{(0)}+\Delta E^a+\frac{\hbar^2}{2\mu a_B^2}m+\Delta E^B+\Delta E^\lambda. \tag{3.1}$$

$E^{(0)}$ is the energy of the electron in the QD of radius b, and ΔE^a, ΔE^B, and ΔE^λ are energetical corrections due to the nonzero internal radius a, the weak magnetic field B (diamagnetism), and the weak electric field, respectively.

The general form of the radial wave function $R_{N,m}(\rho)$ is given by[2,6]

$$R_{N,m}(\rho)=AJ_m(k\rho)+BY_m(k\rho),\quad E^{(0)}+\Delta E(a)=\frac{\hbar^2k^2}{2\mu}, \tag{3.2}$$

where $J_m(x)$ and $Y_m(x)$ are the Bessel and Neuman functions, respectively, and A and B are constants. The boundary conditions (2.2), i.e., $R_{N,m}(a)=R_{N,m}(b)=0$ for the wave function $R_{N,m}(\rho)$, Eq. (3.2), lead to a set of two linear algebraic equations. This yields a transcendental equation for the parameter k:

$$J_m(ka)Y_m(kb)-J_m(kb)Y_m(ka)=0. \tag{3.3}$$

In the leading order approximation ($a=0$) we obtain, from Eq. (3.3),

$$E_{m,N}^{(0)}=\frac{\hbar^2\alpha_{m,N}^2}{2\mu b^2}, \tag{3.4}$$

where $\alpha_{m,N}$ ($N=1,2,3,...$) are roots of the Bessel functions $[J_m(\alpha_{m,N})=0]$, i.e., $\alpha_{0,1}=2.40$, $\alpha_{0,2}=5.52$, and $\alpha_{1,1}=3.83,...$.[14]

It follows from Eq. (3.3) that, under the condition $a\ll b$ and for $m=0$,

$$\Delta E_{m,N}^a=E_{0,N}^{(0)}D_{0,N}\left[\ln\frac{a}{b}+\ln\frac{\alpha_{0,N}}{2}+C\right]^{-1}, \tag{3.5}$$

where

$$D_{0,N}=\frac{\pi Y_0(\alpha_{0,N})}{\alpha_{0,N}J_0'(\alpha_{0,N})}.$$

C is the Euler constant (≈ 0.577). For $m\neq 0$ ($|m|=1,2,3,...$) the corrections to the energy become

$$\Delta E_{m,N}^a=E_{m,N}^{(0)}D_{m,N}\left(\frac{a}{b}\right)^2, \tag{3.6}$$

where

$$D_{m,N}=-\frac{\pi}{|m|!(|m|-1)!}\left(\frac{\alpha_{m,N}}{2}\right)^{2|m|-1}\frac{Y_m(\alpha_{m,N})}{J_m'(\alpha_{m,N})}.$$

The corrections to the energy, ΔE^B and ΔE^λ, are determined by the matrix elements of the diamagnetic ($\sim a_B^{-4}$) and logarithmic [$\sim\ln(\rho/a)$] terms on the left-hand side of Eq. (2.1) calculated with respect to the normalized wave functions

$$R_{\perp N,m}(\rho)=\frac{2^{1/2}}{bJ_{|m|+1}(\alpha_{m,N})}J_m\left(\alpha_{m,N}\frac{\rho}{b}\right). \tag{3.7}$$

We obtain

$$\Delta E_{m,N}^B=\frac{\hbar^2 b^2[2(m^2-1)+\alpha_{m,N}^2]}{24\mu a_B^4\alpha_{m,N}^2} \tag{3.8}$$

and

$$\Delta E_{m,N}^\lambda=\frac{e\lambda}{2\pi\varepsilon_0\varepsilon}\left[\ln\frac{b}{a}-1+\frac{|m|(1-J_0^2(\alpha_{m,N})-2\Sigma_{k=1}^{|m|-1}J_k^2(\alpha_{m,N}))}{\alpha_{m,N}^2 J_{|m|+1}^2(\alpha_{m,N})}\right]. \tag{3.9}$$

The expressions (3.5), (3.6), and (3.8) are valid under the conditions $a\ll b\ll a_B$, a_0. Equation (3.9) implies that $(e\lambda/2\pi\varepsilon_0\varepsilon)\ln(b/a)\ll\hbar^2\pi^2/2\mu b^2$.

A. High QR ($d > a_0$)

In order to solve Eq. (2.5) describing the longitudinal states, it is convenient to introduce the notation

$$u=\frac{2(z-z_0)}{a_0 n},\quad g^2=\frac{4}{a_0^2 n^2}(\rho^2-2\rho\rho_0\cos\varphi+\rho_0^2),$$

$$W_n^{(N,m)}=-\frac{N_{\mathrm{Ry}}}{n^2},$$

where $N_{\mathrm{Ry}}=\hbar^2/2\mu a_0^2$ is the impurity Rydberg constant. Equation (2.5) then becomes

$$\frac{d^2 f_n^{(N,m)}(u)}{du^2}+n\,\mathrm{Av}|(u^2+g^2)^{-1/2}|f_n^{(N,m)}(u)-\frac{1}{4}f_n^{(N,m)}(u)=0, \tag{3.10}$$

where Av indicates the average with respect to the wave functions (2.4) and (3.7). The quantum number n labels the longitudinal states. Due to the coordinate transformation, the boundary condition (2.6) becomes

$$f^{(N,m)}(u_{1,2})=0, \tag{3.11}$$

where $u_{1,2}=(d/a_0 n)(2z_0/d\pm 1)$.

The analysis of Eq. (3.10) is based upon the Hasegava-Howard method[15] more elaborated in Ref. 16. Only an outline of the corresponding analysis will be given below. For $|u|\gg \mathrm{Av}|g|\sim b/a_0 n$ the general solution to Eq. (3.10) is

$$f_n^{(N,m)}(u)=A_\pm W_{n,1/2}(|u|)+B_\pm M_{n,1/2}(|u|), \tag{3.12}$$

where $W_{n,1/2}$ and $M_{n,1/2}$ are the Whittaker functions.[14]

In the region $|u|\ll 1$, an iteration method is performed by the double integration of Eq. (3.10) using the trial function

$$f_{n,0}^{(n,m)}(u)=c_\pm+\alpha_\pm|u|(u^2+g^2)^{1/2}\ln(|u|+(u^2+g^2)^{1/2}), \tag{3.13}$$

where the constants $A_\pm$, $B_\pm$, $C_\pm$, and $\alpha_\pm$ correspond to the regions $u>0$ and $u<0$, respectively. The continuity conditions applied to the function $f_{n,0}^{(N,m)}(u)$, Eq. (3.13), and its first derivative at $u=0$ give the result $c_+=c_-\equiv c$ and $\alpha_+=-\alpha_-\equiv\alpha$. The results of the integration for the region $|u|\gg \mathrm{Av}|g|$ and from the standard expansion of the Whittaker functions involved in Eq. (3.12) for $|u|\ll 1$ (Ref. 14) are compared. When terms of the same order are equated a set of four linear algebraic equations is obtained. The total set of six linear algebraic equations for the coefficients $A_\pm$, $B_\pm$, c, and α consists of these equations and two boundary conditions (3.11) for the function $f_n^{(N,m)}$ Eq. (3.12). This set is solved by the determinantal procedure, yielding a transcendental equation for the quantum number n:

$$\left\{\varphi+\frac{1}{2}[p+\xi-G_1-G_2+\sqrt{(p-\xi)^2+(G_1-G_2)^2}]\right\}\left\{\varphi+\frac{1}{2}[p+\xi-G_1-G_2-\sqrt{(p-\xi)^2+(G_1-G_2)^2}]\right\}=0. \tag{3.14}$$

In the above expression the following notation has been used:

$$\varphi(n)=2C-1+\psi(1-n)+\frac{1}{2n}, \tag{3.15}$$

$$\xi(n)=\ln\frac{b}{a_0 n}+1-\Delta_{m,N}, \tag{3.16}$$

$$\Delta_{m,N}=1-\frac{2|m|}{\alpha_{mN}^2}\,\frac{\frac{1}{2}\left[J_0^2\left(\alpha_{mN}\frac{\rho_0}{b}\right)-J_0^2(\alpha_{mN})\right]+\Sigma_{k=1}^{|m|}\left[J_k^2\left(\alpha_{mN}\frac{\rho_0}{b}\right)-J_k^2(\alpha_{mN})\right]}{J_{|m|+1}^2(\alpha_{mN})}$$
$$-\frac{1}{2}\left(\frac{\rho_0}{b}\right)^2\frac{J_m^2\left(\alpha_{mN}\frac{\rho_0}{b}\right)+J_{|m|+1}^2\left(\alpha_{mN}\frac{\rho_0}{b}\right)-J_{|m|-1}\left(\alpha_{mN}\frac{\rho_0}{b}\right)J_{|m|+1}\left(\alpha_{mN}\frac{\rho_0}{b}\right)-J_{|m|}\left(\alpha_{mN}\frac{\rho_0}{b}\right)J_{|m|+2}\left(\alpha_{mN}\frac{\rho_0}{b}\right)}{J_{|m|+1}^2(\alpha_{mN})}, \tag{3.17}$$

$$p(n)=\frac{2a_0}{\pi\left\{\rho_0^2+\frac{b^2}{3}\frac{\lfloor 2(m^2-1)+\alpha_{mN}^2\rfloor}{\alpha_{mN}^2}\right\}}\int_0^b\rho\,d\rho\,R_{N,m}^2(\rho)(\rho+\rho_0)\left\{E(k)\left[\ln\frac{4\rho_0^2}{a_0^2n^2}+2\ln\left(1+\frac{\rho}{\rho_0}\right)-2+\ln k'\right]+(1+k'^2)K(k)\right\}, \tag{3.18}$$

$$G_{1,2}(n)=\frac{W_{n,1/2}(\nu_{1/2})\Gamma(-n)}{M_{n,1/2}(\nu_{1,2})}, \tag{3.19}$$

where $\nu_1 = u_2$, $\nu_2 = -u_1$, $\Gamma(x)$ is the gamma function, $\psi(x)$ is the psi function (the logarithmic derivative of the gamma function), $k = 4(\rho/\rho_0)(1+\rho/\rho_0)^{-2}$, $k' = (1-k^2)^{1/2}$, and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively. The wave functions $R_{N,m}$ in Eq. (3.18) are defined by expression (3.7). On solving Eq. (3.14), the quantum number n can be found that determines the longitudinal energy $W_n^{(N,m)} \equiv W_n$.

When the impurity center is located at the symmetry plane of the QR ($z_0 = 0$, $\nu_1 = \nu_2$, $G_1 = G_2$) or at any position of the quantum well wire (d, $\nu_{1,2} \to \infty$, $G_{1,2} \to 0$) the longitudinal states possess a definite parity. The levels of even and odd parity correspond to states determined by the second and first curly brackets on the left-hand side of Eq. (3.14), respectively. However, classification of the energy levels W_n into two groups can be made for the impurity center in any plane ($z_0 \neq 0$, $\nu_1 \neq \nu_2$); all states determined by the above mentioned second curly brackets are referred to as quasi-even (g) states while those determined by the first curly brackets are referred to as quasi-odd (u) states. The quasi-even states have quantum number n given by $n = n_0 + \delta n_g$ where $n_0 = 0,1,2,...$, while the quasi-odd states have n given by $n = n_0 + \delta n_u$ where $n_0 = 1,2,3,...$. The ground level ($n_0 = 0$) is nondegenerate and has $n < 1$. The excited states ($n_0 = 1,2,3,...$) have a doublet structure consisting of quasi-even and quasi-odd components.

Notice that Eq. (3.14) consists of elementary functions, well-known special functions, and a one-dimensional integral determining the function $p(n)$ containing both elementary and special functions. This equation can be solved numerically for arbitrary values of the height of the QR, $d > a_0$, and the impurity position ρ_0, z_0 for $d/2 - |z_0| \gg b$. However, explicit dependences of the energy W_n upon the above-mentioned parameters can be found for the limiting cases of a high QR ($d/a_0 > 1$), small displacements z_0 of the impurity from the symmetry plane of the QR ($2z_0/d \ll 1$), small $a \leqslant \rho_0 \ll b$, and a maximum ($\rho_0/b \approx 1$) shift ρ_0 of the impurity from the symmetry axis $\rho = 0$.

For the QR with radius $b \approx 0.4 a_0$, height $d \approx 2a_0$, and impurity positioned close to the midplane $z_0 = 0$ and internal surface $\rho = a$, Eq. (3.14) gives for the ground longitudinal state $n \approx 0.5$. Using in Eq. (3.14) the asymptotic expansion for the Whittaker functions[14] for $d/a_0 n \gg 1$, we obtain the condition

$$\left(\frac{d}{a_0 n}\right)^{2n} \frac{4\sinh\dfrac{2z_0}{a_0 n}}{\exp\left(\dfrac{d}{a_0 n}\right) - 2\cosh\dfrac{2z_0}{a_0 n}} \ll 1. \tag{3.20}$$

The term containing the Whittaker functions under the square root sign can then be safely neglected. Even though the impurity is shifted by a considerable distance $z_0 = d/4$, the term on the left-hand side of Eq. (3.20) is about 0.3. The equation for the quasi-even states then becomes

$$2C + \psi(1-n) + \frac{1}{2n} + \ln\frac{b}{a_0 n} - \Delta_{m,N} - \frac{1}{2}(G_1 + G_2) = 0, \tag{3.21}$$

where

$$G_{1,2}(z_0, d) = \Gamma(-n)\left(\frac{d}{a_0 n}\right)^{2n}\left\{\exp\left[\frac{d}{a_0 n}\left(1 \mp \frac{2z_0}{d}\right)\right] - 1\right\}^{-1}. \tag{3.22}$$

For small displacements $2z_0/a_0 n \ll 1$ the dependence of the quantum number $n(z_0) < 1$ and consequently the energy W_0 of the ground state as a function of the displacement z_0 can be found explicitly from Eq. (3.21) with the result

$$W_0(z_0) = W_1(0) + 2|W_1(0)|\Gamma(1-n_1) \times \left(\frac{2z_0}{a_0 n_1}\right)^2 \left(\frac{d}{a_0 n_1}\right)^{2n_1} \exp\left(-\frac{d}{a_0 n_1}\right)$$

$$n_1 = \sqrt{-\frac{2W_1(0)}{N_{\mathrm{Ry}}}}, \tag{3.23}$$

where $W_1(0) < 0$ is the longitudinal energy of the impurity positioned at the symmetry plane $z_0 = 0$ and at any radii ρ_0.

The effect of the radial displacement ρ_0 is described by the parameter $\Delta_{m,N}$, Eq. (3.17). For $\rho_0 = b$ we have $\Delta_{m,N} = 0$ and with increasing $\rho_0 \Delta_{m,N}$ monotonically increases towards the internal boundary. For $a \leqslant \rho_0 \ll b$ we have, from Eq. (3.17),

$$\Delta_{m,N}(\rho_0) = 1 - \frac{2}{\alpha_{mN}^2 J_{|m|+1}^2(\alpha_{mN})}\left\{|m|\left[\frac{1}{2}[1 - J_0^2(\alpha_{mN})] - \sum_{k=1}^{|m|} J_k^2(\alpha_{mN})\right] + \frac{1}{(|m|+1)!^2}\left(\frac{\alpha_{mN}\rho_0}{2b}\right)^{2(|m|+1)}\right\}. \tag{3.24}$$

For small displacements from the external surface $\rho_0 = b$ for which $4n_0 \Delta_{m,N} \ll 1$, Eq. (3.21) leads to approximate expressions for the quantum number $n(\Delta_{m,N})$ and then for the ground state energy W_0:

$$W_0(\Delta) = W_2(0) - |W_2(0)| \frac{4n_2}{1+2n_2}\Delta_{m,N},$$

$$n_2 = \sqrt{-\frac{2W_2(0)}{N_{\mathrm{Ry}}}}, \tag{3.25}$$

where $W_2(0)$ is the longitudinal energy of the impurity positioned at the external radius $\rho_0 = b$ and for any plane z_0.

In the logarithmic approximation $b/a_0 \ll 1$, $|\ln(b/a_0)| \gg 1$ the quantum number $n < 1$ can be calculated explicitly:

$$\frac{1}{n} = 2\left\{-\ln\frac{b}{a_0} + \Delta_{m,N}(\rho_0) + \frac{1}{2}[G_1(z_0) + G_2(z_0)]\right\}. \tag{3.26}$$

It enables us (see Sec. VI) to derive the dependence of the binding energy on the external radius b of the QR and the position (ρ_0, z_0) of the impurity center in a qualitative way. Below, the logarithmic approximation is used only for the qualitative analysis. The numerical calculations, corresponding figures, and estimates of the expected experimental results are made for realistic relationships between the parameters of the problem.

B. Low QR ($d<a_0$)

In order to find the positive longitudinal energies $W_s = N_{\rm Ry}/s^2$ we introduce in Eq. (2.5) the following notation:

$$t=\frac{2(z-z_0)}{ia_0 s},\quad \tilde{g}^2=\frac{4}{a_0^2 s^2}(\rho^2-2\rho\rho_0\cos\varphi+\rho_0^2),$$

$$\tau_{1,2}=\frac{2}{ia_0 s}\left(\frac{d}{2}\mp z_0\right).$$

In the region $|t|\gg \mathrm{Av}|\tilde{g}|$ the ansatz for the solution of Eq. (2.5) satisfying the boundary conditions $f_s^{(N,m)}(\tau_1)=f_s^{(N,m)}(\tau_2)=0$ is taken in the form

$$f_s(t)=A_+\,\mathrm{Re}\,W_{is,1/2}(t)+B_+\,\mathrm{Im}\,M_{is,1/2}(t),\quad it>0. \tag{3.27}$$

For the region $it<0$ the wave function $f_s(t)$ can be obtained from Eq. (3.27) by replacing A_+ by A_-, B_+ by B_-, and t by $-t$. Using for the region $|t|\ll 1$ the trial function

$$f_s^{(0)}(t)=c+\alpha t\sqrt{t^2-\tilde{g}^2}\,\ln(t+\sqrt{t^2-\tilde{g}^2}) \tag{3.28}$$

and then the iteration and matching procedure described above in Sec. III A, we arrive at the equation for the quantum number s:

$$\left\{\tilde{\varphi}+\frac{1}{2}[\tilde{p}+\tilde{\xi}-\tilde{G}_1-\tilde{G}_2+\sqrt{(\tilde{p}-\tilde{\xi})^2+(\tilde{G}_1-\tilde{G}_2)^2}]\right\}\times\left\{\tilde{\varphi}+\frac{1}{2}[\tilde{p}+\tilde{\xi}-\tilde{G}_1-\tilde{G}_2-\sqrt{(\tilde{p}-\tilde{\xi})^2+(\tilde{G}_1-\tilde{G}_2)^2}]\right\}=0, \tag{3.29}$$

where

$$\tilde{\varphi}(s)=\frac{\tilde{\Gamma}(s)}{2i}\left\{\frac{1}{\Gamma(is)}\left[i\frac{\pi}{2}+2C-1+\psi(1+is)-\frac{1}{2is}\right]-\mathrm{c.c.}\right\}, \tag{3.30}$$

$$\frac{1}{\tilde{\Gamma}(s)}=\frac{1}{2i}\left[\frac{1}{\Gamma(is)}-\frac{1}{\Gamma(-is)}\right], \tag{3.31}$$

$$\tilde{G}_{1,2}(s)=\tilde{\Gamma}(s)\frac{\mathrm{Re}\,W_{is,1/2}(\tau_{1,2})}{\mathrm{Im}\,M_{is,1/2}(\tau_{1,2})}. \tag{3.32}$$

The parameters $\tilde{\xi}(s)$ and $\tilde{p}(s)$ can be obtained from Eqs. (3.16) and (3.18), respectively, by replacing n by s.

We consider the quasi-even states determined by the second curly brackets on the left-hand side of Eq. (3.29). As before, we assume that the impurity center is shifted not too far from the midplane $z_0=0$, which allows us to neglect the Whittaker functions under the square root sign Eq. (3.29). Putting $s\ll 1$ and using the asymptotic expansion for the Whittaker functions,[14] we obtain the equation for the quantum number s:

$$2s\left[\ln\left(\frac{b}{a_0 s}\right)^2+2(C-\Delta_{m,N})\right]+\cot\left(s\ln|\tau_1|+\frac{|\tau_1|}{2}\right)+\cot\left(s\ln|\tau_2|+\frac{|\tau_2|}{2}\right)=0. \tag{3.33}$$

This equation can be solved by the method of iteration. In the zeroth approximation $(d/a_0=0,s=0)$, we find that $s_0^{-1}=(\pi a_0/d)(2j+1)$, $j=0,1,2,\ldots$. After some tedious calculations it is possible to obtain the expression for the energy W_s in the next-order approximation:

$$W_j^{(N,m)}=\frac{\hbar^2\pi^2(2j+1)^2}{2\mu d^2}+\Delta W_j^{(N,m)}, \tag{3.34}$$

where

$$\Delta W_j^{(N,m)}=-2N_{\rm Ry}\left(\frac{a_0}{d}\right)\left\{-4\left[C-\Delta_{m,N}+\ln\frac{\pi b(2j+1)}{d}\right]\times\cos^2\frac{(2j+1)\pi z_0}{d}+\ln\left[\pi^2(2j+1)^2\left(1-\frac{4z_0^2}{d^2}\right)\right]\right\}. \tag{3.35}$$

The expression (3.35) is valid under the conditions

$$\frac{d}{a_0\pi(2j+1)}\ll 1,\quad \frac{\pi b(2j+1)}{d}\ll 1.$$

The energies $W_j^{(N,m)}$, Eq. (3.34), are the even size-quantized energy levels perturbed by the quasi-Coulomb impurity potential $V_{N,m}(z)$, Eq. (2.7).

The quasi-odd states can be considered in much the same way as the quasi-even states. Equating the left-hand brackets of Eq. (3.29) to zero, we obtain, for $s\ll 1$,

$$|\tau_{1,2}|\gg 1,$$

$$4s(C-1+p)+\cot\left(s\ln|\tau_1|+\frac{|\tau_1|}{2}\right)+\cot\left(s\ln|\tau_2|+\frac{|\tau_2|}{2}\right)=0, \tag{3.36}$$

where $p\sim a/b\gg 1$ is defined by Eq. (3.18).

In the zeroth-order approximation $(b/a_0=0,s\ll 1)$ we obtain $s_0^{-1}=(2j\pi a_0/d)$, $j=1,2,\ldots$. The next approximation implies the explicit dependence of the parameter p, Eq. (3.18), on the quantum number $n\equiv s$. This dependence is, however, very difficult to obtain. Nevertheless, we conjecture that in this case the energies $W_j^{(N,m)}$ are the odd size-quantized energy levels shifted towards lower energies by the effect of the impurity potential $V_{N,m}(z)$ in Eq. (2.7).

IV. QUANTUM DISK REGIME: STRONG MAGNETIC AND WEAK ELECTRIC FIELDS

Below we consider the QR with extremely different internal a and external b radii subject to a strong magnetic field B such that $a \ll a_B \ll b, a_0$. In this case the effects of the electric field, impurity center, and confinement are the perturbation to the energy of the radial motion of the electron in the presence of the magnetic field (Landau levels). Employing the imposed conditions the wave function Ψ has the form (2.3) where the function $R_{N,m}(\rho)$ describes the Nth radial state of the electron in the presence of the magnetic field. The longitudinal function $f^{(N,m)}(z)$, in Eq. (2.3) obeys Eq. (2.5) with the boundary conditions (2.6) and with the potential $V_{N,m}(z)$, Eq. (2.7). For the lateral energy $E_{\perp N,m}$ in Eq. (2.8) we have

$$E_{\perp N,m} = E^{(0)} + \Delta E^a + \Delta E^b + \Delta E^\lambda, \tag{4.1}$$

where $E^{(0)}$ are the Landau levels, and ΔE^a, ΔE^b, and ΔE^λ are the corrections to these levels caused by the confinement at the surfaces $\rho = a$, b and the electric field (λ), respectively.

The general form of the radial function is given by[2]

$$R_m(\rho) = \exp\left(-\frac{x}{2}\right)(x)^{|m|/2}[AM(-\gamma,|m|+1,x) + BU(-\gamma,|m|+1,x)], \tag{4.2}$$

where $x = \rho^2/2a_B^2$,

$$\gamma = \frac{\mu a_B^2}{\hbar^2}\left(E^{(0)} + \Delta E(a) + \Delta E(b) - \frac{1}{2}(|m|+m+1)\right),$$

$M(-\gamma,|m|+1,x)$ and $U(-\gamma,|m|+1,x)$ are the confluent hypergeometric functions,[14] and A and B are constants. The boundary conditions $R_m(a) = R_m(b) = 0$ for the wave function (4.2) yield a set of two linear algebraic equations that results in the transcendental equation for the quantum number γ:

$$M(-\gamma,|m|+1,x_1)U(-\gamma,|m|+1,x_2) - M(-\gamma,|m|+1,x_2)U(-\gamma,|m|+1,x_1) = 0, \tag{4.3}$$

where $x_1 = a^2/2a_B^2$ and $x_2 = b^2/2a_B^2$.

Using the limiting expressions for $x_1 \ll 1$ and asymptotic expansions for $x_2 \gg 1$ for the functions M and U,[14] we find from Eq. (4.3) the parameter γ and then the unperturbed Landau levels

$$E_{m,N}^{(0)} = \frac{\hbar^2}{\mu a_B^2}\left(N + \frac{|m|+m+1}{2}\right) \quad N = 0,1,2,\ldots. \tag{4.4}$$

The corrections to these levels caused by the nonzero value of the internal radius a read

$$\Delta E_{m,N}^a = \frac{\hbar^2}{\mu a_B^2}\left[\frac{(|m|-1)!N!|m|!}{(N+|m|)!}x_1^{-|m|} - \ln x_1 - C + \psi(1+|m|)\right]^{-1}, \tag{4.5}$$

and the correction due to the finite value of the external radius b is

$$\Delta E_{m,N}^b = \frac{\hbar^2}{\mu a_B^2}\frac{x_2^{2N+|m|+1}\exp(-x_2)}{N!(N+|m|)!}. \tag{4.6}$$

For $m=0$ the first term in the square brackets on the right-hand side of Eq. (4.5) is dropped. For arbitrary radii a and b of the QR and arbitrary strength of the magnetic field B, Eq. (4.3) can be solved numerically.

It follows from Eqs. (4.2) and (4.3) that in the zeroth approximation ($x_1 = 0, x_2 = \infty$) the normalized unperturbed wave function $R_{N,m}(\rho)$ becomes

$$R_{N,m}(\rho) = \frac{N!}{a_B^2(N+|m|)!}x^{|m|/2}\exp\left(-\frac{x}{2}\right)L_N^{|m|}(x), \tag{4.7}$$

where $L_N^{|m|}(x)$ is the associated Laguerre polynomial.[14]

The energy shift ΔE^λ caused by the electric field is determined by the matrix element of the longitudinal term $\sim \ln(\rho/a)$ on the left-hand side of Eq. (2.1) calculated with respect to the functions (4.7) with the result

$$\Delta E_{m,N}^\lambda(\lambda) = \frac{e\lambda}{2\pi\varepsilon_0\varepsilon}\left[-\ln x_1 + \frac{N!}{(N+|m|)!}\zeta_{N,m}\right], \tag{4.8}$$

where

$$\zeta_{N,m} = \frac{1}{2}\int_0^\infty \exp(-x)x^{|m|}[L_N^{|m|}(x)]^2 \ln x\, dx. \tag{4.9}$$

Equation (4.8) implies that $e\lambda|\ln x_1|/2\pi\varepsilon_0\varepsilon \ll \hbar^2/\mu a_B^2$.

The energy of the longitudinal motion $W^{(N,m)}$ can be obtained by solving Eq. (2.5) in which the potential $V_{N,m}(z)$, Eq. (2.7), is calculated through the averaging procedure with respect to the functions $R_{N,m}(\rho)$, Eq. (4.7).

A. High QR ($d > a_0$)

Using an approach completely analogous to that in Sec. III A, we obtain Eq. (3.14) for the quantum number $n = [-N_{\mathrm{Ry}}/W_n]^{1/2}$. In this equation the functions $\varphi(n)$ and $G_{1,2}(n)$ are defined by Eqs. (3.15) and (3.19), respectively. The functions $\xi(n)$ and $p(n)$ read as follows:

$$\xi(n) = 1 + \frac{1}{2}\left[-\ln\frac{a_0^2 n^2}{2a_B^2} - C + \Lambda_{N,m}(x_0)\right], \tag{4.10}$$

where

$$\Lambda_{N,m}(x_0) = C + \ln x_0 + \frac{N!}{(N+|m|)!}\int_{x_0}^\infty \exp(-x)x^{|m|} \times [L_N^{|m|}(x)]^2 \ln\frac{x}{x_0}dx, \quad x_0 = \frac{\rho_0^2}{2a_B^2}, \tag{4.11}$$

and

$$p(n)=\frac{a_0}{\pi a_B^2\left[\frac{\rho_0^2}{2a_B^2}+2N+|m|+1\right]}\int_0^\infty \rho\, d\rho\, R_{N,m}^2(\rho)(\rho+\rho_0)$$
$$\times\left\{E(k)\left[\ln\frac{4\rho_0^2}{a_0^2n^2}+2\ln\left(1+\frac{\rho}{\rho_0}\right)-2+\ln k'\right]\right.$$
$$\left.+(1+k'^2)K(k)\right\}. \tag{4.12}$$

The parameters k and k' are the same as in Eq. (3.18).

For the ground Landau state $N=m=0$ we have

$$\Lambda_{0,0}(x_0)=C+\ln x_0-\mathrm{Ei}(-x_0), \tag{4.13}$$

where $\mathrm{Ei}(-x_0)$ is the exponential-integral function.[14]

As above, the first (second) curly brackets on the left-hand side of Eq. (3.14) describe the quasi-odd (even) longitudinal states. The ground state ($n<1$) is nondegenerate, and the excited states ($n\approx 1,2,...$) possess a doublet structure. Comparing the above equations for the quantum numbers with those considered in Sec. III, we arrive at the conclusion that the parameter $2^{1/2}a_B/a_0$ and the function $\Lambda_{N,m}(x_0)/2$ in Eq. (4.11) play here the same role as the parameter b/a_0 and the function $\Delta_{m,N}(\rho_0)$, respectively, in the case of the QD regime and weak magnetic field. In principle, the functions $\Lambda_{N,m}(x_0)$ and $p(n)$ can be calculated numerically for an arbitrary value of the parameter x_0. However, explicit dependences of the functions (4.11) and (4.12) on the parameter x_0 can be found for weak ($x_0\ll 1$) or sufficiently large ($x_0\gg 1,\rho_0/a_0\ll 1$) displacements of the impurity from the symmetry axis $\rho=0$. Since we focus on the quasi-even ground state $n<1$, it is appropriate to give the limiting expressions for the function $\Lambda_{N,m}(x_0)$, Eq. (4.11):

$$\Lambda_{Nm}(x_0)=\begin{cases} C+2\zeta_{Nm}\dfrac{N!}{(N+|m|)!}+\dfrac{(N+|m|)!}{N!(|m|+1)!}x_0^{m+1}, \\ \quad x_0\ll 1, \\ C+\ln(x_0), \\ \quad x_0\gg 1. \end{cases} \tag{4.14}$$

Particularly, $\Lambda_{0,0}(x_0)\approx x_0$ for $x_0\ll 1$. For small displacements $2z_0/a_0n\ll 1$, Eq. (3.23) for the energy $W_0(z_0)$ remains valid in the present case. The effect of the radial displacement ρ_0 is described by the function $\Lambda_{N,m}(x_0)$ in Eq. (4.11), with the special cases (4.13) and (4.14). This function and, as a consequence, the energy W_0 monotonically increase with increasing shift of the impurity from the internal surface $\rho_0=a$ to the external surface $\rho_0=b$.

In order to study qualitatively the dependence of the binding energy on the magnetic field strength B and the position of the impurity (ρ_0,z_0) it is profitable to use the logarithmic approximation $a_B/a_0\ll 1$, $|\ln(a_B/a_0)|\gg 1$. In this case the expression for the quantum number n can be obtained from Eq. (3.26) by replacing b by $2^{1/2}a_B$ and $\Delta_{m,N}$ by $-\Lambda_{N,m}/2$. Particularly, it becomes clear that the energy W_0 decreases with increasing magnetic field strength B.

B. Low QR ($d<a_0$)

The methodology to calculate the positive longitudinal energies $W_s=N_{\mathrm{Ry}}/s^2$ is similar to that presented in Sec. III B. The equation for the quantum number s has the form (3.29) where the functions $\tilde{\varphi}(s)$ and $\tilde{G}_{1,2}(s)$ are defined by Eqs. (3.30) and (3.32), respectively. The functions $\tilde{\xi}(s)$ and $\tilde{p}(s)$ are obtained from Eqs. (4.10) and (4.12), respectively, by replacing n by s. The energies of the quasi-even states $W_j^{(N,m)}$ are given by Eq. (3.34) where

$$\Delta W_j^{(N,m)}=-2N_{\mathrm{Ry}}\left(\frac{a_0}{d}\right)\left\{-4\left[\frac{1}{2}(C+\Lambda_{N,m})\right.\right.$$
$$\left.+\ln\frac{\pi\sqrt{2}a_B(2j+1)}{d}\right]\cos^2\frac{\pi z_0(2j+1)}{d}$$
$$\left.+\ln\left[\pi^2(2j+1)^2\left(1-\frac{4z_0^2}{d^2}\right)\right]\right\} \tag{4.15}$$

for $j=0,1,2,...$. The above expression is valid under the conditions

$$\frac{d}{a_0\pi(2j+1)}\ll 1, \quad \frac{\pi a_B(2j+1)}{d}\ll 1.$$

The energies $W_j^{(N,m)}$, Eqs. (3.34) and (4.15), are the even size-quantized energy levels perturbed by the quasi-Coulomb impurity potential $V_{N,m}(z)$, Eq. (2.7). This conclusion holds equally for the quasi-odd states.

V. QUANTUM WELL WIRE REGIME

We consider the QR of width $\delta=b-a$ to be much less than the radii of the QR, a and b, and the impurity Bohr radius a_0 ($\delta\ll a,b,a_0$). Under these conditions and both relatively weak magnetic and electric fields the lateral confinement caused by the boundaries of the QR provides the main contribution to the energy. The solution to Eq. (2.1) possesses the form (2.3) where the normalized wave function

$$R_{N,m}(\rho)=\left(\frac{2}{\delta}\right)^{1/2}\sin\frac{N\pi}{\delta}(\rho-a), \quad N=1,2,3,..., \tag{5.1}$$

describes the Nth radial state in the two-dimensional (2D) quantum well of width δ, corresponding to the angular quantum numbers $m=0,\pm 1,\pm 2,...$ and the energy

$$E_N^{(0)}=\frac{\hbar^2\pi^2N^2}{2\mu\delta^2}, \quad N=1,2,3,... . \tag{5.2}$$

The longitudinal wave functions $f^{(N,m)}(z)$ satisfy the boundary conditions (2.6) and obey Eq. (2.5) with the potential $V_j^{(N,m)}$, Eq. (2.7), determined by the wave functions $R_{N,m}(\rho)$ in Eq. (5.1).

It follows from Eq. (2.1) that for the narrow QR ($\delta \ll a,b$) the lateral component $E_{\perp N,m}$ of the total energy E in Eq. (2.8) can be written in the form

$$E_{\perp N,m}=E_N^{(0)}+\frac{\hbar^2}{2\mu b^2}\left[\left(m+\frac{\Phi}{\Phi_0}\right)^2-\frac{1}{4}\right]+\Delta E^{\lambda}, \quad (5.3)$$

where $\Phi=B\pi b^2$ is the magnetic flux and $\Phi_0=2\pi\hbar/e$ is the quantum unit. The dependence of the lateral energy $E_{\perp N,m}$, Eq. (5.3), on the magnetic field has been calculated in the zeroth approximation $\delta=0$. The energy shift ΔE^{λ} caused by the electric field is determined by the matrix element of the logarithmic term $\ln(\rho/a)\approx(\rho-a)/a$ on the left-hand side of Eq. (2.1) calculated with respect to the wave functions (5.1). As a result, we obtain

$$\Delta E^{\lambda}_{m,N}=\frac{e\lambda}{4\pi\varepsilon_0\varepsilon b}\,\delta. \quad (5.4)$$

Before proceeding with the longitudinal states note that for the approximation $\delta=0$ the potential $V_{N,m}(z)$, Eq. (2.7), in Eq. (2.5) for the narrow QR ($\delta\ll a,b$) becomes

$$V_{N,m}(z)=-2N_{\mathrm{RY}}\mathrm{Av}\frac{a_0}{\left[4b^2\sin^2\frac{\varphi}{2}+(z-z_0)^2\right]^{1/2}}, \quad (5.5)$$

where Av is the average with respect to the functions (2.4) and (5.1).

A. High QR ($d>a_0$) of small radius ($a,b\ll a_0$)

In order to calculate the energy levels $W_n=-N_{\mathrm{Ry}}/n^2$ we follow the procedure developed in Sec. III A to derive the transcendental equation (3.14) for the quantum number n, in which the functions $\varphi(n)$ and $G_{1,2}(n)$ are defined according to Eqs. (3.15) and (3.19), respectively. The functions $\xi(n)$ and $p(n)$ are given by

$$\xi(n)=\ln\frac{b}{a_0 n}+1, \quad (5.6)$$

$$p(n)=\frac{2a_0}{\pi b}\left(\ln\frac{8b}{a_0 n}-1\right). \quad (5.7)$$

As above, the first (second) curly brackets on the left-hand side of Eq. (3.14) describe the quasi-odd (even) longitudinal states. The ground state ($n<1$) is nondegenerate; the excited states ($n\approx1,2,3,...$) are doublets.

The equation for the quantum numbers n of the quasi-even states in explicit form can be derived from Eq. (3.21) by setting $\Delta_{m,N}=0$. For small displacements $2z_0/a_0 n\ll1$ the dependence of the energy of the ground state W_0 on the position of the impurity z_0 coincides completely with that given by Eq. (3.23). In the logarithmic approximation ($b/a_0\ll1$, $|\ln(b/a_0)|\gg1$) the expression for the quantum number $n<1$ can be obtained from Eq. (3.26) at $\Delta_{m,N}=0$. Obviously, the energy W_0 decreases with decreasing radius b of the QR.

B. Low QR ($d<a_0$) of small radius ($a,b\ll a_0$)

We now apply the strategy developed in Sec. III 2 to calculate the positive longitudinal energies $W_s=N_{\mathrm{Ry}}/s^2$, Eq. (3.29), for the quantum number s. In this equation the functions $\tilde{\varphi}(s)$ and $\tilde{G}_{1,2}(s)$ are provided by Eqs. (3.30) and (3.32), respectively, while the functions $\tilde{\xi}(s)$ and $\tilde{p}(s)$ are given by Eqs. (5.6) and (5.7), respectively, by replacing n by s. The dependence of the energy $W_j^{(N,m)}$ of the quasi-even states on the displacement z_0 possesses the form (3.34) and (3.35) taken for $\Delta_{m,N}=0$. The energies $W_j^{(N,m)}$ of the quasi-even (-odd) states are the even (odd) size-quantized levels perturbed by the quasi-Coulomb impurity potential $V_{N,m}$ in Eq. (5.5).

C. High QR ($d>a_0$) of large radius ($a,b\gg a_0$)

In this case it is convenient to present the potential $V_{N,m}(z)$, Eq. (5.5), in closed form

$$V_{N,m}(z)=-2N_{\mathrm{Ry}}\left(\frac{a_0}{b}\right)\frac{1}{\pi}\left[1+\left(\frac{z-z_0}{2b}\right)^2\right]^{-1/2}\times K\left(\left[1+\left(\frac{z-z_0}{2b}\right)^2\right]^{-1/2}\right), \quad (5.8)$$

where $K(x)$ is the complete elliptic integral.[14] This potential possesses a simple form in the limiting cases of small ($|z-z_0|\ll2b$) and large ($|z-z_0|\gg2b$) displacements of the electron from the impurity center:

$$V_{N,m}(z)=-2N_{\mathrm{Ry}}\left(\frac{a_0}{b}\right)\frac{1}{\pi}\left[2\ln2-\ln\frac{|z-z_0|}{2b}\right],\quad \frac{|z-z_0|}{2b}\ll1, \quad (5.9)$$

$$V_{N,m}(z)=-2N_{\mathrm{Ry}}\frac{a_0}{|z-z_0|},\quad \frac{|z-z_0|}{2b}\gg1. \quad (5.10)$$

Below, we restrict ourselves to the quasiclassic calculation of the ground level W_0 using the WKB expression

$$\left(\frac{2\mu}{\hbar^2}\right)^{1/2}\int_{z_1}^{z_2}[W_0-V_{N,m}(z)]^{1/2}dz=\pi, \quad (5.11)$$

where $z_{1,2}$ are the smaller and greater roots, respectively, of the expression under the square root sign. The ground state implies that $|z_{1,2}-z_0|<2b$. It allows to substitute Eq. (5.9) for the potential $V_{N,m}(z)$ into the expression (5.11) with the result

$$W_0=N_{\mathrm{Ry}}\left(\frac{a_0}{b}\right)\frac{1}{\pi}\ln\frac{\pi^2a_0}{2^7b}. \quad (5.12)$$

It is clear that decreasing the radius b of the QR leads to a decreasing energy W_0.

D. Low QR ($d<a$) of large radius ($a,b\gg a_0$)

In this case the longitudinal energy $W_j^{(N,m)}$ of the quasi-even states is given by Eq. (3.34) where the correction to the ground level ΔW_0 is determined by the matrix element of the potential $V_{N,m}(z)$, Eq. (5.9), calculated with respect to the wave function of the ground state $f^{(0)}(z)$ of the electron in the quantum well of width d:

$$f^{(0)}(z)=\left(\frac{2}{d}\right)^{1/2}\cos\frac{\pi z}{d}. \tag{5.13}$$

The shift ΔW_0 can be found in analytical form

$$\Delta W_0=\frac{1}{\pi}N_{\rm Ry}\left(\frac{a_0}{b}\right)\left[\ln\frac{a_0^2}{2^4 b^2}+\ln\frac{d^2}{2^4 a_0^2}+\gamma(z_0)\right], \tag{5.14}$$

where

$$\gamma(z_0)=\ln\left(1-\frac{4z_0^2}{d^2}\right)+\frac{2z_0}{d}\ln\frac{1+\frac{2z_0}{d}}{1-\frac{2z_0}{d}}-2$$
$$+\frac{1}{\pi}\sin\frac{2\pi z_0}{d}\left[\mathrm{ci}\,\pi\left(1+\frac{2z_0}{d}\right)-\mathrm{ci}\,\pi\left(1-\frac{2z_0}{d}\right)\right]$$
$$-\frac{1}{\pi}\cos\frac{2\pi z_0}{d}\left[\mathrm{si}\,\pi\left(1+\frac{2z_0}{d}\right)+\mathrm{si}\,\pi\left(1-\frac{2z_0}{d}\right)+\pi\right], \tag{5.15}$$

where $\mathrm{ci}(x)$ and $\mathrm{si}(x)$ are the integral cosine and sine, respectively.[14]

It follows from Eq. (5.14) that if the radius b of the QR increases, the energy ΔW_0 increases. The less the height d of the QR the less is the energy ΔW_0. The energy ΔW_0 increases with the displacement z_0 of the impurity from the midplane $z_0=0$. In particular for small shifts $2z_0/d \ll 1$ the function $\gamma(z_0)$, Eq. (5.15), becomes $\gamma(z_0)=-3.18+5.80(2z_0/d)^2$.

VI. DISCUSSION

A. QD regime, weak magnetic field, high QR

Since the contribution of the second term on the right-hand side of Eq. (3.23) to the binding energy is less than that of the size-quantized energy $\sim 1/d^2$ [see Eq. (2.9)], the binding energy decreases with increase of the height of the QR, $d>a_0$. It is clear from Eq. (3.26) that the binding energy increases with decrease of the radius b of the QR and diverges at b/a_0. Expression (3.23) shows that the shift of the impurity center z_0 from the midplane of the QR, $z=0$, leads to a decrease of the binding energy E_b, Eq. (2.9). In the narrow QR ($b<a_0$) the effective longitudinal radius $a_0 n$ becomes small and the electron density is concentrated close to the impurity center. This leads to a decrease with respect to the influence of the boundary planes at $z=\pm d/2$ as well as in case of increasing the height of the QR, d. The impurity being positioned at the midplane $z=0$ produces the greatest binding energy. The dependence of the correction ΔE_b on the displacement of the impurity z_0 for different heights d and radii b is shown in Figs. 1(a) and 1(b), respectively. The

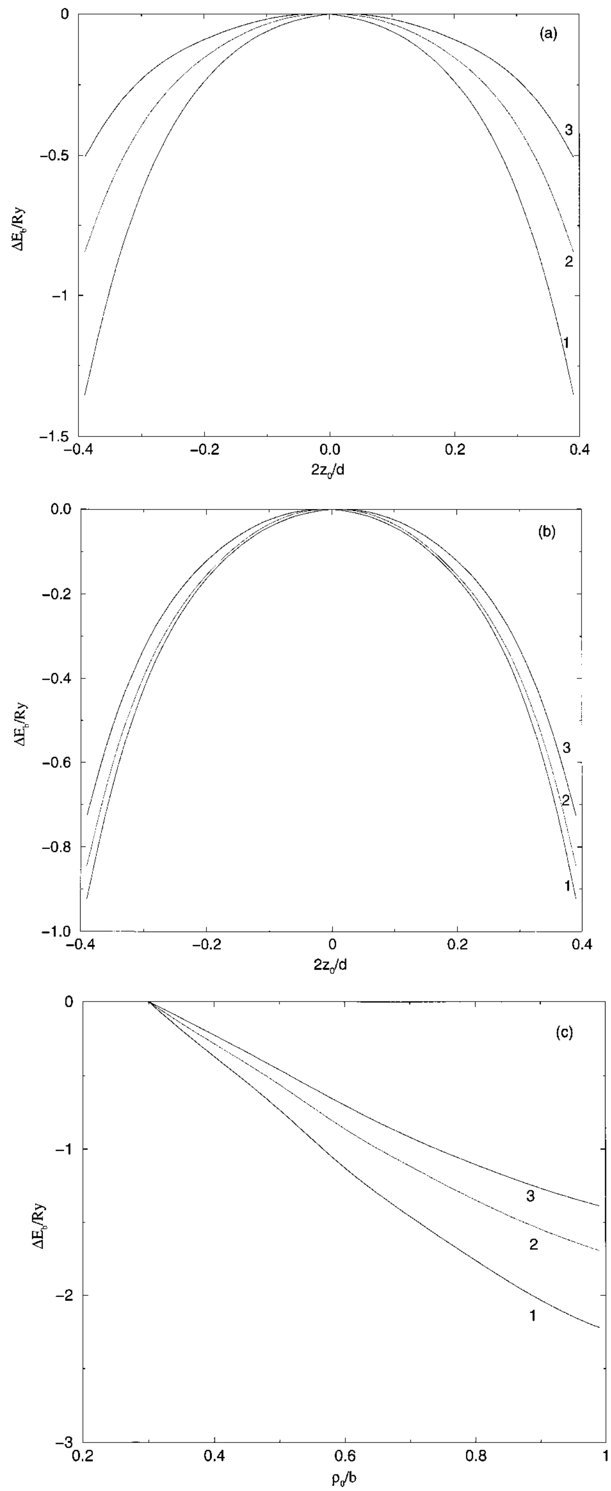

FIG. 1. Correction $\Delta E_b/N_{\rm Ry}$ ($N_{\rm Ry}$ is the impurity Rydberg constant) as a function of (a) the longitudinal displacement z_0 of the impurity from the midpoint $z_0=0$ of the QR for the $\rho_0/b=0.5$; $b/a_0=0.6$ and $d/a_0=2.0$ (1), 2.5 (2), 3.0 (3); (b) the longitudinal displacement z_0 of the impurity from the midpoint $z_0=0$ of the QR for $\rho_0/b=0.5$; $d/a_0=2.5$ and $b/a_0=0.4$ (1), 0.6 (2), 0.8 (3); (c) the radial displacement ρ_0 of the impurity from the internal boundary $\rho_0=a$ towards the external boundary $\rho_0=b$ for $z_0=0$ with $d/a_0=2.5$ and $b/a_0=0.4$ (1), 0.6 (2), (0.8) (3). Case: QD regime of high QR, weak magnetic field.

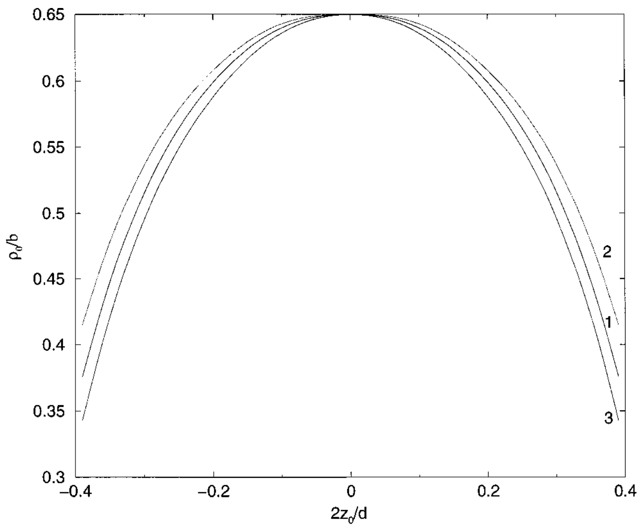

FIG. 2. Impurity binding energy contour plots as function of the impurity position (ρ_0,z_0) for the parameters $(d/a_0=2.5, b/a_0=0.6;\ E_b=5.41N_{\mathrm{Ry}})$ (1); $(d/a_0=3.0,\ b/a_0=0.8;\ E_b=4.57N_{\mathrm{Ry}})$ (2); $(d/a_0=2.0,\ b/a_0=0.4;\ E_b=6.93N_{\mathrm{Ry}})$ (3). Case: QD regime of high QR, weak magnetic field.

higher (d increases) or the narrower (b decreases) the QR is, the less is the effect caused by the shift z_0 of the impurity.

The radial shift of the impurity center ρ_0 from the internal surface $\rho=a$ towards the external surface $\rho=b$ produces the same effect as that induced by the displacement z_0 from the midplane $z=0$. If the parameter ρ_0 increases, then the binding energy monotonically decreases because of a small radial electron density $\sim|R_{m,N}(\rho)|^2$, Eq. (3.7), close to the external surface $\rho=b$. The wider QR (b increases) is the less is the effect associated with the radial displacement ρ_0. The dependence of the correction ΔE_b on the shift ρ_0 for different radii b of the QR is depicted in Fig. 1(c).

It follows from the above that the corrections to the binding energy induced by the displacements from the middle circle $\rho_0=(a+b)/2$, $z_0=0$ to the region $a\leqslant\rho_0\leqslant(a+b)/2$, $-d/2\leqslant z_0\leqslant+d/2$ can be balanced. The binding energy contour plots determined by the conditions $E_b(\rho_0,z_0)=\mathrm{const}$ are presented in Fig. 2.

B. QD regime, weak magnetic field, low QR

It follows from Eqs. (2.10) and (3.35) that the binding energy increases with decrease of the height $d<a_0$ and the radius b of the QR. These results coincide with those obtained numerically by Li *et al.*[17] and Branis *et al.*[18] for the impurity located at the center of the QD and at the axis of the quantum well wire (QWW), respectively, in the presence of a magnetic field. For the small displacements $2z_0/d\ll 1$ from the midplane $z_0=0$ we obtain, from Eqs. (2.10) and (3.35),

$$E_b(z_0)=E_b(0)+2N_{\mathrm{Ry}}\left(\frac{a_0}{d}\right)\left(\frac{2z_0}{d}\right)^2\left\{-1+\frac{\pi^2(2j+1)^2}{2}\times\left[C-\Delta_{m,N}+\ln\frac{\pi b(2j+1)}{d}\right]\right\}, \tag{6.1}$$

where $j=0,1,2,...$ and where $E_b(0)$ is the binding energy of the impurity positioned at the midplane $z_0=0$. From Eq. (6.1) it follows that the shift of the impurity from the point $z_0=0$ leads to a decrease with respect to the binding energy. The less the external radius b of the QR $(b\ll d)$, the greater is the shift of the binding energy caused by the displacement z_0. The foregoing is valid for the binding energy of the quasi-odd states.

The dependence of the binding energy of the quasi-even states on the radial position ρ_0 can be derived from Eqs. (2.10) and (3.35):

$$E_b(\Delta_{m,N})=E_b(0)+8N_{\mathrm{Ry}}\left(\frac{a_0}{d}\right)\Delta_{m,N}\cos^2\frac{\pi z_0(2j+1)}{d}, \tag{6.2}$$

where $\Delta_{m,N}(\rho_0)$ is defined by Eq. (3.17) with $\Delta_{m,N}(b)=0$. It is clear that the binding energy decreases if the radial shift ρ_0 of the impurity center tends towards the external boundary $\rho_0=b$. A decrease of the binding energy with increasing the radial sizes of the structure and with a shifting the impurity center from the symmetry axis is typical for systems of both the cylindrical and rectangular[19] cross sections. The shift of the binding energy associated with the parameter $\Delta_{m,N}$ reaches a maximum for the impurity center positioned at the plane $z_0=0$ and subsequently decreases with increasing displacement from this plane. For a high QR the corrections to the binding energy induced by the radial (ρ_0) and vertical (z_0) displacements can be chosen such that they cancel each other: i.e., in this case there is no resulting change of the energy for specific shifts from the middle circle $\rho_0=(a+b)/2$, $z_0=0$ to the region $a\leqslant\rho_0\leqslant(a+b)/2$, $-d/2\leqslant z_0\leqslant+d/2$.

It was pointed out in Ref. 2 that the binding energy E_b reaches a maximum at $z_0=0$ and for the QD regime at $\rho_0=a$. This is completely in line with our results [see Figs. 1(a), 1(b), and 1(c)] and Eqs. (2.9), (3.23), and (6.1)]. The dependences of the binding energy E_b on the radial (ρ_0) and longitudinal (z_0) displacements and the shape of the E_b contour plots (Fig. 4, below) qualitatively coincide with those obtained in Ref. 2.

Bruno-Alfonso and Latge studied in Ref. 2 the dependence of the maximum of the binding energy $E_{b\,\max}$ of the ground state $m=0$ on the internal radius a of the QR. Particularly, they have found that for the QD regime $(a\ll b)$ the maximum binding energy $E_{b\,\max}$ decreases with increasing internal radius a. This result is explained by Eq. (3.17), which becomes, for $m=0$,

$$\Delta_{0,N}=1-\frac{1}{2J_1^2(\alpha_{0N})}\left(\frac{a}{b}\right)^2. \tag{6.3}$$

It follows from the above that increasing a leads to a decrease with respect to $n^{-1}=(-W_n/N_{\mathrm{Ry}})$, Eq. (3.26), and a decrease of the binding energy $E_{b\,\max}$ given by Eq. (2.9) for the high QR or by Eq. (6.2) for the low QR with $E_b=-\Delta W_j^{(N,m)}$. In order to explain the dependence of the energy $E_{b\,\max}$ on the radius a for the QWW regime obtained in Ref.

2, Eq. (2.1) should be solved in the next-order approximation with respect to the finite width δ of the QR.

Although the confinement caused by the internal surface $\rho = a$ in case of the QD regime as well as weak magnetic and electric fields has minor effects on the binding energy, it influences the total energy E [see Eq. (2.8)]. It follows, from Eq. (3.1) that the magnetic field B leads to paramagnetic ($\sim m a_B^{-2}$) and diamagnetic, Eq. (3.8), shifts of the energy levels. They are shifted by an amount ΔE^a [see Eqs. (3.5) and (3.6)] by the confinement, associated with the internal surface of the QR.

It is clear from Eq. (3.9) that the energy shift ΔE^λ induced by the electric field of the charged wire depends on the ratio of the radii of the QR. With increasing ratio b/a the shift $|\Delta E^\lambda|$ increases. Note that, in principle, the electric field allows us to influence the binding energy E_b. It displaces the maximum of the electron density with respect to the position of the impurity center that leads to a change of the binding energy. This effect becomes more pronounced in the presence of sufficiently strong electric fields, implying the necessity of a numerically exact solution to the problem of the electron in the QR subjected to an axially symmetric electric field.

C. QD regime, strong magnetic field

The expression for the binding energy E_b possesses the form (2.9) where $W^{(N,m)}$ is the longitudinal energy of the impurity electron in the presence of a strong magnetic field calculated in Sec. IV. If the impurity center is shifted towards the external surface $\rho_0 = b$ or displaces from the midplane $z_0 = 0$, the binding energy decreases. If the magnetic field strength B increases, the binding energy increases also. This coincides with the results obtained by the variational procedure for the impurities in the QW's (Ref. 17) and QWW's (Ref. 18). The dependences of the correction to the binding energy ΔE_b on the displacement z_0 for different heights $d > a_0$ are qualitatively the same as those depicted in Fig. 1(a). The corrections to the binding energy ΔE_b as functions of the shift z_0 and radial displacement ρ_0 for different magnetic field strengths B are shown in Figs. 3(a) and 3(b) respectively. The contour plots of the binding energy determined by the condition $E_b(\rho_0, z_0) = \text{const}$ are given in Fig. 4.

As expected, the radial confinement as well as a weak electric field has minor effects on the binding energy E_b. However, the total energy E in Eq. (2.8) depends on both the radii of the QR and the linear electrical density λ. If the internal (external) radius a (b) decreases (increases), the corrections to the energy ΔE^a in Eq. (4.5) [ΔE^b in Eq. (4.6)] decrease and vice versa. It follows from Eq. (4.8) that a decrease of the internal radius a leads to an increase of the energy shift $|E^\lambda|$ induced by the electric field. For a negatively charged wire ($\lambda < 0$) and for the special case $\Delta E^a + \Delta E^b - |E^\lambda| = 0$ the energy shifts associated with the confinement and the electric field cancel. If the electric field effects become equally important as the magnetic field effects, it is natural that the binding energy will show an equally strong dependence on both field strengths. In order to study the corresponding dependencies a numerically exact solution to the problem of the electron in the QR in the presence of magnetic and crossed radially directed electric fields is desirable.

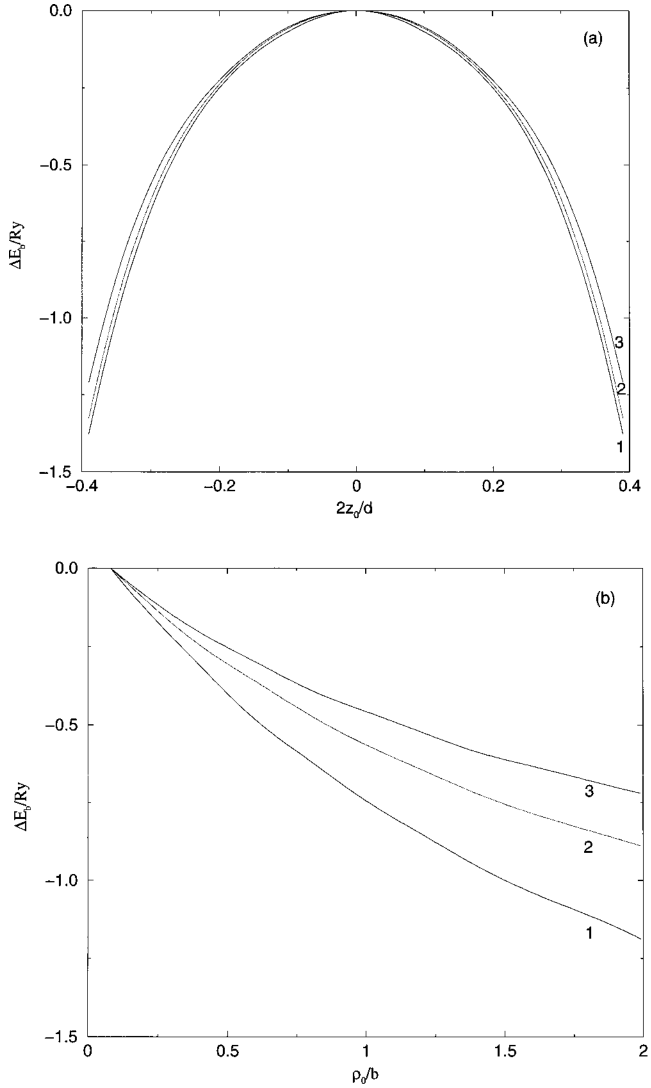

FIG. 3. Correction $\Delta E_b / N_{\text{Ry}}$ to the impurity binding energy (N_{Ry} is the impurity Rydberg constant) as a function of (a) the longitudinal displacement z_0 of the impurity from the midpoint of the QR $z_0 = 0$ for $\rho_0^2 / 2a_B^2 = 0.5$; $d/a_0 = 2.0$ and $2^{1/2} a_B / a_0 = 0.4$ (1), 0.6 (2), 0.8 (3); (b) the radial displacement of the impurity ρ_0 from the internal boundary $\rho_0 = a$ towards the external boundary $\rho_0 = b$ for $z_0 = 0$; $d/a_0 = 2.5$ and $2^{1/2} a_B / a_0 = 0.4$ (1), 0.6 (2), 0.8 (3). Case: QD regime of high QR, strong magnetic field.

D. QWW regime, QR of small radius

The binding energy E_b is given by Eq. (2.9) where $W^{(N,m)}$ is the longitudinal energy of the impurity electron in the thin ($\delta \ll a, b$) QR calculated in Sec. V A. If the impurity center is shifted from the midplane ($z_0 = 0$) of the QR, the binding energy decreases. The binding energy increases for a decrease of the radius b of the QR. For the high QR, the smaller the radius b, the less is the effect induced by the

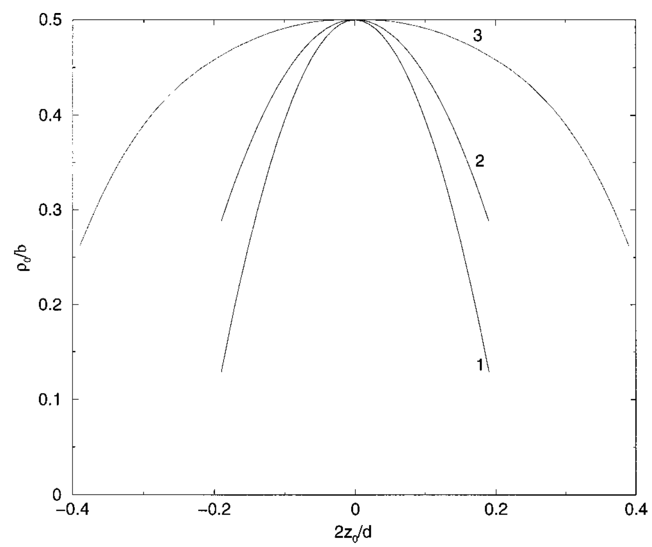

FIG. 4. Impurity binding energy contour plots as a function of the impurity position (ρ_0, z_0) for the parameters $(d/a_0 = 2.0,\ 2^{1/2}a_B/a_0 = 0.8;\ E_b = 4.90N_{\rm Ry})$ (1); $(d/a_0 = 2.5,\ 2^{1/2}a_B/a_0 = 0.6;\ E_b = 4.80N_{\rm Ry})$ (2); $(d/a_0 = 3.5,\ 2^{1/2}a_B/a_0 = 0.3;\ E_b = 5.57N_{\rm Ry})$ (3). Case: QD regime of high QR, strong magnetic field.

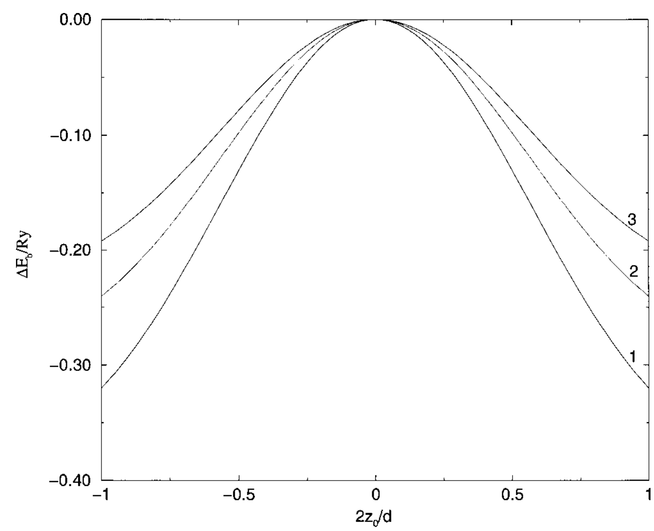

FIG. 5. Correction $\Delta E_b/N_{\rm Ry}$ to the impurity binding energy ($N_{\rm Ry}$ is the impurity Rydberg constant) induced by the longitudinal displacement of the impurity z_0 from the midpoint of the QR for $b/a_0 = 3$, 4, 5. Case: QWW regime of low QR of large radius.

displacement z_0. The dependence of the correction ΔE_b on the displacement z_0 for different heights d are qualitatively the same as those shown in Fig. 1(a). Also, the shifts ΔE_b as a function of z_0 for different radii b look qualitatively similar to those depicted in Fig. 1(b).

E. QWW regime, QR of large radius

For a high QR $(d > a_0)$ the binding energy E_b is determined by Eq. (2.9) where the energy $W^{(N,m)}$ for the ground longitudinal state possesses the form (5.12). If the radius b of the QR increases, the binding energy decreases. For a low QR $(d < a_0)$ the binding energy E_b of the ground state [see Eq. (2.10)] coincides apart from the sign with the correction to the size-quantized energy ΔW_0 in Eq. (5.14). With increasing radius b of the QR the binding energy decreases. The binding energy increases for a decreasing height d of the QR. If the impurity center moves from the midplane $(z_0 = 0)$ towards the bottom $(z_0 = -d/2)$ or top $(z_0 = +d/2)$ of the QR, then the binding energy decreases (see Fig. 5).

The QWW regime is favorable to demonstrate the approximately periodic oscillations of the ground level E, Eq. (2.8), as a function of the flux Φ with period Φ_0 that is a manifestation of the magnetostatic Aharonov-Bohm effect. The total energy E in Eq. (2.8) is the oscillating lateral level $E_{\perp N,m}$ in Eq. (5.3) shifted towards lower energies by the amount $W^{(N,m)}$. The impurity potential $V_{N,m}$ in Eq. (5.5) and consequently the binding energy E_b do not depend on the magnetic field strength B. Thus in the case being considered here the impurities do not effect the dependence of the total energy on the magnetic field and do not change the persistent current. Following Ref. 2 in which the above circumstance was revealed first in the framework of a variational approach, the influence of the impurities on the Aharonov-Bohm effect can be described by a modified trial function. From our investigation it is clear that the reason for the missing effect of an impurity in the QWW regime under the condition $\delta \ll a_B$ is that the confinement dominates the influence of the magnetic field on the longitudinal states. Therefore, the energy $W^{(N,m)}$ is unaffected by the magnetic field. As the magnetic field strength B and the width δ of the QR increase, the effect of the impurities on the Aharonov-Bohm oscillations becomes more pronounced. In this case the dependence of the lateral energy $E_{\perp N,m}$ on the magnetic field strength is contained in higher-order terms in the expansion for $\delta = 0$.

Barticevic *et al.*[6] calculated the lateral energy $E_{\perp m}$ of the electron in the QR of width $\delta = 50$ Å, 100 Å as a function of the mean radius $\overline{\rho}$ of the QR and a strength of the magnetic field up to $B = 16$ T. They reported that for $B = 0$ and $m = 0$ and in the region $\overline{\rho} > \delta$ the lateral energy does not depend on the radius $\overline{\rho}$ and obeys the dependence $E_{\perp 0} \sim 1/\delta^2$. Also, it was found that for magnetic quantum numbers $m \neq 0$ the lateral energy $E_{\perp m}$ increases with increasing $|m|$ and rapidly decreases with increase of the radius $\overline{\rho}$. For $B \neq 0$ the degeneracy of the states with the positive $(+m)$ and negative $(-m)$ quantum numbers is lifted and the state with negative $m < 0$ becomes the ground state. The correction to the lateral energy of the state $m = 0$ induced by the magnetic field shows the dependence $\Delta E_{\perp 0} \sim B^2$. All listed results obtained numerically in Ref. 6 are in complete accordance with those resulting from the analytical expression for the lateral energy $E_{\perp N,m}$ in Eq. (5.3).

In order to estimate the values to be expected in an experiment we take the parameters for the GaAs material $\mu = 0.067m_0$, $\varepsilon = 12.5$, $a_0 = 98.7$ Å, and $N_{\rm Ry} = 5.83$ meV. Also, we take the realistic sizes of the QR's typically used in the theoretical papers and prepared in experiments.[2–7] For the QD regime of the QR of radii $a = 50$ Å, $b = 500$ Å subjected to a weak magnetic field the blueshift $\Delta E^a_{0,1}$, Eq. (3.5), induced by the nonzero internal radius a can be balanced by the redshift $\Delta E^{\lambda}_{0,1}$, Eq. (3.9), produced by the electric field of the effective linear charge density $\lambda = -0.7$ pC m^{-1}. This density corresponds to a linear electron density $n_e = 0.43\times 10^5$ cm^{-1}. For the same regime and a strong magnetic field $B = 5.4$ T the total blueshift of the ground Landau level $(N = m = 0)$ caused by the nonzero radius $a = 50$ Å, Eq. (4.5), and finite radius $b = 275$ Å, Eq. (4.6), is $\Delta E^a_{0,0} + \Delta E^b_{0,0} = 9.4$ meV. This shift is balanced by the effective electric field corresponding to a linear charge density $\lambda = -3.2$ pC m^{-1} ($n_e = 2.0\times 10^5$ cm^{-1}). For a low QR of height $d = 40$ Å and of radii $a \approx b = 300$ Å (QWW regime) the correction to the binding energy $\Delta E_b(z_0) = -\Delta W_0$, Eq. (5.14), associated with the displacement of the impurity by $z_0 = 15$ Å is $\Delta E_b = -1.45$ meV and can therefore be detected experimentally. Note that the lateral energy $E_{\perp 0,0}$, Eq. (5.3), of the electron in the GaAs QR of width $\delta = 50$ Å and radius $b = 100$ Å in the absence of the external fields is $E_{\perp 0,0} = 222.5$ meV, i.e., very close to the value $E_{\perp 0,0} = 210$ meV calculated in Ref. 6. An experimental study of QR's requires substantial efforts. In particular, the ring topology is hardly provided by the QD regime of the QR's of small radii[2] less than the impurity radius (see Sec. III). Also, the application of a radially directed electric field to these QR's calls for further experimental effort. Nevertheless, we believe that recent advances in the fabrication of semiconductor nanostructures and corresponding experimental techniques for their investigation provide the basis for studies of the topological effects for the different regimes of the QR's subjected to external fields.

VII. CONCLUSION

In summary, we have performed a comprehensive analytical investigation of the problem of an impurity electron arbitrarily located in a QR in the presence of crossed homogeneous magnetic and radially directed electric fields. The QWW and QD regimes and weak and strong magnetic fields as well as low and high QR's are under consideration. The dependences of the total and binding energies of the impurity electron on the strengths of the external fields, the parameters of the QR, and the position of the impurity center within the QR are derived explicitly. It is shown that if the QR confinement and/or the magnetic field strength increase, the binding energy also increases. The binding energy reaches a maximum for the impurity center positioned at the midplane of the QR. For the QD regime the binding energy decreases for a shift of the impurity from the symmetry axis towards the outer part of the ring. It is found that the effects of the confinement and the magnetic field can be energetically canceled by those caused through the axially symmetric electric field. We demonstrate that for a relatively narrow QR the impurity influences only insignificantly the oscillations of the ground electronic level as a function of the magnetic field (magnetostatic Aharonov-Bohm effect). Estimates of the linear electron densities needed to bring in balance the blue energy shifts caused by the ring confinement and magnetic fields and changes of the binding energy induced by the displacement of the impurity are made for the parameters of a GaAs QR. In view of the increasing interest on optoelectronic and transport properties of nanostructured systems subjected to external fields the present analytical approach can be considered as a basis for the further understanding of the physics of such systems as well as precise numerical calculations.

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