

Displacement Profile of Charge Density Waves and Domain Walls at Critical Depinning

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(Received 1 April 2003; published 25 June 2004)

The influence of a strong surface potential on the critical depinning of an elastic system driven in a random medium is considered. If the surface potential prevents depinning completely the curvature C of the displacement profile exhibits at zero temperature a pronounced rhombic hysteresis curve of width $2f_c$ with the bulk depinning threshold f_c . The hysteresis disappears at nonzero temperatures if the driving force is changed adiabatically. If the surface depins by the applied force or thermal creep, C is reduced with increasing velocity. The results apply, e.g., to driven magnetic domain walls, fluxline lattices, and charge-density waves.

DOI: 10.1103/PhysRevLett.92.257205

PACS numbers: 75.60.-d, 74.25.Qt

The driven viscous motion of an interface in a medium with random pinning forces is one of the paradigms of condensed matter physics [1,2]. This problem arises, e.g., in the domain wall motion of magnetically or structurally ordered systems with impurities [3] or when an interface between two immiscible fluids is pushed through a porous medium [4]. Closely related problems are the motion of other elastic systems like a vortex line in an impure superconductor [5], of a dislocation line in a solid [6] or driven charge density waves (CDWs) [7]. For a constant external driving force this problem has been considered close to the zero temperature critical depinning threshold [8–12] and in the creep region [6,13].

It was a tacit assumption of these investigations that the motion of the elastic system is not hindered by effects from surfaces or internal grain boundaries. Surface barriers are, however, known to be relevant in all cases mentioned above. In superconductors they prevent the penetration of new flux lines into the probe [14]. In CDWs normal electrons have to be converted into those condensed in the CDW by a phase-slip mechanism which is essentially a nucleation process [15,16]. The motion of domain walls may be hindered by a variation of the width of the sample such that positions of minimal width are preferred, etc. Experimental [17] and numerical [18] studies of CDWs with contact effects revealed hysteretic behavior of the polarization.

It is the aim of the present Letter to consider the effect of a strong surface pinning potential in addition to the weak bulk random pinning. It turns out that, starting from a flat interface, at $T = 0$ and increasing the driving force f to $f > f_c$ the mean curvature C of the averaged (parabolic) displacement profile behaves as $C(f) \propto (f - f_c)$. In more general situations $C(f, t)$ exhibits a pronounced hysteretic behavior. At nonzero temperatures $C(f, t)$ increases with time and reaches asymptotically its behavior of the pure system $C(f) \propto f$. We further determine the reduction of the curvature in the case that the surface is depinned due to a sufficiently large driving

force or due to thermally activated processes at the surface. The latter mimic also phase slip processes in CDWs.

Model and zero temperature critical depinning.—We focus on a simple realization of the problem. The equation of motion of a D -dimensional field $\varphi(\mathbf{x}, t)$ describing an interface profile in the case of domain walls or a phase profile in the case of CDWs is given by $\frac{1}{\gamma}(\partial\varphi/\partial t) = -(\delta\mathcal{H}/\delta\varphi)$, where \mathcal{H} denotes the Hamiltonian of the system:

$$\mathcal{H} = \int d^Dx \left\{ \frac{\Gamma}{2} (\nabla\varphi)^2 - f \cdot \varphi + V(\mathbf{x}, \varphi) \right\}. \quad (1)$$

γ and Γ denote the mobility and the stiffness constant of the elastic object, respectively, and f is the driving force which is assumed to change only adiabatically. The potential includes a random force and a surface contribution

$$V(\mathbf{x}, \varphi) = - \int_0^\varphi d\varphi' g(\mathbf{x}, \varphi') [1 - \rho(\mathbf{x})] + \frac{\Gamma}{a^2} V_s(\varphi) \rho(\mathbf{x}). \quad (2)$$

The random force $-g(\mathbf{x}, \varphi)$ is assumed to be Gaussian distributed with $\langle g \rangle_d = 0$ and $\langle g(\mathbf{x}, \varphi) g(\mathbf{x}', \varphi') \rangle_d = \delta^{(D)}(\mathbf{x} - \mathbf{x}') \Delta_0(\varphi - \varphi')$ where $\langle \dots \rangle_d$ denotes the random average. For domain walls $\Delta_0(\varphi) = \Delta_0(-\varphi)$ is an analytical monotonically decreasing function of φ which decays to zero over a finite distance l . For CDWs $g \propto \sin[\varphi - \alpha(\mathbf{x})]$ with a random phase $\alpha(\mathbf{x}) \in [0, 2\pi]$ and therefore $\Delta_0(\varphi)$ is periodic with $\Delta_0(\varphi) = \Delta_0(\varphi + 2\pi\mathbb{Z})$. The factor $[1 - \rho(\mathbf{x})]$ is essentially 1 in the bulk and drops to zero in the vicinity ($a \ll L$) of $x_1 = 0$ and $x_1 = L$, e.g., $\rho(\mathbf{x}) = e^{-x_1/a} + e^{(x_1-L)/a}$, where the surface potential $V_s(\varphi)$ is assumed to act, which favors the values of $\varphi(0, \mathbf{x}_\perp)$ and $\varphi(L, \mathbf{x}_\perp)$ at $2\pi\mathbb{Z}$.

The details of the interaction between the elastic system and the surface depend on the specific system under consideration. We will restrict ourselves here to a periodic surface potential which has applications in type-II superconductors and may also serve as a first step for the treatment of conversion phenomena in CDWs.

The case $V_s \equiv 0$ was considered previously in [8,9,11,12]. It was shown that at zero temperature the system undergoes a depinning transition at a critical value f_c . For $f > f_c$ the velocity $v = \langle \dot{\varphi} \rangle$ increases as $v \sim (f - f_c)^\beta$ with the critical exponent β calculated in an expansion in $D = 4 - \epsilon$ dimensions. The average displacement profile is macroscopically flat. At nonzero temperatures the depinning transition is smeared out and goes over into a creep motion for $f \ll f_c$ [6].

In this Letter we will consider the opposite case where a *strong* surface potential V_s , obeying $\max\{V_s'\} \gg f_c a^2/\Gamma$, slows down or prevents completely the motion of the elastic object. We study the history-dependent curvature $C(f, t)$ of the parabolic displacement profile. The steady state solution for the average phase is given by

$$\varphi_0 \equiv \langle \varphi \rangle = vt + \frac{C_s(f)}{2}(L - x_1)x_1, \quad (3)$$

where $C_s(f) = C_s(f, t \rightarrow \infty)$ is the saturation value of the curvature.

Infinite surface barriers.—We begin with the case $V_s \rightarrow \infty$, where the depinning transition is suppressed. To determine $C(f, t)$ we first apply perturbation theory. Using the decomposition $\varphi(\mathbf{x}) = \varphi_0(\mathbf{x}) + \varphi_1(\mathbf{x})$ with $\langle \varphi_1(\mathbf{x}) \rangle_d = 0$ in the equation of motion and expanding $g(\mathbf{x}, \varphi_0 + \varphi_1)$ to linear order in φ_1 we get after averaging over the disorder

$$\frac{1}{\gamma} \dot{\varphi}_0 = -\Gamma C(t) + f + \langle g_\varphi(\mathbf{x}, \varphi_0(\mathbf{x}, t)) \varphi_1(\mathbf{x}, t) \rangle_d, \quad (4)$$

where $g_\varphi(\mathbf{x}, \varphi) = (\partial/\partial\varphi)g(\mathbf{x}, \varphi)$. Calculating φ_1 also to first order of g we get from (3) and (4) $C_s = C_0 = f/\Gamma$ since $\Delta'_0(0) = 0$, i.e., there seems to be no influence of the disorder. Here, the situation is completely analogous to that at the conventional depinning transition [8]. However, as we know from critical depinning, this is the situation below the Larkin scale L_p .

Next we discuss renormalized perturbation theory starting from a situation where $C_s = 0$. As long as $f \leq f_c$, the elastic object is pinned and boundary pinning does not matter, hence $C(f, t) = 0$. At $f = f_c$ the elastic object is in the same critical state as at the depinning transition. Therefore we can use the results of the previous renormalization group calculation in this case. As a result γ and $\Delta_0(z)$ are replaced there by the renormalized, momentum p dependent quantities

$$\gamma(p) \approx \gamma(pL_p)^{-2+z}, \quad (5a)$$

$$\Delta_p(\varphi) \approx K_D^{-1}(\Gamma l/L_p^\zeta)^2 p^{4-D-2\zeta} \Delta^*(\varphi(pL_p)^\zeta/l). \quad (5b)$$

z is the dynamical exponent and ζ denotes the roughness exponent which was calculated for domain walls to order $\epsilon = 4 - D$ in [8,9] and recently to $\mathcal{O}(\epsilon^2)$ [12]. For CDWs $\zeta = 0$ [9]. The most important feature of $\Delta_p(\varphi)$ is that $\Delta^*(\varphi)$ has a cusplike singularity at the origin. The renormalized equation for $C(f, t \rightarrow \infty)$ is given by

$$\Gamma C(f, t) = f + f_p \omega_p \int_0^\infty dt' \int_0^1 d\tilde{p} \times \tilde{p}^{1+z-\zeta} e^{-\omega_p \tilde{p}^2 t'} \times \Delta^{*'}\left(\left[-C(t) + C(t-t')\right] \frac{x_1}{2}(x_1 - L)\right), \quad (6)$$

where $\tilde{\mathbf{p}} = \mathbf{p}L_p$, $\omega_p = \gamma f_p/l$, and $f_p = l\Gamma L_p^{-2}$. After having increased f adiabatically to a fixed value slightly larger than f_c , $C(f, t)$ saturates for $t \rightarrow \infty$ and hence the difference $C(t) - C(t-t')$ vanishes. As a result the argument of $\Delta^{*'}$ also vanishes and the right-hand side of (6) becomes independent of x_1 . Since $C(t) > C(t-t')$ the argument of $\Delta^{*'}$ approaches zero from positive values. Thus we get for the saturation value $C_s(f)$

$$C_s(f) = \frac{f - f_c}{\Gamma} = \frac{l}{L_p^2} \frac{f - f_c}{f_p}, \quad f_c = \frac{f_p}{2 - \zeta} \Delta^{*'}(0^+). \quad (7)$$

One can understand this result in the following way: Using the decomposition $\varphi = \varphi_0 + \varphi_1$ in the asymptotic region, where $C(t)$ saturates, the equation of motion can be written as

$$\frac{1}{\gamma} \dot{\varphi}_1 = \Gamma \nabla^2 \varphi_1 + f - \Gamma C + g_1(\mathbf{x}, \varphi_1), \quad (8)$$

where $g_1(\mathbf{x}, \varphi_1) = g(\mathbf{x}, \varphi_0(\mathbf{x}) + \varphi_1(\mathbf{x}))$. $g_1(\mathbf{x}, \varphi)$ and $g(\mathbf{x}, \varphi)$ have the same statistical properties. According to (8) the force acting on the field φ_1 is now reduced by the curvature force $-\Gamma C$. The depinning of the φ_1 field seems hence to occur at $f \nearrow f_c = f_c + \Gamma C_s$. However, since the boundary conditions fix $\varphi_1(0) = \varphi_1(L) = 0$ and hence $\langle \dot{\varphi}_1 \rangle = 0$ for all values of f , the system *is always at its depinning transition*, which implies (7). Starting from some $f < f_c$ and $C_s = 0$, C_s will stay at this value until f reaches f_c . For $f > f_c$, C_s obeys (7). The same argument can be used for negative forces $f < 0$. Then we find for $f < -f_c$: $\Gamma C_s(f) = f + f_c = -(|f| - f_c)$ since $\Delta^{*'}(0^-) = -\Delta^{*'}(0^+)$.

A *scaling argument* supports the validity of Eq. (7) to all orders in g : Close to the $V_s = 0$ depinning transition the correlation length ξ diverges as $\xi \approx L_p((f - f_c)/f_c)^{-\nu}$. For $L' < \xi$ the *roughness*—the mean square displacement of a piece of linear size L' of the elastic object—scales as $w^2(L') \approx l^2(L'/L_p)^{2\zeta}$ [8,9,11,12]. If we choose the system size $L \approx \xi$ we expect that the roughness scales as the height of the parabolic φ profile on the same scale, $w(\xi) \approx C\xi^2$, which is indeed fulfilled if we use the scaling law $\nu = 1/(2 - \zeta)$

$$C = \frac{w(\xi)}{\xi^2} = \frac{l}{L_p^2} \left(\frac{f - f_c}{f_c} \right)^{\nu(2-\zeta)} \approx \frac{f - f_c}{\Gamma}. \quad (9)$$

Hysteresis.—Next we consider the case that we increase f adiabatically from $f \leq f_c$ to a value f_{\max} , where $C(f, t)$ reaches C_{\max} , and then *decrease* f again. In this case $C(f, t) < C(f, t-t')$ and hence the argument of $\Delta^{*'}$ becomes negative. Instead of (7) we get from (6)

$$\Gamma C_{\max} \equiv f_{\max} - f_c = f + f_c. \quad (10)$$

The effective force acting on the elastic object is now given by $f - \Gamma C_{\max}$. Further decreasing f , there is no change of $C(f, t)$ until the effective force reaches the threshold $-f_c = f - \Gamma C_{\max} = f - (f_{\max} - f_c)$. According to the last relation this happens at $f = \tilde{f}_{\max} = f_{\max} - 2f_c$. Analogous arguments can be used for reversing the fields from $\tilde{f} < 0$ to $\tilde{f} > 0$. Thus C_s undergoes a *hysteresis* which consists of the two parallel segments given by $C_s \Gamma = (f \mp f_c)$ and two horizontal segments determined by $C_{\max} = (f_{\max} - f_c)/\Gamma$ and $C_{\min} = (f_{\min} + f_c)/\Gamma$, respectively. Indeed, similar hysteresis effects of the strain have been observed in CDWs [17,19]. Note, that in [17] the polarization is measured, which is proportional to C_s .

These findings are fully supported by numerical simulations, as shown in Fig. 1. For integrating the equation of motion, the \mathbf{x} coordinate is discretized with a lattice constant α and the simulation time is measured in units of a time τ_0 (the dimensionless lattice Laplacian for $D = 1$ is given by $\nabla^2 \varphi_i = \varphi_{i+1} + \varphi_{i-1} - 2\varphi_i$, with lattice sites $i = 0, \dots, L$). α and τ_0 are chosen such that $(\tau_0 \gamma \Gamma / \alpha^2) = 1$ and the dimensionless stochastic forces $\tau_0 \gamma g(\mathbf{x}, \varphi) \in [-1/2, 1/2]$ (the dimensionless driving force is $\tau_0 \gamma f$).

Curvature at finite temperature.—Next we want to consider the problem of finite temperatures. Changing f only adiabatically we may use equilibrium statistical mechanics. It is convenient to go over to the field $\tilde{\varphi}(\mathbf{x}) = \varphi(\mathbf{x}) + (f/2\Gamma)x_1(x_1 - L)$. The Hamiltonian rewritten in $\tilde{\varphi}$ has the same statistical properties as the initial one (1), since $V_R(\mathbf{x}, \varphi) = -\int \varphi d\varphi' g(\mathbf{x}, \varphi')$ is a *random* function of both arguments. This can most easily be seen by using the replica method [3]. The disorder averaged free en-

thalpy follows from the *replica Hamiltonian*

$$\mathcal{H}_n = \frac{\Gamma}{2} \sum_{a,b=1}^n \int_{\mathbf{x}} \left\{ (\nabla \tilde{\varphi}_a)^2 \delta_{a,b} - \frac{\Gamma}{T} R(\tilde{\varphi}_a - \tilde{\varphi}_b) \right\}, \quad (11)$$

with $\langle V_R(\mathbf{x}, \varphi) V_R(\mathbf{x}', \varphi') \rangle_d = \delta^{(D)}(\mathbf{x} - \mathbf{x}') R(\varphi - \varphi')$. Apparently, the replica Hamiltonian is the same as that following from (1). It is worth to mention that this is true only if the random potential $V_R(\mathbf{x}, \varphi)$ is strictly uncorrelated in \mathbf{x} direction. The application of surface barriers implies therefore $C = f/\Gamma$ and $\langle (\tilde{\varphi}(\mathbf{x}) - \tilde{\varphi}(\mathbf{x}'))^2 \rangle_d^{1/2} \simeq l(L/L_p)^{\xi}$ where ξ denotes the equilibrium roughness exponent corresponding to Hamiltonian (11). Thus the displacement profile is the same as in the pure case. For nonadiabatic changes of f , traces of the $T = 0$ hysteresis are expected to be seen at nonzero temperatures (cf. Fig. 2).

The numerical solution of the equation of motion with thermal noise at finite temperatures and $V_s = \infty$ is in agreement with these analytical considerations. Figure 2 shows the coefficient $C(t)$ as it approaches its saturation value $C_s = f/\Gamma$ with time. Strictly speaking, we are not in a steady state until $C(t)$ has reached its saturation value and hence the phase profile deviates slightly from the parabolic shape. In Fig. 2, $C(t)$ is the least square fit to the profile. Note, that for *low* temperatures ($T < 5.0$ in the simulation, where T is the dimensionless variance of the thermal noise) this approach is very slow, noticeable by the occurring steps, triggered by avalanches, even at large times. For high T ($T = 5.0$), one sees that $C(t)$ fluctuates around the saturation value due to thermal noise. Therefore the $T = 0$ hysteresis of C vanishes at finite temperatures.

Critical depinning.—So far the surface potential was assumed to fix the value of φ at the surfaces $x_1 = 0$ and $x_1 = L$. We will now assume that the surface potential is reduced such that a macroscopic motion of the elastic object is possible. To determine the mutual interaction

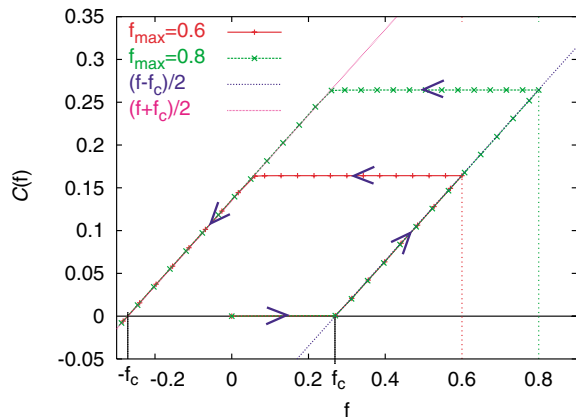


FIG. 1 (color online). Hysteresis of $C(f)$ at $T = 0$ for a one-dimensional interface. The driving force is first increased to $f_{\max} = 0.6$ or $f_{\max} = 0.8$, respectively, and then decreased to $-f_c \approx -0.27$. The arrows show the direction of the hysteresis. The numerical simulation was done for a one-dimensional interface with length $L = 1000$ and averaged over 300 disorder configurations.

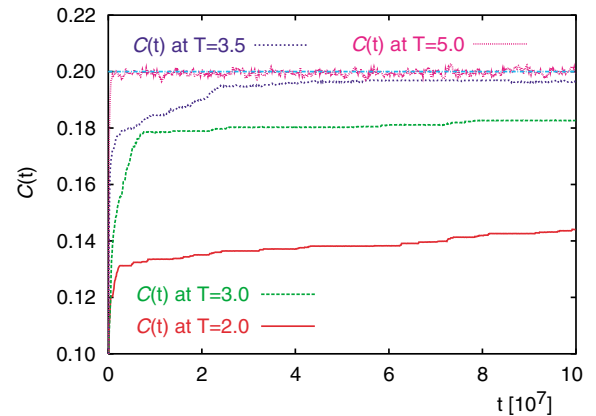


FIG. 2 (color online). Simulation-time-resolved coefficient $C(t)$ for a driving force of $f = 0.2$ at various temperatures. The simulation was done for a system of length $L = 1000$ and for one disorder configuration of CDW type for each temperature (see text).

between the bulk and the surface we have to consider the effective equation of motion of the surface. Denoting $\varphi(0, \mathbf{x}_\perp) = \varphi_s(\mathbf{x}_\perp)$ the effective equation of motion of the surface field can be written as (a is assumed to be of the order of the lattice spacing)

$$\frac{1}{\gamma} \dot{\varphi}_s = \Gamma C \frac{L}{2a} + \Gamma \nabla_\perp^2 \varphi_s + f - \frac{\Gamma}{a^2} V'_s(\varphi_s). \quad (12)$$

An analogous equation can be written for $\varphi(L, \mathbf{x}_\perp)$. In (12) we have replaced the force resulting from the displacement in the bulk by the corresponding average force. In the steady state $\nabla_\perp^2 \varphi_s = 0$ and Eq. (12) has a depinning threshold $f_{s,c} \gg f_c$ determined by

$$\Gamma C(f_{s,c}) \frac{L}{2a} + f_{s,c} - \frac{\Gamma}{a^2} \max V'_s(\varphi) = 0. \quad (13)$$

For $f > f_{s,c} \gg f_c$ the macroscopic velocity is given by the steady state solution $v = \dot{\varphi}_s$ which follows from integrating (12) with $\nabla_\perp^2 \varphi_s = 0$. The corresponding solution

$$v(t) = v_p \Phi\left(\frac{\Gamma C(t) \frac{La}{2} + fa^2}{\Gamma V'_{s,\max}}, t\right) \quad (14)$$

depends of course on the specific form of the surface potential, $v_p = \gamma f_p$. Equation (14) has to be combined with the effective equation for the bulk ($f > f_c$) [8]

$$\left(\frac{v(t)}{v_p}\right)^{1/\beta} = \frac{f - f_c}{f_p} - C(t) \frac{L_p^2}{l}, \quad (15)$$

which follows from (5a) and (7). Note that we used the condition $a \ll L_p$ such that the surface potential does not change the bulk depinning threshold. Eqs. (14) and (15) determine both the velocity and the curvature C as a function of the driving force. If we increase f from $f = 0$ with $C = 0$, C remains zero until we reach f_c . For $f_c < f < f_{s,c}$, C obeys (7). At $f_{s,c}$ the elastic object is depinned and with increasing velocity the curvature is reduced compared to a nonmoving object which is subject to the same force, as follows from (15). If the surface potential is periodic, also $v(t)$ will be periodic and the bulk depinning transition is slightly smeared out [20]. We will assume that this effect is weak. In principal it can be avoided by adding some randomness to the surface potential.

Nucleation and creep.—At finite but low temperatures the surface field may exhibit a creep motion even if $f \ll f_{s,c}$. Creep proceeds via the formation of droplets at the surfaces $x_1 = 0$ and $x_1 = L$, inside which φ is changed by 2π with respect to the bulk value of φ . The droplet consists of a cylindrical piece (the cylinder axis is perpendicular to $x_1 = 0$) in the surface layer of height a and radius R and an attached semisphere with the same radius. The width of the droplet wall confining the cylinder is of the order $a' = a/\sqrt{V''_s}$.

Keeping only the leading order terms we get for the energy of the droplet $E_{\text{dp}}(R) = 2\Gamma R^{D-2} \{ \sqrt{V''_s} + \ln \frac{R}{a'} -$

$\pi CRL - fR^2/\Gamma \}$. The critical droplet size $R_c \ll L$ follows as $R_c \approx \sqrt{V''_s}/(CL)$ or $R_c = (CL)^{-1}$ in $D = 3$ or $D = 2$, respectively. In deriving E_{dp} , we have neglected the contribution from the disorder which is correct as long as $R_c < L_p$, i.e., $L \gg L_p$. The nucleation rate of droplets and hence the creep velocity is given by

$$\frac{v_{D=3}}{v_p} = A \exp\left(-B \frac{V''_s \Gamma}{CLT}\right), \quad \frac{v_{D=2}}{v_p} = A' \left(\frac{CLa'}{\sqrt{V''_s}}\right)^{B'\Gamma/T}, \quad (16)$$

which replaces (14) in the case $f \ll f_{s,c}$, $T > 0$ (cf. [15]). The present treatment is too crude to give the coefficients A, A', B , and B' . Again, (16) has to be considered together with (15) to determine C and v . In CDWs, where φ can be multivalued, nucleation processes also occur deep in the bulk [15]. The droplet energy then does not contain a term $\sim f$, leaving the relations (16) essentially unchanged.

To conclude we have shown that surface pinning of impure elastic systems lead to an onset of curvature C only above a threshold value f_c of the random force. In general, C exhibits a pronounced hysteresis. The curvature is reduced above the surface depinning transition or at finite temperatures when nucleation processes at the surface allow for creep motion.

We thank B. Rosenow, S. Brazovskii, and in particular T. Emig for fruitful discussions. The authors acknowledge support by Sonderforschungsbereich 608.

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