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# A new dust-dynamics-induced interchange instability in dusty plasmas

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**Abstract**

A nonuniform dusty plasma held in equilibrium in a magnetic field against gravity has a polarization electric field $\boldsymbol{E}_0$ in the direction of gravity for negatively charged dust grains. It is shown that the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift of the plasma which carries a positive current induces, in combination with compressible dust dynamics, a novel flute-like interchange instability that is different from the usual Rayleigh–Taylor mode.

## 1. Introduction

The situation where an inverted density distribution of a dusty plasma may be held in a magnetic field against gravity can occur in many space and astrophysical scenarios. Such a configuration is well known to be Rayleigh–Taylor (R–T) unstable for a fully ionized plasma [1]. The presence of massive dust particles, which generally get negatively charged, alters the situation somewhat. D'Angelo [2] has, for instance, reported a reduction in the growth of the R–T instability in the presence of stationary charged dust particulates. The present authors [3] have found a similar reduction in a somewhat more generalized treatment of the dusty R–T instability. However, both previous works [2,3] have neglected the dust dynamics. The treatment thus amounts to freezing the dust particles and with it the electronic charge they have taken up from the plasma. The reduction in the growth rate arises essentially from the reduction in the charge density perturbation $(n_{i1} - n_{e1})$ resulting therefrom. Such an approximation may be valid for somewhat high-frequency perturbations but is physically unacceptable, as in a physical situation all frequencies of the perturbation are available. It is, therefore, necessary to abondon this restriction. Furthermore, relaxing the above restriction and thereby rendering the dust particles mobile also necessitates that the equilibrium question be also addressed properly including the response of the dust component under the gravity force. Earlier, it was tacitly assumed that the dust particles were held fixed and were not allowed to respond to the gravity force.

In this Letter, we incorporate dust dynamics in the study of the interchange instability and demonstrate the existence of a new dust-dynamics-induced instability, which is different from the usual R–T mode [1–3].

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## 2. Equilibrium

Consider a dusty plasma held in a magnetic field under gravity. The plasma particles, both the electrons and ions, are assumed to be magnetized, while the micron-sized massive dust particles are unmagnetized. The plasma and dust momentum balance equation in equilibrium are then

$$-\nabla p + \frac{1}{c}\boldsymbol{J}_0 \times \boldsymbol{B}_0 + e(n_{i0} - n_{e0})\boldsymbol{E}_0 + n_{i0}m_i\boldsymbol{g} = 0, \tag{1}$$

and

$$q_d n_d \boldsymbol{E}_0 + n_d m_d \boldsymbol{g} = 0, \tag{2}$$

where $p$ and $\boldsymbol{J}_0$ are, respectively, the sum of the electron and ion pressures and current densities, $c$ is the speed of light, $B_0\hat{z}$ is the external magnetic field, $\hat{z}$ is the unit vector along the $z$ axis, $n_{i0} = n_{e0} + Z_d n_{d0}$, $q_d = -Z_d e$, $e$ is the magnitude of the electron charge and $Z_d$ is the number of the charge residing on the dust grains, which are assumed to be negatively charged. In the dust momentum balance equation (2), the dust particles are held in balance under the action of the gravitational force $m_d\boldsymbol{g} \equiv -m_d g\hat{x}$, and the dc electric field $\boldsymbol{E}_0 = -E_0\hat{x}$ is produced by the charge imbalance. The latter is, of course, created by the pull on the dust particles by the gravity.

The equation of continuity of the plasma is

$$\nabla \cdot \boldsymbol{J}_{0\perp} = 0, \tag{3}$$

assuming that the parallel (to $B_0\hat{z}$) component of the current density is zero, while the cold dust component has zero velocity in equilibrium.

As already noted in Ref. [2], any plasma pressure or density distribution, which is uniform in the $\boldsymbol{g} \times \boldsymbol{B}_0$ direction (which because of (2) is also the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction) is a solution of Eq. (1).

## 3. Perturbation analysis

The equilibrium given by (1)–(3) is disturbed in the presence of electrostatic waves in the frequency regime $\nu_{ch}$, $\omega_{cd} \ll \omega \ll \omega_{ci}$, where $\nu_{ch}$ is the charging frequency of the dust grains, $\omega_{cd}$ ($\omega_{ci}$) is the dust gyro- (ion gyro-) frequency. In a low-beta plasma, the wave electric field vector is given by $\boldsymbol{E}_1 = -\nabla\phi$, where $\phi$ is the electrostatic potential. The perpendicular (to $\hat{z}$) component of the electron and ion fluid velocities in the drift approximation ($|\partial_t| \ll \omega_{ci}$) are, respectively,

$$\boldsymbol{v}_{e\perp} \approx \boldsymbol{v}_E + \boldsymbol{v}_{De}, \tag{4}$$

and

$$\boldsymbol{v}_{i\perp} \approx \boldsymbol{v}_{E1} + \boldsymbol{v}_{Di} + \boldsymbol{v}_g + \boldsymbol{v}_{pi} \equiv \boldsymbol{v}_{il} + \boldsymbol{v}_{pi}, \tag{5}$$

where $\boldsymbol{v}_{E1} = (c/B_0)\hat{z} \times \nabla\phi$ is the $\boldsymbol{E}_1 \times \boldsymbol{B}_0$ drift velocity, $\boldsymbol{v}_{De} = -(c/eB_0 n_e)\hat{z} \times \nabla(n_e T_e)$ is the electron diamagnetic drift, $\boldsymbol{v}_{Di} = (c/eB_0 n_i)\hat{z} \times \nabla(n_i T_i)$ is the ion diamagnetic drift, $\boldsymbol{v}_{pi} = -(c/B_0\omega_{ci})[\partial_t + (\boldsymbol{v}_{E0} + \boldsymbol{v}_{D0} + \boldsymbol{v}_g)\cdot\nabla)]\nabla_\perp\phi$ is the ion polarization drift, $\boldsymbol{v}_{E0} = (c/B_0)\boldsymbol{E}_0 \times \hat{z}$, $\boldsymbol{v}_{D0} = (c/eB_0 n_{i0})\hat{z} \times \nabla(n_{i0}T_{i0})$, and $\boldsymbol{v}_g = (g/\omega_{ci})\hat{y}$ is the equilibrium ion fluid drift caused by the gravitational field. Here, $n_j$ and $T_j$ are the number density and the temperature of the particle species $j$ ($j$ equals e for electrons, i for ions and d for dust grains). We note that the electron gravitational drift is smaller by a factor $m_e/m_i$ compared to $\boldsymbol{v}_g$, where $m_e$ is the electron mass. For convenience, the ion motion along the magnetic field direction has been ignored.

Substituting Eq. (4) into the electron continuity equation, we have

$$(\partial_t + \boldsymbol{v}_{E0} \cdot \nabla)n_{e1} + \boldsymbol{v}_{E1} \cdot \nabla n_{e0} = 0, \tag{6}$$

where $n_{e1}$ ($\ll n_{e0}$) is the electron number density perturbation and the electron motion parallel to the magnetic field direction has been ignored.

On the other hand, inserting expression (5) for $\boldsymbol{v}_{i\perp}$ into the ion continuity equation, we obtain

$$[\partial_t + (v_0 + v_g)\partial_y]n_{i1} + \boldsymbol{v}_{E1} \cdot \nabla n_{i0} - \frac{n_{i0}c}{B_0\omega_{ci}} D_t \nabla_\perp^2 \phi = 0, \tag{7}$$

where $n_{i1}$ ($\ll n_{i0}$) is the ion number density perturbation, $D_t = \partial_t + (v_0 + v_g + v_{i*})\partial_y$, $v_0 = (c/B_0)E_0$, $v_g = g/\omega_{ci} \equiv v_{ti}^2/R$, $v_{i*} = (c/eB_0 n_{i0})d_x\nabla[n_{i0}(x)T_{i0}(x)]$, $v_{ti}$ is the ion thermal velocity, and $R$ is the radius of curvature.

The dynamics of the unmagnetized cold charged dust particulates is governed by

$$\partial_t n_{d1} + n_{d0}\nabla \cdot \boldsymbol{v}_d = 0, \tag{8}$$

and

$$m_d \partial_t \mathbf{v}_d = -q_d \nabla \phi, \tag{9}$$

where $n_{d1}$ ($\ll n_{d0}$) is the dust number density perturbation.

Subtracting (6) from (7) and imposing the quasi-neutrality condition ($n_{e1} + Z_d n_{d1} = n_{i1}$), which is valid for plasmas in which the ion plasma frequency is much larger than the ion gyro-frequency, we obtain

$$v_g \partial_y (n_{e1} + Z_d n_{d1}) - \frac{c Z_d}{B_0} d_x n_{d0} \partial_y \phi + (\partial_t + v_0 \partial_y)(Z_d n_{d1}) - \frac{n_{i0} c}{B_0 \omega_{ci}} D_t \nabla_\perp^2 \phi = 0. \tag{10}$$

The combination of (8) and (9) leads to

$$\partial_{tt} n_{d1} + \frac{Z_d e n_{d0}}{m_d} \nabla^2 \phi = 0. \tag{11}$$

Eqs. (6), (10) and (11) are the desired equations for studying interchange instabilities in nonuniform dusty magnetoplasmas containing mobile charged dust grains.

In order to obtain the local dispersion relation, we assume that all the physical variables (viz., $n_{e1}$, $\phi$ and $n_{d1}$) vary as $\exp(iky - i\omega t)$, where $\mathbf{k} = \hat{y}k$ and $\omega$ are the wavevector and the frequency, respectively. Thus, Fourier analyzing (6), (10) and (11) and combining the resultant equations, we obtain the linear dispersion relation

$$\Omega^2 - \Omega \omega_* + g \epsilon \kappa_e - \frac{Z_d \omega_{LD}^2 \Omega}{\omega^2} \left( \Omega - \frac{\omega^2 \kappa_d}{k \omega_{cd}} \right) = 0, \tag{12}$$

where $\Omega = \omega - kg/\omega_{cd}$, $\omega_* = k v_{i*}$, $\epsilon = n_{e0}/n_{i0}$, $\kappa_e = d_x \ln n_{e0}$, $\kappa_d = d_x \ln n_{d0}$ and $\omega_{LD}^2 = \omega_{cd}\omega_{ci}$. We have also substituted for $E_0$ in terms of $g$ from the equilibrium condition (2). It is observed that $v_0 = g/\omega_{cd} \gg v_g \equiv g/\omega_{ci}$, so that $v_0 + v_g \approx v_0$.

The first three terms of (12) essentially give the usual Rayleigh–Taylor instability incorporating the finite Larmor radius effects, the growth rate of which is reduced because of the factor $\epsilon$ in the third term [2, 3]. The only role the dust plays in this instability is to make a fraction $(1 - \epsilon)$ of the electrons inert by attaching them to itself. The second set of two terms (the fourth and the fifth terms on the left-hand side of (12)) are both multiplied by the dust charge number $Z_d$, and have their origin directly in the dust dynamics. When the last two terms dominate over the first three terms on the left-hand side of (12), then we may separately equate them to zero, whereby they yield a new kind of dust interchange instability, entirely different from the R–T mode. The new interchange mode dispersion relation thus reads

$$\omega^2 - \frac{k \omega_{cd}}{k_d} \omega + \frac{g k^2}{k_d} = 0, \tag{13}$$

where

$$\omega = \tfrac{1}{2} \omega_{cd} \frac{k}{k_d} \pm \tfrac{1}{2} \omega_{cd} \frac{k}{k_d} \left( 1 - \frac{4 g k_d}{\omega_{cd}^2} \right)^{1/2}. \tag{14}$$

It follows from (14) that there appears an instability if

$$4 g k_d > \omega_{cd}^2. \tag{15}$$

Inequality (15) can also be written as

$$\tfrac{1}{2} g \tau_{cd}^2 > \tfrac{1}{2} \pi^2 L_d. \tag{16}$$

The quantity $\frac{1}{2} g \tau_{cd}^2$ is the distance $\lambda_g$ through which a dust particle would free fall under gravity from rest. Condition (16) for instability has the interesting physical meaning that the dust density variation length $L_d \equiv k_d^{-1}$ be less than $2/\pi^2$ times the dust free fall distance $\lambda_g$, over a dust gyro-period $\tau_{cd} \equiv \omega_{cd}^{-1}$. If $4 g k_d \gg \omega_{cd}^2$, then

$$\omega \approx \omega_{cd} \frac{k}{k_d} \pm i \left( \frac{g k^2}{k_d} \right)^{1/2}. \tag{17}$$

Eq. (17) predicts an oscillatory instability with an increment $(g/k_d)^{1/2} k$. Since the latter is proportional to the wavenumber $k$, it is likely that short wavelength perturbations could be rapidly driven via the present instability mechanism. Furthermore, the present growth rate, in general, could also be larger than the usual R–T growth rate of this system.

## 4. Summary

The equilibrium of a dusty plasma supported by a magnetic field under gravity involves an equilibrium polarization electric field in the direction of gravity which holds the negatively charged unmagnetized massive dust particles from falling freely under gravity. Such an equilibrium electric field $\boldsymbol{E}_0$ produces the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift of the electrons and ions, which exceeds the ion gravity drift by a dust to ion mass ratio.

In this paper, we have studied the instability of a self-consistent dusty plasma equilibrium state against electrostatic perturbations employing the multi-fluid model. The latter holds provided that the grain sizes as well as the inter-grain spacing are much smaller than the characteristic scale lengths (viz. the plasma Debye length, gyro-radii, etc.). It has been shown that there appears a new type of interchange instability that is primarily induced by the dynamics of the dust particles. The physical mechanism of the newly found instability can then be understood as follows: Because of the charge imbalance $(n_i - n_e)$ in the dusty plasma, the $\boldsymbol{E} \times \boldsymbol{B}_0$ drift now carries a positive current [4]. A transverse plasma density perturbation with a wavevector in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction coupled with a longitudinal (compressible) negative charged dust density perturbation with the same wavevector then produces a charge wave in a plasma with an inhomogeneity in the direction of $\boldsymbol{E}_0$ (or equivalently $\boldsymbol{g}$). The $\boldsymbol{E}_1 \times \boldsymbol{B}_0$ drift resulting from the charge wave then enhances the latter, if the equilibrium density is inverted with respect to the gravity. Accordingly, the present interchange sets in.

The result of the instability would be to make the plasma move along the direction of $\boldsymbol{E}_0$, so as to nullify it. This will then allow the dust to fall further under gravity, pulling the plasma again with it through the newely found instability. This then provides a proper mechanism through which a dusty plasma will fall under gravity determined essentially by the massive dust component.

Our investigation assumes constant charge on the dust grains. In reality, the dust charge fluctuations induced by wave motions may cause a small damping [5,6]. Accordingly, the growth rate of our interchange instability could be somewhat reduced.

The results of our investigation should obviously be relevant in a variety of low-temperature laboratory as well as space and astrophysical plasmas which are held under the combined effects of gravity and Lorentz forces.

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