Magnetic field effect on the polarizability of bound polarons in quantum nanocrystallites

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We have studied the simultaneous effects of magnetic and electric fields on the ground-state energy of a donor impurity confined in a polar CdSe quantum nanocrystallite embedded in a nonpolar matrix. Calculations are performed in the framework of the effective-mass approximation using the Hassé variational approach. We describe the effect of the quantum confinement by a finite deep potential and we take onto account the interaction between the charge carriers (electron and ion) and the confined longitudinal optical phonons (LO phonons). It is found that the corrections due to the LO phonons on the binding energy, the diamagnetic coefficient, and the polarizability are very important and cannot be neglected or ignored.

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I. INTRODUCTION

The quantum nanocrystallites (QNC's) are nanostructures of spherical shape which may be obtained by precipitation in either isolating[1] or semiconducting matrices[2] or synthesized as colloidal suspensions in organic liquids.[3,4] The tridimensional nanoscale confinement of the carriers gives rise to a full quantum nature to these structures. So many novel physical and optical effects appear with promising potentialities in finding novel applications in the nanotechnology area. In addition, their possible design flexibility allows one to develop artificial atoms or microelectronic units which justify the great technological interest and the intensive investigation in basic and applied research.[5]

In the past decade, the understanding of the electronic and optical properties of impurities in the QNC was the subject of many investigations. Porras-Montenegro and Perez-Merchano[6] have calculated the impurity binding energy as a function of the radius and the impurity position as well as the density of impurity states in a GaAs/GaAlAs QNC. Zhu and Chen[7] have reported theoretical results on the energy levels and binding energies of an off-center donor in a spherical GaAs/GaAlAs QNC. Silva-Valencia and Porras-Montenegro[8] have studied the optical absorption spectra with a transition between the $n=1$ valence level and the donor impurity band.

The effect of an applied electric field on the properties of shallow donors has also been a subject of much interest. Indeed, in a confined medium, the electric field leads to a quantum confined Stark effect[9–12] characterized by a redshift, many times greater than the electron-hole binding energy. In our previous works,[13,14] by using a variational approach and neglecting the polaronic effect, we have studied the simultaneous effects of magnetic and electric fields on the binding energy of a donor confined in an infinite potential. We have shown that the magnetic field reduces the spatial extension of the wave function and leads to a decrease of the donor polarizability. We have also reported the calculation of the polarizability of a shallow donor placed anywhere in a spherical quantum dot. We have shown that for a donor placed at the center of the sphere, the energy level shifts to lower energies. However, for an off-center donor, the shift depends strongly on the orientation of the electric field.

The materials commonly used in the fabrication of the QNC's are ionic semiconductors. These materials show a strong electron-phonon coupling leading to significative modifications in the donor spectra due to the polaronic effects. As pointed out by several authors,[15–24], the polaronic effects are more complicated in low dimensional semiconductors than in bulk ones due to the occurrence of interface and confined modes. Several theoretical studies have been devoted to the electron and donor optical-phonon coupling in QNC's but they were restricted to an infinitely deep potential model. Marini *et al.*[17,18] have determined the Huang-Rhys factor in the model of a donorlike exciton located at the center of a CuCl QNC. Fedorov and Baranov[25] have studied the effect of LO and surface optical (SO) phonons on the donor energy. Fliyou *et al.*[26] have calculated the binding energy of a hydrogenic impurity in a GaAs QNC using the Lee, Low, and Pines transformation[27] and the variational method. In a ZnSe QNC, Xie, and Chen[28] have shown that the phonon contribution to the binding energy is dependent on the position of the donor. Their numerical results prove that the magnitude of the SO phonon contribution is quite small and plays a relatively unimportant role in the binding energy of a Coulomb impurity.

In this paper, we study the effects of the coupling of the charge carriers (electron and ion) with LO phonons on a donor impurity placed at the center of a polar QNC embedded in a nonpolar matrix and submitted to the joint effects of magnetic and electric fields. The confinement will be described by a finite barrier, so the probability of penetration in the host material is not negligible, which leads to some modifications of the energy behavior. In the following section, we outline our theoretical approach used to determine the state of the bound polaron in the presence of magnetic and electric fields. In Sec. III we present and discuss our

numerical results by comparing them with the case without phonons.

II. THEORY

We consider a donor impurity D^0 located at the center of a QNC with a finite potential barrier submitted to the joint effects of electric and magnetic fields applied along the z axis. In the frame of the effective-mass approximation and the Fröhlich[29] interaction Hamiltonian for electron-LO phonon coupling, the Hamiltonian of the system D^0 reads

$$H = H_{el} + H_{ph} + H_{int}. \tag{1}$$

The electronic Hamiltonian is given by

$$H_{el} = H_0 + V_w + W + M, \tag{2}$$

where the unperturbed Hamiltonian reads

$$H_0 = T + V = -\frac{\hbar^2}{2m_e^*}\Delta - \frac{e^2}{\varepsilon_\infty r}. \tag{3}$$

where m_e^* is the electron effective band mass, ε_∞ denotes the high-frequency dielectric constant, and r is the electron position relative to the donor located at the center of the QNC of radius R. We assume that the confinement potential energy is modelized by a square well

$$V_w = \begin{cases} 0, & r < R \\ V_0, & r \geqslant R. \end{cases} \tag{4}$$

The potential well depth V_0 is equal to the conduction bands offset between the nanocrystallite and the host material. The energy due to the external electric field $\vec{F}$ is given by

$$W = -e\vec{F}\cdot\vec{r} = -eFr\cos\theta. \tag{5}$$

In the present study, we do not take into account the possible spin-orbit coupling as well as the Zeeman effect, restricting ourselves to the diamagnetic contribution. Therefore, our results may be interpreted as ''mean'' results independently of a possible energy splitting. In these conditions, the diamagnetic contribution M due to the magnetic field B reads, using the Coulomb gauge,

$$M = \frac{\hbar^2 e^2}{2m_e^* c^2} B^2 r^2 \sin^2\theta. \tag{6}$$

The Hamiltonian of the noninteracting LO phonon can be written as[15,16]

$$H_{ph} = \sum_{qlm} \hbar\omega_{LO}[a_{lm}^+(q)a_{lm}(q) + 1/2], \tag{7}$$

where $a_{lm}^+(q)$ and $a_{lm}(q)$ are the creation and annihilation operators, respectively, of a LO phonon of wave number q with quantum numbers l,m and frequency ω_{LO}. The electron-LO-phonon interaction Hamiltonian writes as

$$H_{int} = -\sum_{qlm} [V_l(q) j_l(qr_e) Y_{lm}(\theta,\varphi) a_{lm}(q) + \text{H.c.}] + \sum_{qlm} [V_l(q) j_l(0) Y_{lm}(\theta,\varphi) a_{lm}(q) + \text{H.c.}], \tag{8}$$

where H.c stands for the Hermitian conjugates. The first term describes the interaction between the confined electron and the longitudinal-optical vibration mode. The second term accounts for the interaction of the positively charged donor center with LO phonon. $j_l(qr)$ and $Y_{lm}(\theta,\varphi)$ are the spherical Bessel functions and the spherical harmonics, respectively. The summations run over $l = 0,1,2,\ldots,\infty$; $m = -l,\ldots,+l$. q is deduced from the roots of j_l. The interaction amplitude $V_l(q)$ may be written in the form

$$V_l(q) = -\left[\frac{4\pi e^2 \hbar\omega_{LO}}{j_{l+1}^2(qR) R^3 q^2}\right]^{1/2}\left[\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right]^{1/2}, \tag{9}$$

where ε_0 is the static dielectric constant.

In order to separate out the static lattice deformation induced by the positive donor center and to introduce a proper screening of the electron-donor coulomb interaction,[18,24,30] we apply to Hamiltonian (1) the Platzman transformation[31] by means of the unitary operator

$$U_p = \exp - \sum_{qlm}\left[\frac{V_l(q)}{\hbar\omega_{LO}} a_{lm}^+(q) - \text{H.c.}\right]. \tag{10}$$

The effect of this transformation is to displace the equilibrium position of the ions. So, the interaction of the positively charged donor with LO phonons, the second term in Eq. (8) is replaced by a screening potential

$$V_{scr} = \frac{e^2}{r}\left(1 - \frac{r}{R}\right)\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right), \tag{11}$$

which cancels for $r = R$. In the limit when R becomes infinite, potential V_{scr} added to the Coulombic potential leads to screen the electron-donor center interaction by the static dielectric constant ε_0.

In order to separate the Hamiltonian in an electronic and an ionic contribution, we apply a second canonical transformation elaborated by Pekar[32] for bulk materials. This approach is expected to be well justified when the motion of the electrons is much faster than that of the heavier ions. This may arise in the following two cases: (1) in the case of a strong electron-phonon coupling where a self-localization occurs, i.e., fast electron oscillations; (2) in the case of a microsphere with a small radius R, where the quantum confinement produces an orbital shrinking which increases the electron speed. This situation corresponds to a strong quantum confinement. Moreover, we remark that in the cases of applied electric or magnetic fields, the electron velocity is also increased, so that the above approximation becomes more suitable.

In the present study, we focus on the second situation, i.e., QNC with small radius in magnetic and electric fields. In the

case of moderate electron-phonon coupling, we expect that our method becomes less accurate for large QNC's, three-dimensional (3D) limit.

Within the adiabatic approximation, the transformed Hamiltonian writes

$$H_T = H_{el} + V_{scr} + \langle V_{ep} \rangle_\Psi , \tag{12}$$

where the mean value of the electron-LO phonon interaction is given by

$$\langle V_{ep} \rangle_\Psi = -2 \sum_{qlm} \frac{V_l(q)}{\hbar \omega_{LO}} |\rho_{lm}(q)|^2 . \tag{13}$$

$\rho_{lm}(q)$ is the Fourier transform of the electron charge distribution

$$\rho_{lm}(q) = \int_{sphere} d\vec{r} j_l(qr) Y_{lm}(\theta, \varphi) |\Psi(\vec{r})|^2 . \tag{14}$$

It is worth mentioning that since the ground state is of spherical symmetry, only the term with $l=m=0$ is nonzero. Moreover, we remark that the convergence of the summation over q is very fast due to the strong localization of the ground states.

The ground-state wave function of the neutral bound polaron is solution of the Schrödinger equation:

$$H_T \Psi(r) = E^{ph} \Psi(r). \tag{15}$$

In the following, all expressions will be given in the effective units: $a^* = \varepsilon_\infty \hbar^2 / m_e^* e^2$ for length and $R^* = m_e^* e^4 / 2\varepsilon_\infty^2 \hbar^2$ for energy. Furthermore, we introduce the dimensionless parameters $f = (e a_D / R_D) |\vec{F}|$ and $\gamma = \hbar \omega_c / 2R_D$ characterizing the strength of the electric and magnetic fields, respectively. $\omega_c = eB/m_e^* c$ is the effective cyclotron frequency.

In order to solve numerically Eq. (15), we use Hasse's variational method[33] adapted to QNC's in our previous works.[13,14] The trial wave function is given by the following expression:

$$\Psi = \Psi_0 [1 + (\mu + \lambda r) \vec{f} \cdot \vec{r}], \tag{16}$$

i.e.,

$$\Psi = \Psi_0 + (\Psi_1 + \Psi_2) \vec{f} \cdot \vec{r}, \tag{17}$$

where $\Psi_1 = \mu \Psi_0$ and $\Psi_2 = \lambda r \Psi_0$, λ and μ are variational parameters. Ψ_0 describes the wave function of the unperturbed donor confined in a QNC:

$$\Psi_0 = \begin{cases} A \dfrac{\sin(ar/R)}{r} \exp(-\alpha r), & r < R \\ C \dfrac{\exp(-br)}{r} \exp(-\alpha r), & r \geqslant R. \end{cases} \tag{18}$$

Here α is a variational parameter, a and b are determined through the two following equations deduced from the continuity conditions for Ψ_0 and Ψ_0' at $r=R$:

$$\begin{aligned} a \cot\ a &= -bR \\ a^2 + b^2 &= V_0 . \end{aligned} \tag{19}$$

The two normalization constants A and C are related by

$$C \exp(-bR) = A \sin a. \tag{20}$$

Thus the donor ground-state energy is obtained by minimizing the expectation value of H_T with respect to the variational parameters λ and μ for a fixed value of α:

$$E^{ph} = \min \frac{\langle \Psi | H_T | \Psi \rangle}{\langle \Psi | \Psi \rangle}. \tag{21}$$

Taking into account the expressions of the transformed Hamiltonian and the wave function, the total energy reduces to

$$E^{ph} = \frac{E^{ph}(\alpha) + E_1^{ph}(\alpha) f^2}{1 + \dfrac{N_2}{N_0} f^2}. \tag{22}$$

$E^{ph}(\alpha)$, $E_1^{ph}(\alpha)$, and N_2/N_0 depend explicitly on the variational parameters λ and μ. They may be expressed as functions of the following integrals:

$$I_n = \int_0^R r^n \sin^2 ar\ \exp(-2\alpha r) dr,$$

$$J_n = \int_0^R r^n \sin^2(ar) \exp(-2\alpha r) dr,$$

$$K_n = \int_R^\infty r^n \exp -2(b+\alpha) r dr,$$

$$L_n = \int_0^R \exp -2\alpha r \sin^2(ar) j_0\left(\frac{n\pi r}{R}\right) dr. \tag{23}$$

It may be verified that the ratio N_2/N_0 reduces to

$$\frac{N_2}{N_0} = \frac{\mu^2 (A^2 I_2 + C^2 K_2) + \lambda^2 (A^2 I_4 + C^2 K_4) + 2\mu\lambda (A^2 I_3 + C^2 K_3)}{3[A^2 I_0 + C^2 K_0]}. \tag{24}$$

We first solve the problem without any electric field ($f=0$). The upper bound for the ground-state energy obtained from Eq. (12) and Eq. (22) writes

$$E^{ph}(\alpha)=E_0^{ph}+\gamma^2 D^{ph}, \tag{25}$$

where E_0^{ph} is the bound polaron energy, without any electric or magnetic field

$$E_0^{ph}=\frac{1}{4\pi(A^2I_0+C^2K_0)}\Bigg\{4\pi C^2V_0K_0 - 8\pi[A^2I_{-1}+C^2K_{-1}]+8\pi\left(1-\frac{\varepsilon_\infty}{\varepsilon_0}\right)A^2\left[I_{-1}-\frac{I_0}{R}\right] - \frac{32\pi^2}{R}A^4N_0^{-2}\left(1-\frac{\varepsilon_\infty}{\varepsilon_0}\right)\sum_n L_n^2\Bigg\}, \tag{26}$$

and D^{ph} is the diamagnetic coefficient

$$D^{ph}=\frac{A^2I_2+C^2K_2}{6(A^2I_0+C^2K_0)}. \tag{27}$$

In the case of a weak electric field expression (22) becomes

$$E^{ph}=E^{ph}(\alpha)-\frac{p^{ph}}{2}f^2, \tag{28}$$

where p is the polarizability of the bound polaron given by

$$p^{ph}=2\left[\frac{N_2}{N_0}E^{ph}(\alpha)-E_1^{ph}(\alpha)\right]. \tag{29}$$

III. RESULTS AND DISCUSSION

We have done our numerical calculation for a CdSe semiconductor with a relatively high electron-phonon coupling constant ($\alpha=0.46$). This value of α does not correspond, stricktly speaking, to the strong coupling limit. So our method becomes certainly questionable in the 3D limit. However, we expect that it will be suitable for small QNC in electric and magnetic fields.

We have used the following material parameters[18] for the electron band mass $m_e^*=0.13m_0$, the dielectric constants $\varepsilon_0=9.56$, $\varepsilon_\infty=6.23$, and the LO-phonons energy $\hbar\omega_{LO}=26$, 46 meV. The effective units of length and energy are $a^*=25.3$ Å and $R^*=45.55$ meV respectively. We note that since the potential depth value is finite, then the probability of penetration in the barrier region is not negligible.

We have first determined the influence of the phonons corrections on the binding energy of the impurity donor in the absence of the magnetic and electric fields. In order to estimate the influence of the quantum confinement on the Coulomb correlations, we introduce the binding energies $E_b^{0,ph}$ defined as the difference between the electron ground-state energy without the Coulomb interaction and the impurity ground-state energy. They read

$$E_b^0=E_e^0-E_0^0, \tag{30}$$

without taking into account the interaction with the phonons, and

$$E_b^{ph}=E_e^{ph}-E_0^{ph}, \tag{31}$$

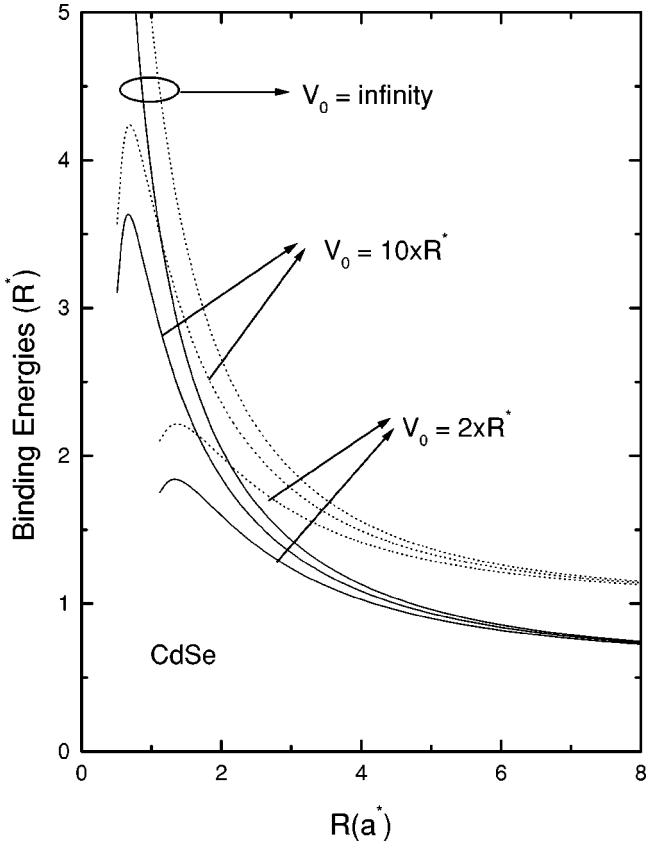

FIG. 1. Variation of the binding energies of the impurity donor as function of QNC radius R for $f=0$ and $\gamma=0$, and for three barrier heights $V_0(R^*)=2$, 10 and infinity. The solid and dashed curves represent the binding energies with (E_b^{ph}) and without (E_b^0) the phonons corrections, respectively.

when this interaction is taken into account. We note that the energy of the confined electron is equal to $E_e^0=a^2$ and $E_e^{ph}=a^2+\langle V_{ep}\rangle_{\Psi_0}$ without and with phonon interaction, respectively. E_0^0 and E_0^{ph} correspond to the impurity ground state energies without electric and magnetic with and without the phonon interaction. In Fig. 1, the binding energies are plotted versus the crystallite radius R. We can see that with finite barriers, the binding energy (dashed curves) increases as the radius R decreases and reaches a maximum at R_c. Before this threshold R_c and at small values of $R<R_c$ discrete levels vanish in the well and the electron wave function is extended even outside the well. On the other hand, the shape of the curves obtained by taking into account the phonons correction (solid curves) is similar to those obtained by neglecting the phonons effects (dashed curves). The importance of this contribution is more pronounced for $R=R_c$ and for $R>2a^*$. Indeed, the coupling with the phonons leads to a weakening of the binding energy, which results to a significative screening of the coulombic potential. This behavior is analogous to that obtained by Zhao and Liang[34] by using the Lee, Low, and Pines variational method and a parabolic quantum well. In order to analyze the influence of the phonons interaction with the charge carriers (electrons and ions) on the ground-state energy, we draw in Fig. 2 the sum of the two terms due to this interaction, i.e., ΔE_{ph}

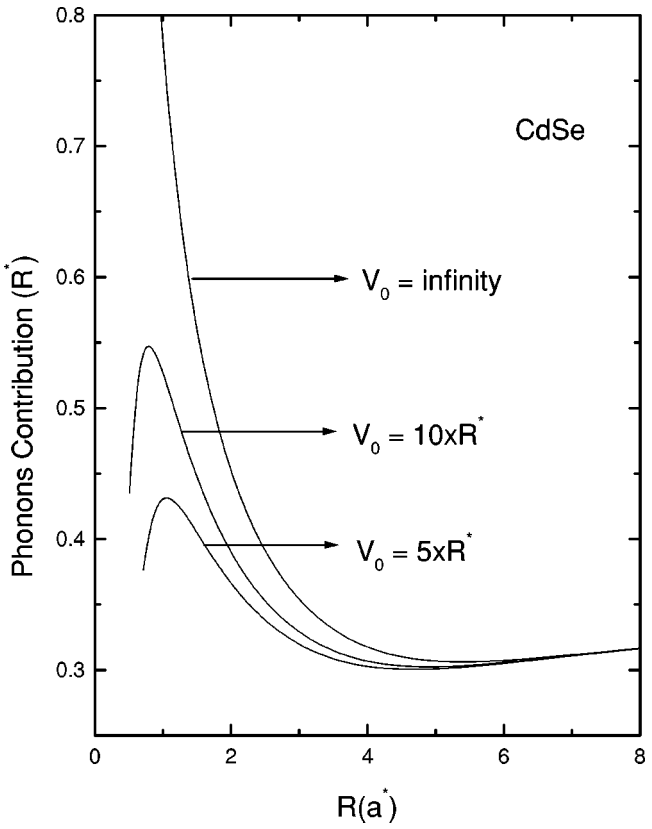

FIG. 2. Variations of the phonons contribution ($\Delta E_{ph} = \langle V_{scr}\rangle_\Psi + \langle V_{ep}\rangle_\Psi$) as function of radius R and for different degrees of confinement.

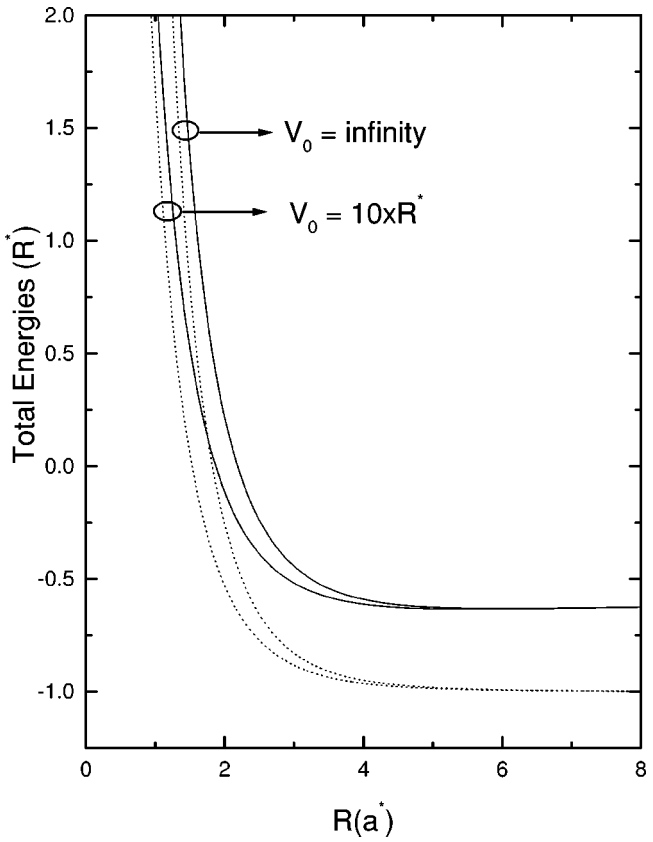

FIG. 3. The ground-state energy of the impurity donor as function of QNC radius R for $f=0$ and $\gamma=0$. The solid and dashed curves represent the total energies with (E_0^{ph}) and without (E_0^0) the phonons corrections, respectively.

$=\langle V_{scr}\rangle_\Psi + \langle V_{ep}\rangle_\Psi$ as a function of the radius R. We remark that in the case of a strong confinement, the phonons contribution increases and reaches a maximum which depends strongly on the height of the potential barrier, then decreases at increasing radius and exhibits a minimum localized between $R=4a^*$ and $R=6a^*$. For $R>6a^*$ the contribution of the phonons increases only slightly and reaches its saturation value corresponding to the 3D limit.

In order to investigate the influence of electric and magnetic fields on the bound polaron binding energy, we rewrite the total energy E^{ph}, given in Eq. (28) taking into account relation (25). It becomes, without phonons

$$E^0 = E_0^0 + \gamma^2 D^0 - \frac{p^0}{2} f^2, \tag{32}$$

and with phonons

$$E^{ph} = E_0^{ph} + \gamma^2 D^{ph} - \frac{p^{ph}}{2} f^2. \tag{33}$$

Equations (32) and (33), valid only for low electric fields, show that for given electric (f) and magnetic (γ) fields, the determination of the total energy E^{ph} (E^0) require the knowledge of the energy of the unperturbed system E_0^{ph} (E_0^0), the diamagnetic coefficient D^{ph} (D^0), and the polarizability p^{ph} (p^0).

In Fig. 3, we plot the unperturbed total energies E_0^{ph} (solid curves) and E_0^0 (dashed curves) corresponding to the cases with and without phonon coupling, respectively. It can be seen that for small radii this energy diverges, and decreases rapidly reaching the bulk value for large radii R. In addition, we remark that the phonons correction shifts the total energy to higher values. This shift is obvious even for radii neighboring unity and becomes important at increasing QNC sizes.

The influence of the magnetic field is estimated by computing the diamagnetic contribution $M=\gamma^2 D$. In Fig. 4, the diamagnetic coefficient D is drawn against the QNC radius R. We remark that D increases at increasing radius R and converges asymptotically to the bulk limit for large values of R. Indeed, when R decreases, the electronic orbital is more localized and becomes less sensitive to the influence of the magnetic field. The solid curves in Fig. 4 demonstrate the importance of the polaron effects which shifts up the magnetic contribution, mainly for large QNC sizes. This results from the screening of the electron-donor coulomb interaction due to the interaction with the phonons. This effect leads to a decrease of the correlation between the electrons and the ions.

To describe the confinement effect on the magnetic contribution, we present in Fig. 5 the variations of the diamagnetic coefficient as a function of the radius R for different

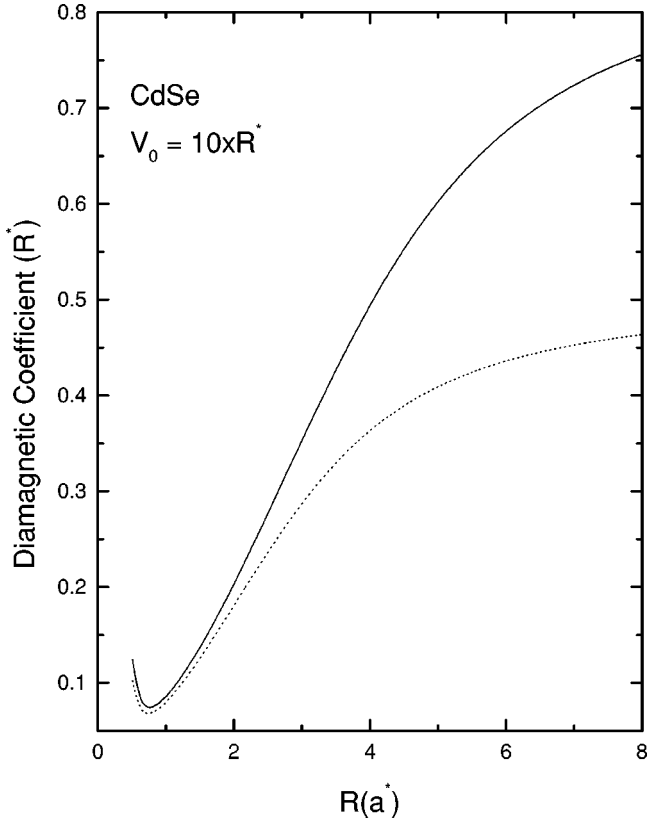

FIG. 4. The diamagnetic coefficient of impurity donor as function of the radius R without (dashed curve) and with (solid curve) phonons contribution.

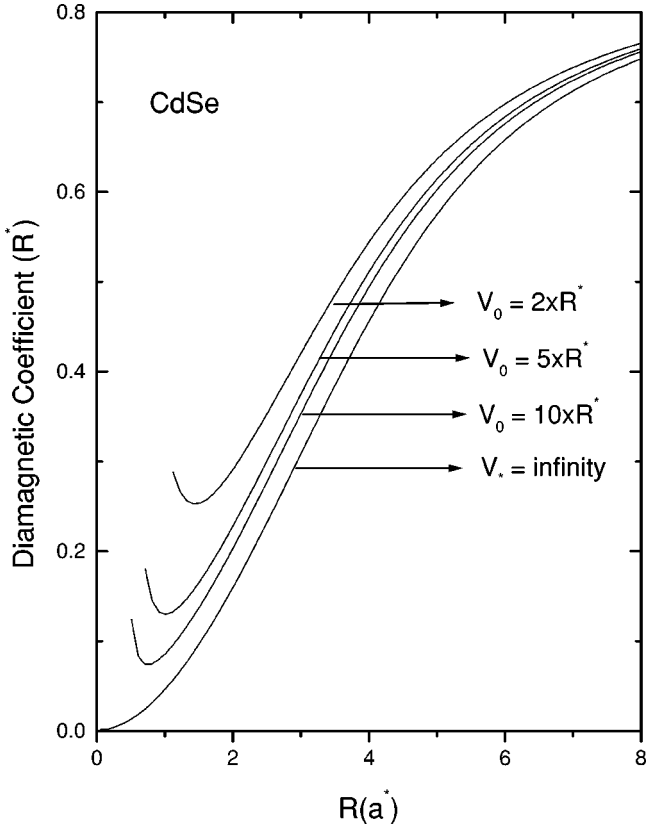

FIG. 5. Variations of the diamagnetic coefficient of the bound polaron as function of the radius R and for different height barrier $V_0(R^*)=2$, 5, 10, and infinity.

heights of the barriers. We observe that the effect of the confinement tends to reduce the diamagnetic field, namely, for the small sizes.

Now, we discuss the dependence of the polarizability on the optical lattice polarization. Figure 6 illustrates the variations of the bound polaron polarizability in a polar QNC of CdSe as a function of the radius R and for a significative value of the magnetic field ($\gamma=0.5$). We analyze both cases: without (dashed lines) and with (solid lines) phonons correction. Some interesting phenomena can be observed: the dashed curve shows that the polarizability increases at increasing crystallite volume. For small sizes, the confinement effect is predominant in comparison with the magnetic-field effect. On the other hand, the influence of the magnetic field on the polarizability is more pronounced for large radius. This is due to the fact that the electronic orbitals are more extended and because the magnetic field gives rise to an additive lateral confinement leading to a reduction of the spatial extension of the electronic orbitals and therefore an increase of the binding between the electrons and the ionized donor. The solid lines show that the phonons correction tends to increase considerably the polarizability of the donor, so we expect a large shift of the polarizability in comparison to the without phonon contribution case.

In Fig. 7, the polarizability versus the dot radius for different degrees of confinement is plotted. Two main aspects can be observed: (i) the polarizability increases strongly with the QNC sizes if the barrier height is large. On the other hand, for small potential depths, it has a minimum for a particular value of the QNC and increases for small radii as a consequence of the wave function spreading in to the material barrier; (ii) the polarizability decreases when the confinement potential increases. This result reflects the fact that the electronic density becomes more localized in the well for height values of the confinement potential. This behavior is in good agreement with that observed in a recent work due to Seufert *et al.*[35] in a single CdSe/ZnSe cylindrical quantum dot.

It must be stressed that the results we obtain in the 3D limit ($R\rightarrow\infty$) may be less accurate in order to describe the actual situation arising in a CdSe bulk crystal, mainly because its coupling constant do not correspond to a strong electron-phonon coupling. Nevertheless, we may verify the accuracy of our calculations by comparing the limit values we obtained for the total energy when $R\rightarrow\infty$, with that we may deduce from the expression obtained[32] using a Coulomb-type trial wave function $E_{D^0}^{3D}=-[11/16(\varepsilon_\infty/\varepsilon_0)+5/16]$. Without the phonon coupling, we get the usual value $E_{D^0}^{3D}=-1$, obtained with $\varepsilon_\infty=\varepsilon_0$.

In conclusion, we have studied the effect of the coupling of charge carriers (electron and ion) with LO phonons on a donor impurity in polar QNC embedded in nonpolar matrix and submitted to weak uniform electric and magnetic fields. Our calculations are performed within the effective-mass approximation and using the Hasse variational treatment. Our

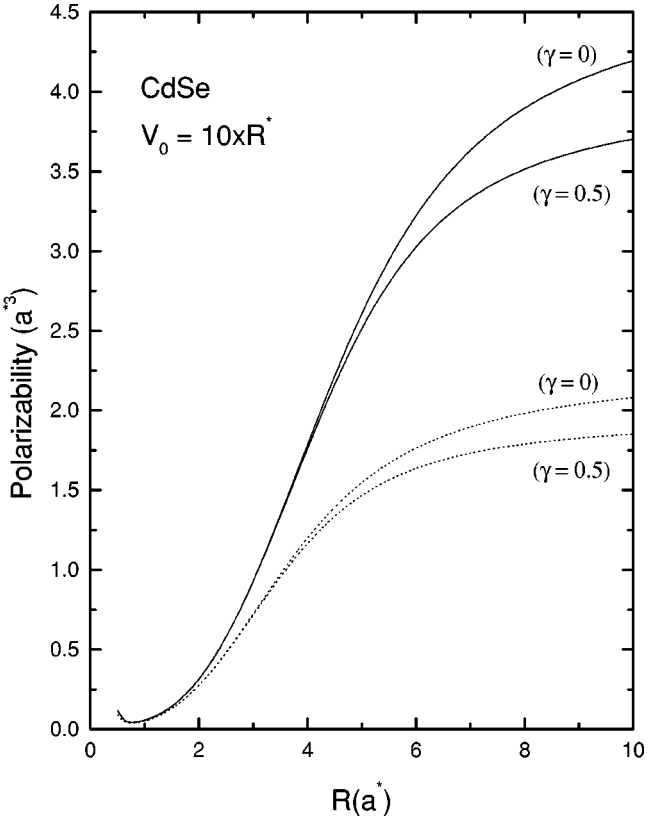

FIG. 6. The polarizability of the impurity donor plotted against the crystallite radius R without (dashed curve) and with (solid curve) phonons correction for two values of magnetic fields $\gamma=0$ and 5.

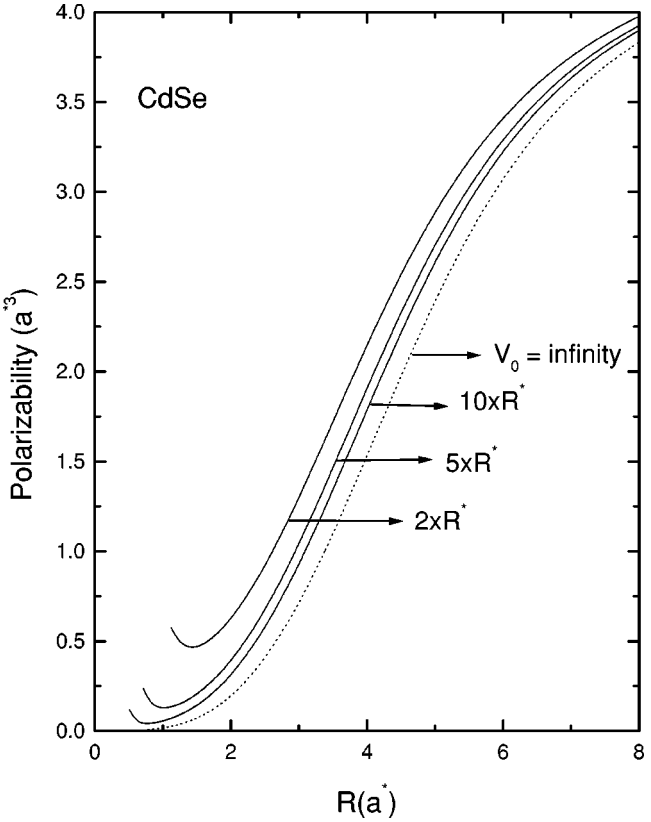

FIG. 7. The polarizability of the bound polaron plotted as function of radius R for different values of the well depths $V_0(R^*)$ $=2$, 5, 10, and infinity.

results show that the polaronic effects are important and cannot be neglected. As it can be shown in the different figures, the phonons corrections depend on the QNC sizes and on the potential depth of the wells. Our results show that the polarizability is underestimated when the phonons correction is not taken into account. In addition, we show that the polarizability decreases with increasing confinement potential which is in good agreement with recent results obtained in single CdSe/ZnSe cylindrical quantum dots.[35] Before closing this paper, it must be stressed that the simplicity of our approach constitutes an advantage owing to the fact that it gives us the possibility to estimate the order of magnitude of the diamagnetic contribution, the stark shift, the polarizability, and the binding energy for given weak magnetic and electric fields.

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