

# Experimental observation of negative lateral displacements of microwave beams transmitting through dielectric slabs

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## Abstract

It was theoretically predicted that when a beam of light travels through a thin slab of optically denser medium in the air, the emerging beam from the slab will suffer a lateral displacement that is different from the prediction of geometrical optics, that is, the Snell's law of refraction and can be zero and negative as well as positive. These phenomena have been directly observed in microwave experiments in which large angles of incidence are chosen for the purpose of obtaining negative lateral displacements.

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It is well known that the Snell's law of refraction determines the propagation direction of refracted beam when a light beam is incident obliquely on a plane interface between two different dielectric media and underlies all considerations of optical lenses and systems based on geometrical optics. According to this law, when a light beam impinges onto a plane parallel slab of optically denser medium of refractive index  $n$  in the air, the emerging beam from the slab will suffer a lateral displacement [1],  $d_g = a \cdot \tan \theta$ , along the surface from the position at which the beam is incident on the left surface, as is shown in Fig. 1, where  $a$  is the thickness of the slab,  $\theta$  is the refraction angle of the beam within the slab that is determined by Snell's law,  $n_0 \sin \theta_0 = n \sin \theta$ ,  $n_0$  is the refractive index of the air, and  $\theta_0$  is the incidence angle of the beam.

This displacement is always positive for slabs of positively refractive normal media. But it was shown [2,3] that

when the thickness of the slab is comparable with the wavelength of the light, the lateral displacement of the transmitted beam through the slab is different from the prediction of geometrical optics, that is, the Snell's law of refraction. It was further predicted [4] that this displacement could be zero and negative as well as positive, depending on the relation between the refractive index and thickness of the slab, the angle of incidence, and the wavelength of the light. For the lateral displacement to be negative as is depicted by the dashed line in Fig. 1, the angle of incidence should be large enough and satisfy the following equation [4],

$$\cos \theta_0 < \left( \frac{n^2 - 1}{2} \right)^{1/2}. \quad (1)$$

In this paper, we report on the first experimental measurements that verify the negative lateral displacement of a microwave beam passing through a thin slab of positively refractive material. This negative lateral displacement has nothing to do with the negatively refractive left-handed material [5,6]. As a matter of fact, when a light beam

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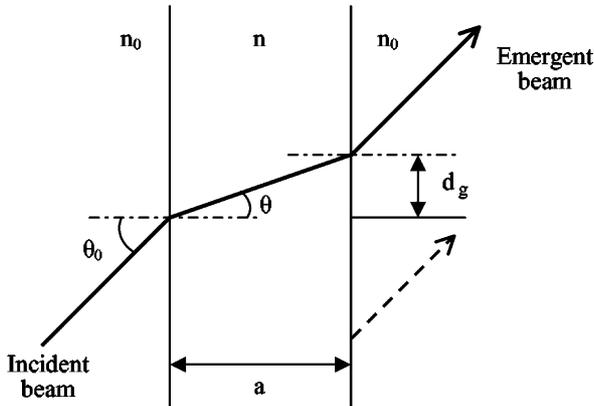


Fig. 1. Schematic diagram of lateral displacements suffered by the transmitted light beam through a dielectric slab along the surface from the position at which the beam is incident at angle of incidence,  $\theta_0$ , on the left surface. The solid line on the right side denotes the position of the emerging beam that is predicted by Snell's law of refraction and undergoes a positive displacement,  $d_g = a \cdot \tan \theta$ . The dashed line denotes the position of the emerging beam that undergoes a negative displacement.

travels through a slab of negatively refractive material, the emerging beam can undergo positive lateral displacements [7].

Our microwave experimental setup for measuring the lateral displacement is shown in Fig. 2. Microwaves of free-space wavelength  $\lambda_0 = 3.28$  cm are generated by a semiconductor signal source (DH1121C) operating at modulation frequency of 1 kHz. The signal is fed by waveguides through an isolator and an attenuator to a 40 cm parabolic transmitting horn antenna. The waveguide is held by a flange which can be rotated about its axis for the purpose of varying the state of microwave beam's polarization. The measured half waist width of the microwave beam produced from the transmitter is  $w_0 = 2.4\lambda_0$ . The beam, after propagating through the dielectric slab, is received by a 10.5 cm  $\times$

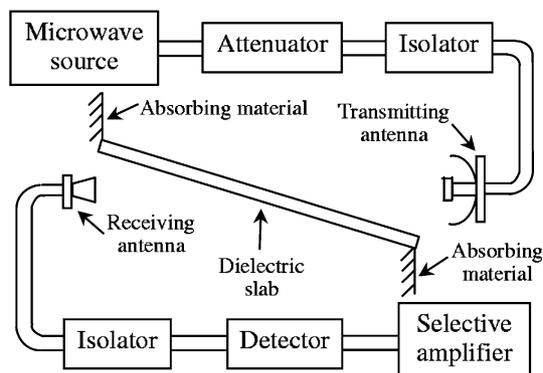


Fig. 2. Diagram of our microwave experimental setup. Microwaves of free-space wavelength  $\lambda_0 = 3.28$  cm are generated by a semiconductor signal source (DH1121C) operating at modulation frequency of 1 kHz. The signal is fed by waveguides through an isolator and an attenuator to a 40 cm parabolic transmitting horn antenna. The beam, after propagating through the dielectric slab, is received by a 10.5 cm  $\times$  14.0 cm rectangular horn antenna and is terminated through an isolator in a square-law diode detector. The detected signal is fed into a selective amplifier (DH388A0).

14.0 cm rectangular horn antenna and is terminated through an isolator in a square-law diode detector. Standing waves are avoided by the isolators on both sides. The detected signal is fed into a selective amplifier (DH388A0). We have six different Plexiglass slabs with refractive index  $n = 1.637$ . Their thicknesses are, respectively, 1.0, 1.5, 1.9, 2.8, 5.0, and 10.0 mm. The width of all these slabs are 50.0 cm. The length of 1.0 and 1.5 mm slabs is 150.0 cm, and the length of 1.9, 2.8, 5.0, and 10.0 mm slabs is 180.0 cm.

To keep the slabs flat in experiments, a wooden frame is made with grooves slotted in the inner up and down sides. The slab is inserted along the grooves into the frame that is put vertically on a round precision ball bearing platform, which can be turned about its axis perpendicular to the plane of incidence for the purpose of varying the angle of incidence. The receiving horn antenna is mounted onto a specially designed support that allows it to be moved right and left or up and down perpendicular to the propagation direction of the incident beam. The receiver is 2.9 m far away from the transmitter so that there is adequate space to rotate the slab in between them. The peak of the transmitted beam is determined in such a way that it corresponds to the largest output of the detector when we scan the receiver perpendicular to the propagation direction of the incident beam in the plane of incidence.

The lateral displacement,  $d$ , of the transmitted beam through the slab is measured by first fixing the angle of incidence to be zero, setting the beam in TE polarization, and scanning the receiver perpendicular to the propagation direction of the incident beam in the plane of incidence. Then we rotate the slab to a desired position by turning the ball bearing platform and repeat the scanning of the receiver. All the results presented here are the averages of 10 runs.

In order to obtain negative lateral displacements, the angle of incidence should be larger than  $23.6^\circ$  according to Eq. (1) and the value of refractive index of the slab. Since shorter slabs are only 150.0 cm in length, the angle of incidence is chosen to be  $\theta_0 = 75^\circ$ , so that their projection onto the surface of the transmitter is about the diameter of the transmitter,  $150.0 \text{ cm} \times \cos 75^\circ = 38.8 \text{ cm}$ . The measured results for slabs of different thicknesses are shown in Fig. 3, which are in good agreement with the result of numerical simulation [4]. It can be seen that the lateral displacements for 1.0, 1.5, 1.9, 2.8, 5.0, and 15.0 mm slabs are negative, that for 12.8 mm slab is about zero, and that for 10.0 mm slab is positive. The 12.8 mm slab is obtained by putting 10.0 and 2.8 mm slabs together in parallel, and the 15.0 mm slab is obtained by putting 10.0 and 5.0 mm slabs together in parallel.

We note that apart from the fact that the transmitted beam suffers a lateral displacement from the position predicted by geometrical optics, it also undergoes [4] an angular deflection,  $\delta$ , from the propagation direction predicted by geometrical optics, the Snell's law of refraction. The meaning of the angular deflection is depicted in Fig. 4.

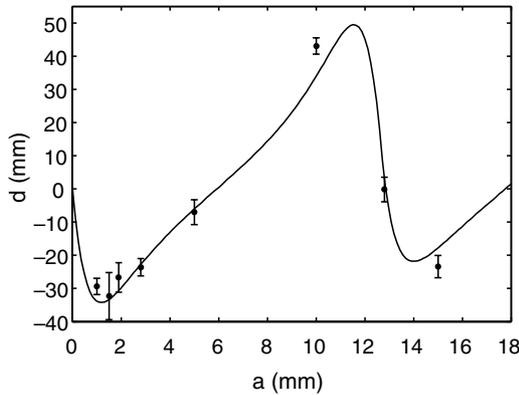


Fig. 3. Measured lateral displacements at incidence angle of  $\theta_0 = 75^\circ$  for TE-polarized microwave beam. The discrete points with error bars are experimental data for slabs of thicknesses of 1.0, 1.5, 1.9, 2.8, 5.0, 10, 12.8, and 15.0 mm, where the free-space wavelength is  $\lambda_0 = 3.28$  cm, the refractive index of the slab is  $n = 1.637$ , and the half waist width of the microwave beam is  $w_0 = 2.4\lambda_0$ . Each point represents the average of 10 experimental runs, and the error bar is one standard deviation about the average. For comparison, the result of numerical simulation is shown by the solid curve.

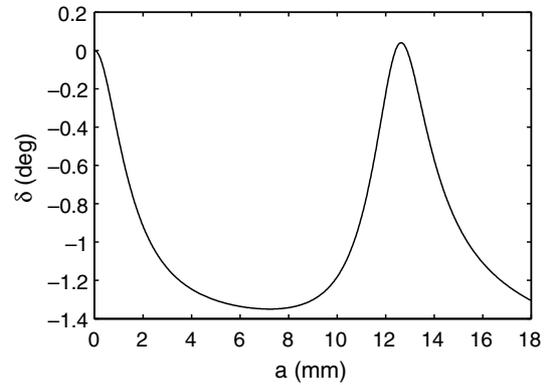


Fig. 5. Dependence of the angular deflection of transmitted beam from the propagation direction predicted by Snell's law of refraction on the thickness of slabs, where all the physical parameters and polarization state are the same as in Fig. 3.

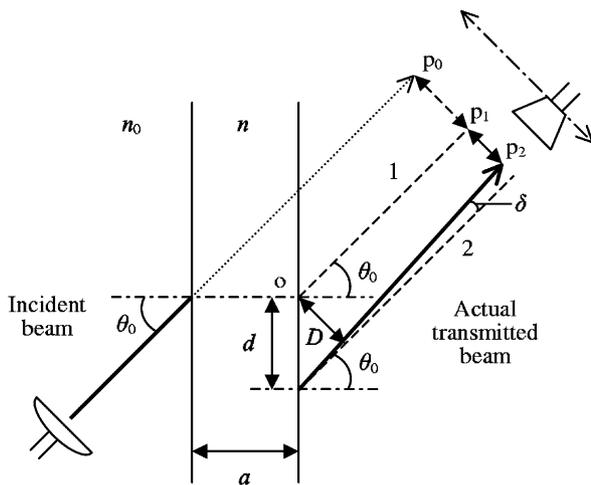


Fig. 4. The physical meaning of angular deflection, where dashed line 2 represents the propagation direction predicted by Snell's law of refraction, the solid line on the right-handed side of the slab represents the actual transmitted beam that is deflected by  $\delta$ , and the dotted line represents the incident microwave beam.

According to Snell's law, the refraction angle of the beam on the right-handed side of the slab should be  $\theta_0$ , as is indicated by the dashed line 2. But the actual refraction angle of transmitted beam is  $\theta_0 + \delta$ , as is shown by the solid line on the right-handed side of the slab. It has been predicted [4] that the angular deflection of transmitted beam depends closely on the waist width of the beam. The wider the waist width of the beam is, the smaller the angular deflection is. The dependence of the angular deflection on the thickness of the slab is shown in Fig. 5, where all the physical parameters are the same as in Fig. 3. It is seen that the angular deflection can be as large as  $1.35^\circ$  for half waist width of  $2.4\lambda_0$ . From the measured distance between

points  $p_0$  and  $p_2$ , it is easy to obtain the distance between points  $p_1$  and  $p_2$  by subtracting the distance,  $a \cdot \sin \theta_0$ , between points  $p_0$  and  $p_1$  (the distance between the dotted line and the dashed line 1). Due to the angular deflection, the distance  $\overline{p_1 p_2}$  is different from the distance  $D$ , between dashed lines 1 and 2, that is related to the defined lateral displacement by  $D = d \cos \theta_0$ . The difference is about  $\overline{op_1} \cdot \tan \delta \approx \overline{op_1} \cdot \delta$  so that  $D = \overline{p_1 p_2} + \overline{op_1} \cdot \delta$ . The impact of this angular deflection has already been taken into account in the measured lateral displacements in Fig. 3, where the agreement of measured data with those of numerical simulation implies the confirmation of the theoretical prediction of the angular deflection.

We also measured the dependence of the lateral displacement of the transmitted beam on the angle of incidence for 2.8 mm slab. The result is shown in Fig. 6. It is clear that the measured displacements for the selected angles of incidence between  $50^\circ$  and  $75^\circ$  are all negative and

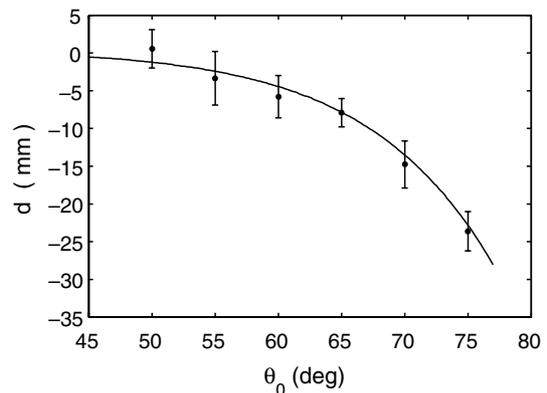


Fig. 6. Measured lateral displacements for slab of thickness  $a = 2.8$  mm and refractive index  $n = 1.637$  at several angles of incidence, which are in good agreement with the result of numerical simulation shown by the solid curve, where the beam is in TE polarization, the free-space wavelength is  $\lambda_0 = 3.28$  cm, and the half waist width of the microwave beam is  $w_0 = 2.4\lambda_0$ . The error bar stands for one standard deviation about the average.

are in good agreement with the result of numerical simulation. Of course, the measured result in Fig. 6 also takes into account the influence of the angular deflection.

The reflected beam also undergoes a lateral displacement from the position predicted by geometrical reflection [3,4,8]. In a symmetrical configuration where both sides of the slab are the same dielectric media, such as the case considered here, the reflection displacement is the same as the transmission one [4] according to the stationary-phase approach [9–11]. On the other hand, when a beam of light is reflected from and transmitted through an asymmetrical double-prism configuration where two prisms of different dielectric media are put adjacently in such a way that a thin air gap is formed between their hypotenuses, the reflection displacement is different from the transmission one and can also be positive, zero, and negative when the angle of incidence is smaller than but close to the critical angle for total reflection [12]. These anomalous lateral displacements need further experimental investigations. They have potential applications in optical devices, such as novel modulators and switches, and in integrated optics, and should be taken into account in optical system designs when thicknesses of optical layers are comparable with the wavelength of the light.

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## References

- [1] K.D. Möller, Optics, University Science Books, Mill Valley, California, 1988.
- [2] L.A.A. Read, G.E. Reesor, Can. J. Phys. 57 (1979) 1409.
- [3] C.W. Hsue, T. Tamir, J. Opt. Soc. Am. A 2 (1985) 978.
- [4] C.-F. Li, Phys. Rev. Lett. 91 (2003) 133903.
- [5] R.A. Shelby, D.R. Smith, S. Schultz, Science 292 (2001) 77.
- [6] J.A. Kong, B.-L. Wu, Y. Zhang, Microwave Opt. Technol. Lett. 33 (2002) 136.
- [7] X. Chen, C.-F. Li, Phys. Rev. E 69 (2004) 066617.
- [8] L.A.A. Read, Man Wong, G.E. Reesor, J. Opt. Soc. Am. 68 (1978) 319.
- [9] K. Artmann, Ann. Physik (Leipzig) 2 (1948) 87.
- [10] J.J. Cowan, B. Aniciu, J. Opt. Soc. Am. 67 (1977) 1307.
- [11] A.M. Steinberg, R.Y. Chiao, Phys. Rev. A 49 (1994) 3283.
- [12] C.-F. Li, Q. Wang, Phys. Rev. E 69 (2004) 055601 (R).