

## Cyclotron-Resonance Harmonics in the ac Response of a 2D Electron Gas with Smooth Disorder

I. A. Dmitriev,<sup>1,\*</sup> A. D. Mirlin,<sup>1,2,†</sup> and D. G. Polyakov<sup>1,\*</sup>

<sup>1</sup>*Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany*

<sup>2</sup>*Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany*

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The frequency-dependent conductivity  $\sigma_{xx}(\omega)$  of 2D electrons subjected to a transverse magnetic field and smooth disorder is calculated. The interplay of Landau quantization and disorder scattering gives rise to an oscillatory structure that survives in the high-temperature limit. The relation to recent experiments on photoconductivity by Zudov *et al.* and Mani *et al.* is discussed.

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The magnetotransport properties of a high-mobility 2D electron gas (2DEG) in semiconductor heterostructures are of great importance from the point of view of both fundamental physics and applications. Important information about the dynamical and spectral properties of the system is provided by its response to a microwave field. Within the quasiclassical Boltzmann theory, the dissipative ac conductivity  $\sigma_{xx}(\omega) = \sigma_+(\omega) + \sigma_-(\omega)$  of a noninteracting 2DEG in a magnetic field  $B$  is given by the Drude formula (we neglect spin for simplicity),

$$\sigma_{\pm}^{(D)}(\omega) = \frac{e^2 \nu_0 v_F^2 \tau_{tr,0}}{4[1 + (\omega_c \pm \omega)^2 \tau_{tr,0}^2]}, \quad (1)$$

where  $v_F$  is the Fermi velocity,  $\nu_0 = m/2\pi$  (with  $\hbar = 1$ ) the density of states (DOS),  $\tau_{tr,0}$  the transport relaxation time at  $B = 0$ ,  $\omega_c = eB/mc$  the cyclotron frequency, and  $m$  the electron effective mass. For a sufficiently clean sample,  $\omega\tau_{tr,0} \gg 1$ , Eq. (1) predicts a sharp cyclotron resonance (CR) peak at  $\omega_c = \omega$ .

It has been shown by Ando [1,2] that the Landau quantization of the orbital electron motion leads, in combination with disorder, to the emergence of harmonics of the CR at  $\omega = n\omega_c$ ,  $n = 2, 3, \dots$ . Indeed, such a structure was experimentally observed [3]. The analytical calculations of Ref. [1] were performed, however, only for fully separated Landau levels with pointlike scatterers [4].

Very recently, great interest in the transport properties of a 2DEG in a microwave field has been caused by experiments on photoconductivity of exceptionally high-mobility samples by Zudov *et al.* [5] and Mani *et al.* [6], where oscillations controlled by the ratio  $\omega/\omega_c$  were observed. Remarkably, these oscillations persisted down to  $B$  as low as 10 mT, an order of magnitude smaller than the field at which the Shubnikov-de Haas oscillations were damped. The experiments [5,6] triggered an outbreak of theoretical activity. Durst *et al.* [7] proposed (see also Refs. [8,9]) that the oscillations are governed by the following mechanism: an electron is excited by absorbing a photon with energy  $\omega$  close to  $n\omega_c$  and is scattered by disorder. In view of the oscillatory structure of the DOS, this leads to an extra contribution to the dc conductivity.

In fact, a very similar mechanism was proposed long ago [10] for the case of a strong dc electric field.

While the proposal of Ref. [7] is very appealing, calculations there involve a number of assumptions and approximations. First, the consideration of Ref. [7] is restricted to the case of white-noise disorder with  $\tau_{tr,0} = \tau_{s,0}$ , where  $\tau_{s,0}$  is the single-particle relaxation time at  $B = 0$ . On the other hand, the experiments are performed on high-mobility samples with smooth disorder,  $\tau_{tr,0}/\tau_{s,0} \approx 50$ . Second, Ref. [7] neglects all vertex corrections and discards inelastic processes. As we argue below, the inelastic relaxation is of central importance for the magnitude of the photoconductivity.

The development of the full theory of the oscillatory photoconductivity remains thus a challenging task, which we postpone to a future work. In this Letter we address the problem of the ac response of a 2DEG with smooth disorder. On top of theoretical importance and experimental relevance this problem possesses on its own, it is closely related to the photoconductivity. Indeed, the key ingredient of the photoconductivity mechanism — absorption of a photon — is governed by the dissipative ac conductivity  $\sigma_{xx}(\omega)$ . We will return to this relation in the end of this Letter, where we discuss implications of our results for the photoconductivity.

We consider a 2DEG subjected to magnetic field  $B$  and a random potential  $U(\mathbf{r})$  characterized by a correlation function  $\langle U(\mathbf{r})U(\mathbf{r}') \rangle = W(|\mathbf{r} - \mathbf{r}'|)$  of a spatial range  $d$ . The total and the transport relaxation rates induced by disorder at  $B = 0$  are given by

$$\left. \begin{array}{l} \tau_{s,0}^{-1} \\ \tau_{tr,0}^{-1} \end{array} \right\} = 2\pi\nu_0 \int \frac{d\phi}{2\pi} \tilde{W}\left(2k_F \sin\frac{\phi}{2}\right) \times \left\{ \begin{array}{l} 1 \\ (1 - \cos\phi) \end{array} \right\},$$

where  $\tilde{W}(\mathbf{q})$  is the Fourier transform of  $W(\mathbf{r})$ . While we are mainly interested in the experimentally relevant case of smooth disorder, when impurities are separated from the 2DEG by a spacer of width  $d \gg k_F^{-1}$ , with  $\tau_{tr,0}/\tau_{s,0} \sim (k_F d)^2 \gg 1$ , our results are valid for arbitrary  $d$  (including short-range disorder with  $\tau_{tr,0}/\tau_{s,0} \sim 1$ ).

To calculate (the real part of) the conductivity, we use the Kubo formula,

$$\sigma_{xx}(\omega) = -\frac{e^2}{4\pi V} \int_{-\infty}^{\infty} \frac{d\varepsilon}{\omega} (f_\varepsilon - f_{\varepsilon+\omega}) \times \text{Tr} \overline{\hat{v}_x (G_{\varepsilon+\omega}^A - G_{\varepsilon+\omega}^R) \hat{v}_x (G_\varepsilon^A - G_\varepsilon^R)}, \quad (2)$$

where  $f_\varepsilon$  is the Fermi distribution,  $G^{R,A}$  are the retarded and advanced Green functions, the bar denotes impurity averaging, and  $V$  is the system area. We will treat disorder within the self-consistent Born approximation (SCBA) [2], which is justified provided the disorder correlation length satisfies  $d \ll l_B$  and  $d \ll v_F \tau_{s,0}$ , where  $l_B = (c/eB)^{1/2}$  is the magnetic length [11]. The Green function in the Landau level (LL) representation,  $G_n^R = (G_n^A)^*$ , is given by the SCBA equations [2,11],

$$G_n^R(\varepsilon) = (\varepsilon - \varepsilon_n - \Sigma_\varepsilon)^{-1}, \quad \Sigma_\varepsilon = \frac{\omega_c}{2\pi\tau_{s,0}} \sum_n G_n^R(\varepsilon), \quad (3)$$

where  $\varepsilon_n = (n + \frac{1}{2})\omega_c$  is the  $n$ th LL energy [Fig. 1(a)].

We will assume throughout the paper that  $\omega, \omega_c \ll \varepsilon_F$ , so that the relevant LL indices are large,  $n \simeq \varepsilon_F/\omega_c \gg 1$ . Further, we will concentrate on the regime of strongly overlapping LLs,  $\omega_c \tau_{s,0}/\pi \ll 1$ ; the opposite case will be briefly discussed in the end. To evaluate the self-energy in Eq. (3), we use the Poisson formula,  $\sum_n F_n = \sum_k \int dx F(x) \exp(2\pi i k x)$ . The  $k=0$  term yields then the  $B=0$  result,  $\Sigma^{(0)} = i/2\tau_{s,0}$ , while the  $k=\pm 1$  contributions provide the leading correction,

$$\Sigma_\varepsilon^{(1)} = (i/2\tau_{s,0}) [1 - 2\delta \exp(-2\pi i \varepsilon/\omega_c)], \quad (4)$$

with  $\delta = \exp(-\pi/\omega_c \tau_{s,0})$  serving as a small parameter of the expansion. According to Eqs. (3) and (4), the oscillatory correction to the DOS reads

$$\Delta\nu(\varepsilon)/\nu_0 = -2\delta \cos(2\pi\varepsilon/\omega_c). \quad (5)$$

The conductivity (2) is given diagrammatically by an electronic bubble with a vertex correction, i.e., by a sum of ladder diagrams, Figs. 1(b) and 1(c). We evaluate first the bare bubble  $\sigma^b$  (which is sufficient for the case of white-noise disorder). Making use of velocity matrix elements in the LL representation, we get for  $\varepsilon_F \gg \omega, \omega_c$ :

$$\sigma_\pm^b(\omega) = \frac{e^2 v_F^2 \nu_0}{4} \int \frac{d\varepsilon}{\omega} (f_\varepsilon - f_{\varepsilon+\omega}) \text{Re}(\Pi_\pm^{RA} - \Pi_\pm^{RR}), \quad (6)$$

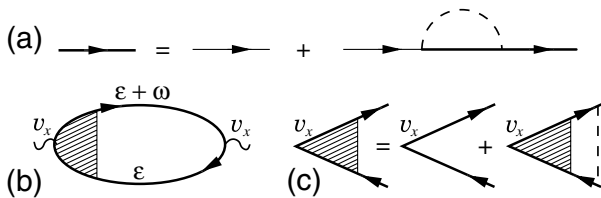


FIG. 1. (a) SCBA equation for the Green function; (b) the dynamical conductivity with vertex correction (c).

$$\Pi_\pm^{RR(RA)} = \frac{\omega_c}{2\pi} \sum_n G_{n\pm 1}^R(\varepsilon + \omega) G_n^{R(A)}(\varepsilon). \quad (7)$$

Using again the Poisson formula, we find

$$\Pi_\pm^{RR} = \frac{\tau_{s,0}(\Sigma_\varepsilon^{(1)} - \Sigma_{\varepsilon+\omega}^{(1)})}{\omega \pm \omega_c + \Sigma_\varepsilon^{(1)} - \Sigma_{\varepsilon+\omega}^{(1)}}, \quad (8)$$

$$\Pi_\pm^{RA} = \frac{\tau_{s,0}(\Sigma_\varepsilon^{(1)*} - \Sigma_{\varepsilon+\omega}^{(1)})}{\omega \pm \omega_c + \Sigma_\varepsilon^{(1)*} - \Sigma_{\varepsilon+\omega}^{(1)}}.$$

For the case of smooth disorder we have to take into account the vertex correction [Fig. 1(c)] while averaging in Eq. (2). This is a nontrivial task since the disorder mixes strongly the LLs, thus seriously complicating a direct calculation in the LL representation. We choose instead a different way, which is suggested by the quasiclassical nature of the problem,  $\varepsilon_F \tau_{s,0} \gg 1$ . It is instructive to recall first how the vertex correction is calculated at  $B=0$ . The vertex function  $\Gamma_\pm^{RA}(\mathbf{r}_1, \mathbf{r}_2)$  [the average of  $G^R v_\pm G^A$  with  $v_\pm = (v_x \pm i v_y)/2$ ] depends then on  $\mathbf{r}_1 - \mathbf{r}_2$  only, yielding  $\Gamma_\pm^{RA}(\mathbf{p})$  in the Fourier space. In the quasiclassical regime the momentum integrals are dominated by the vicinity of the Fermi surface, reducing  $\Gamma_\pm^{RA}(\mathbf{p})$  to  $\Gamma_\pm^{RA}(\phi)$ , where  $\phi$  is the polar angle of velocity on the Fermi surface. The equation for  $\Gamma_\pm^{RA}(\phi)$  is then easily solved, yielding  $\Gamma_\pm^{RA}(\phi) = (v_F/2) e^{\pm i\phi} \tau_{tr,0}/\tau_{s,0}$ .

We are now going to generalize this quasiclassical calculation onto the case of a finite  $B$ . In this situation the vertex functions  $\Gamma_\pm^{RR(RA)}(\mathbf{r}_1, \mathbf{r}_2)$  are, however, neither gauge nor translationally invariant. We define a gauge and translationally invariant vertex function by introducing a phase factor induced by the vector potential  $\mathbf{A}(\mathbf{r})$  on a straight line connecting  $\mathbf{r}_1$  and  $\mathbf{r}_2$ ,

$$\tilde{\Gamma}_\pm^{RR(RA)}(\mathbf{r}) = \exp\left[-i\frac{e}{c} \mathbf{A}(\mathbf{R}) \mathbf{r}\right] \Gamma_\pm^{RR(RA)}(\mathbf{r}_1, \mathbf{r}_2), \quad (9)$$

where  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$  and  $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ . After the Fourier transformation,  $\mathbf{r} \rightarrow \mathbf{p}$  [note that  $\mathbf{p}$  has the meaning of the kinematic rather than canonical momentum, in view of the transformation (9)], we get then the following equation for the dressed vertex:

$$\tilde{\Gamma}_\pm^{RR(RA)}(\mathbf{p}) = p_\pm/m + 4 \sum_n (-1)^n G_{n\pm 1}^R(\varepsilon + \omega) G_n^{R(A)}(\varepsilon) \times \int \frac{d^2 p'}{(2\pi)^2} \tilde{W}(\mathbf{p} - \mathbf{p}') e^{-l_B^2 p'^2} \times L_n^1(2l_B^2 p'^2) \tilde{\Gamma}_\pm^{RR(RA)}(\mathbf{p}'), \quad (10)$$

with  $p_\pm = p_x \pm i p_y/2$ . Using the asymptotic behavior of the Laguerre polynomial  $L_n^1(x)$  at  $n, x \gg 1$ , one can show that the following replacement is justified [12] in Eq. (10):

$$(-1)^n e^{-l_B^2 p'^2} L_n^1(2l_B^2 p'^2) \rightarrow \delta(2l_B^2 p'^2 - 4n). \quad (11)$$

The sum over  $n$  in Eq. (10) is then dominated by a narrow band of width  $\delta n/n \sim 1/\varepsilon_F \tau_{s,0}$  around the Fermi surface.

Exploiting the SCBA condition  $d/v_F\tau_{s,0} \ll 1$ , we finally reduce Eq. (10) to the form

$$\tilde{\Gamma}_{\pm}^{RR(RA)}(\phi) = \frac{v_F}{2} e^{\pm i\phi} + 2\pi\nu_0\Pi_{\pm}^{RR(RA)} \int \frac{d\phi'}{2\pi} \tilde{W}\left(2k_F \sin\frac{\phi - \phi'}{2}\right) \tilde{\Gamma}_{\pm}^{RR(RA)}(\phi').$$

Therefore, the inclusion of the vertex correction leads to a replacement of  $\Pi_{\pm}^{RR(RA)}$  in Eq. (6) by

$$\Pi_{\pm, \text{tr}}^{RR(RA)} \equiv [(\Pi_{\pm}^{RR(RA)})^{-1} - (\tau_{s,0}^{-1} - \tau_{\text{tr},0}^{-1})]^{-1}. \quad (12)$$

Evaluating Eq. (6) with this substitution to first order in  $\delta$ , we get the following result for the ac conductivity at zero temperature,  $T = 0$ :

$$\sigma_{\pm}^{(1)}(\omega) = \sigma_{\pm}^{(D)}(\omega) \left\{ 1 - 2\delta \cos(2\pi\varepsilon_F/\omega_c) \left[ \frac{2\alpha_{\pm}^2}{\alpha_{\pm}^2 + 1} \frac{\sin(2\pi\omega/\omega_c)}{2\pi\omega/\omega_c} + \frac{3\alpha_{\pm}^2 + 1}{\alpha_{\pm}^2 + 1} \frac{\sin^2(\pi\omega/\omega_c)}{\alpha_{\pm}\pi\omega/\omega_c} \right] \right\}, \quad (13)$$

where  $\alpha_{\pm} \equiv \tau_{\text{tr},0}(\omega \pm \omega_c)$ . Let us stress that the single-particle time  $\tau_{s,0}$  enters Eq. (13) only through the damping factor  $\delta$ ; everywhere else it has been replaced by the transport time  $\tau_{\text{tr},0}$  due to the vertex correction [13].

Since the correction in Eq. (13) oscillates with  $\varepsilon_F$ , it becomes damped at finite  $T$  by the factor  $X/\sinh X$  with  $X = 2\pi^2 T/\omega_c$ . If  $T$  is higher than the Dingle temperature  $T_D \equiv 1/2\pi\tau_{s,0}$ , the temperature smearing becomes the dominant damping factor. In ultraclean systems of the type used in the experiments [5,6] the Dingle temperature is as low as  $T_D \sim 100$  mK, so that for characteristic measurement temperatures  $T \sim 1$  K the first-order correction (13) will be completely suppressed. We will show, however, that there exists a correction, oscillatory in  $\omega/\omega_c$ , which is not affected by the temperature. To obtain it, we have to extend our calculation to second order in  $\delta$ . Analyzing all arising terms, we find that the required contribution is generated only by the expansion of  $\Pi_{\pm, \text{tr}}^{RA}$ , Eq. (12), with  $\Pi_{\pm}^{RA}$  given by Eq. (8), to second order in  $\delta$ ,  $(\Sigma_{\varepsilon}^{(1)*} - \Sigma_{\varepsilon+\omega}^{(1)})^2 \rightarrow 2\delta^2 \exp(-2\pi i\omega/\omega_c)$ . Note that there is no need to calculate  $\Sigma_{\varepsilon}$  to second order, neither to take  $\Pi_{\pm}^{RR}$  into account, since the corresponding terms oscillate with  $\varepsilon$ . We thus get the following result for the leading quantum correction at  $T \gg T_D$ :

$$\sigma_{\pm}^{(2)}(\omega) = \sigma_{\pm}^{(D)}(\omega) \left\{ 1 + 2\delta^2 \left[ \frac{\alpha_{\pm}^2(\alpha_{\pm}^2 - 3)}{(\alpha_{\pm}^2 + 1)^2} \cos\frac{2\pi\omega}{\omega_c} + \frac{\alpha_{\pm}(3\alpha_{\pm}^2 - 1)}{(\alpha_{\pm}^2 + 1)^2} \sin\frac{2\pi\omega}{\omega_c} \right] \right\}. \quad (14)$$

The regime which is most interesting theoretically and relevant experimentally is that of long-range disorder,  $\tau_{\text{tr},0}/\tau_{s,0} \gg 1$ , and a classically strong magnetic field,  $\omega_c, \omega \gg \tau_{\text{tr},0}^{-1}$ . In this situation  $|\alpha_{\pm}| \gg 1$  and Eq. (14) acquires a remarkably simple form (Fig. 2),

$$\sigma_{\pm}^{(2)}(\omega) = \sigma_{\pm}^{(D)}(\omega) [1 + 2e^{-2\pi/\omega_c\tau_{s,0}} \cos(2\pi\omega/\omega_c)]. \quad (15)$$

We turn now to the limit of separated LLs, when the LL width  $2\Gamma = 2(2\omega_c/\pi\tau_{s,0})^{1/2}$  [11] is small compared to  $\omega_c$ . This calculation can be done either by using Eqs. (6) and (12) or directly in the LL representa-

tion. We will only briefly present the main results (illustrated in Fig. 2); details will be presented elsewhere [14]. The conductivity  $\sigma_{xx}(\omega)$  is nonzero only for  $\omega$  in intervals  $[M\omega_c - 2\Gamma, M\omega_c + 2\Gamma]$  with an integer  $M$ . At the center of the  $M = 1$  interval we find a CR peak of height  $\sigma_{xx}(\omega = \omega_c) = (e^2\nu_0 v_F^2/\pi\Gamma)\tau_{\text{tr},0}/\tau_{s,0}$  and width  $\sim \Gamma\tau_{s,0}/\tau_{\text{tr},0}$ . All other peaks ( $M \neq 1$ ) are much smaller [in view of  $\tau_{\text{tr},0}/\tau_{s,0} \sim (k_F d)^2 \gg 1$ ]:

$$\sigma_{xx}(\omega = M\omega_c) = \frac{4e^2\nu_0 v_F^2 \Gamma \tau_{s,0}}{3\pi\omega_c^2 \tau_{\text{tr},0}} \frac{M^2 + 1}{(M^2 - 1)^2}. \quad (16)$$

In fact, the whole dependence  $\sigma_{xx}(\omega)$  can be described as a Drude-type structure with a renormalized DOS,  $\nu(\varepsilon) = \nu_0\tau_{s,0}[\Gamma^2 - (\varepsilon - \varepsilon_n)^2]^{1/2}$  for  $|\varepsilon - \varepsilon_n| \leq \Gamma$ , and the transport time  $\tau_{\text{tr}}(\varepsilon) = \tau_{\text{tr},0}\nu_0/\nu(\varepsilon)$ :

$$\sigma_{\pm}(\omega) = \frac{e^2 v_F^2}{4\omega} \int \frac{d\varepsilon (f_{\varepsilon} - f_{\varepsilon+\omega}) \nu(\varepsilon) \tau_{\text{tr}}^{-1}(\varepsilon + \omega)}{[\tau_{\text{tr}}^{-2}(\varepsilon) + \tau_{\text{tr}}^{-2}(\varepsilon + \omega)]/2 + (\omega \pm \omega_c)^2}.$$

Finally, we discuss the relation to the photoconductivity oscillations. For this purpose, we present first a simple way to understand the oscillatory structure in the ac conductivity (15). Since  $\frac{1}{2}\sigma_{xx}(\omega)E_{\omega}^2$  is the power absorbed in a linearly polarized ac field  $E_{\omega}$ , we have

$$\sigma_{xx}(\omega) \sim \omega^2 V |M(\omega)|^2 \langle \nu(\varepsilon) \nu(\varepsilon + \omega) \rangle_{\varepsilon}, \quad (17)$$

where  $M(\omega)$  is the dipole matrix element, and  $\langle \cdots \rangle_{\varepsilon}$  denotes the energy averaging with the corresponding

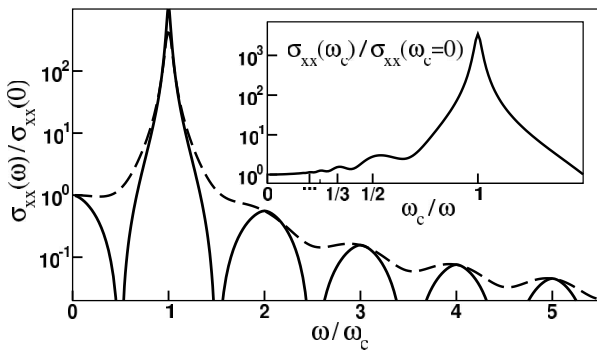


FIG. 2. Magneto-oscillations of the dynamical conductivity for a system with smooth disorder,  $\tau_{\text{tr},0}/\tau_{s,0} = 10$ . Solid line: separated LLs,  $\omega_c\tau_{s,0}/\pi = 3.25$ ; dashed line: overlapping LLs,  $\omega_c\tau_{s,0}/\pi = 1$ . Inset:  $\sigma_{xx}$  for fixed  $\omega\tau_{s,0}/2\pi = 1$  as a function of  $\omega_c$ .

Fermi factors [see Eq. (2)]. Using Eq. (5) for the DOS oscillations, we get  $\langle \nu(\varepsilon)\nu(\varepsilon + \omega) \rangle_\varepsilon \approx \nu_0^2 [1 + 2\delta^2 \times \cos(2\pi\omega/\omega_c)]$  at  $T \gg T_D$ , reproducing the oscillatory quantum correction in Eq. (15). In a similar way, we now consider the photoconductivity. The concentration of photoexcited electrons is  $n \sim [\sigma(\omega)/\omega] E_\omega^2 \tau_{\text{in}}$ , where  $\tau_{\text{in}}$  is the *inelastic* relaxation time, and we assumed that the amplitude  $E_\omega$  of the ac field is sufficiently weak. A correction to the dc conductivity induced by these electrons due to the energy dependence of the DOS can then be estimated as

$$\begin{aligned} \sigma_{\text{ph}} &\sim \omega (eE_\omega l_{\text{in}})^2 V |M(\omega)|^2 \langle \nu(\varepsilon)\nu'(\varepsilon + \omega) \rangle_\varepsilon \\ &\sim -\frac{(eE_\omega l_{\text{in}})^2}{\omega \omega_c} \sigma_{xx}^{(D)}(\omega) e^{-2\pi/\omega_c \tau_{s,0}} \sin \frac{2\pi\omega}{\omega_c}, \end{aligned} \quad (18)$$

where  $l_{\text{in}} = v_F(\tau_{\text{tr},0}\tau_{\text{in}})^{1/2}/\omega_c \tau_{\text{tr},0}$  is the inelastic length and we used Eqs. (15) and (17). Note that the steady state to linear order in the radiation power is reached only due to inelastic processes, leading to  $\sigma_{\text{ph}} \propto \tau_{\text{in}}$ . We believe that Eq. (18) describes the leading contribution to  $\sigma_{\text{ph}}$  induced by the LL quantization in the limit  $\tau_{\text{in}} \gg \tau_{s,0}$ . Let us stress that this contribution comes from an oscillatory correction to the distribution function, as opposed to the mechanism of Ref. [7], related to the effect of microwaves on the collision integral.

Although Eq. (18) agrees with the experiment as far as the period [5,6] and the phase [6] of the oscillations are concerned, there is a considerable disagreement in the damping of oscillations at low  $B$ . Specifically, our consideration predicts a damping factor  $\delta^2 = e^{-2\pi/\omega_c \tau_{s,0}}$  [same as in the ac conductivity, Eqs. (14) and (15)], so that if the experimental data for the damping of the photoconductivity oscillations are fitted to the form  $e^{-\pi/\omega_c \tau_{\text{ph}}}$ , one should find  $\tau_{\text{ph}}/\tau_{s,0} = \frac{1}{2}$ . On the other hand, the experiments yield much larger values,  $\tau_{\text{ph}}/\tau_{s,0} \approx 13 \text{ ps}/2.5 \text{ ps} = 5.2$  [5] and  $\tau_{\text{ph}}/\tau_{s,0} \approx 18 \text{ ps}/11 \text{ ps} \approx 1.6$  [6]. In other words, the photoconductivity oscillations are observed at such low fields that the contribution due to the above mechanism should be completely suppressed.

This suggests that, at least at lower  $B$ , another mechanism, not related directly to the LL quantization, might govern the observed oscillatory photoconductivity. A possible candidate is quasiclassical memory effects. They have been shown to induce strong dc magnetoresistance [15] and generate harmonics of the CR [16] in models with smooth disorder and/or strong scatterers. It is thus natural that the memory effects induce also quasiclassical oscillations in the photoconductivity; work in this direction is in progress [14,17].

To summarize, we have studied the ac magnetoconductivity  $\sigma_{xx}(\omega)$  of a 2DEG with smooth disorder characteristic of high-quality semiconductor structures. The interplay of Landau quantization and disorder induces a contribution oscillating with  $\omega/\omega_c$  (the CR harmonics). The effect is suppressed both in the classical limit

$\omega_c \tau_{s,0} \rightarrow 0$  and in the clean limit  $\omega_c \tau_{s,0} \rightarrow \infty$ , and can be best observed in the crossover range,  $\omega_c \tau_{s,0} \sim 1$ . We have discussed the relation to the recent experiments on photoconductivity of ultraclean samples [5,6].

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\*Also at A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

†Also at Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia.

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