Quantum Oscillations of Electrons and of Composite Fermions in Two Dimensions: Beyond the Luttinger Expansion

S. Curnoe[1,3] and P. C. E. Stamp[1,2,4]
[1]Department of Physics & Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
[2]Canadian Institute for Advanced Research, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
[3]Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[4]Grenoble High Magnetic Field Laboratory, MPI-FKF and CNRS, Grenoble 38042, France
(Received 3 December 1997)

Quantum oscillation phenomena, in conventional two-dimensional electron systems and in the fractional quantum Hall effect, are usually treated in the Lifshitz-Kosevich formalism. This is justified in three dimensions by Luttinger's expansion, in the parameter ω_c/μ. We show that in two dimensions this expansion breaks down, and we derive a new expression, exact in the limit where rainbow graphs dominate the self-energy. Application of our results to the fractional quantum Hall effect near half-filling shows very strong deviations from Lifshitz-Kosevich behavior. We expect that such deviations will be important in any strongly interacting two-dimensional electronic system. [S0031-9007(98)05770-6]

PACS numbers: 71.10.Pm, 73.40.Hm

Quantum oscillation (QO) phenomena (in which Landau quantization causes all thermodynamic and transport properties of conductors to oscillate with $1/B$, where B is the sample induction) for four decades have been among the most powerful tools in solid state physics [1,2] in two and three dimensions. The recent composite fermion (CF) theory [3,4] of the fractional quantum Hall effect (FQHE) predicts similar oscillations in $1/b$, where $b = B - B_{1/2}$ and $B_{1/2} = 2n_e/e$ is the mean "statistical field" coming from double fluxons attached to the CF's. Intense experimental interest in FQHE systems near half-filling [5] has given strong evidence for the CF theory (e.g., from Shubnikov–de Haas QO experiments [6,7], analogous oscillations in acoustic absorption [8], and compressibility [9]). The oscillations have been fit using Lifshitz-Kosevich (LK) formulas [1,10], usually with an impurity scattering Dingle temperature (sometimes assumed energy dependent), and an "effective mass" m^*. The CF cyclotron frequency $\omega^*_{\mathrm{CF}} = eb/m^*$ increases rapidly with b near $b = 0$ (i.e., near half-filling), because of strong infrared divergent gauge interactions [11–14].

The LK formulas (and generalizations of them, incorporating low energy fluctuations [15,16]) rely fundamentally on an expansion of the *oscillatory part* of the thermodynamic potential $\Omega(B)$, in powers of ω_c/μ (where $\omega_c = eB/m$, and μ is an upper cutoff, equal to the chemical potential in the simplest models), given by Luttinger [17]. He wrote the one-particle self-energy [18] as $\Sigma(B) = \bar{\Sigma} + \Sigma_{\mathrm{osc}}(B)$, where Σ_{osc} contained all contributions oscillating in $1/B$ (and analogously the fermionic Green's function $\mathcal{G} = \bar{\mathcal{G}} + \mathcal{G}_{\mathrm{osc}}$). Expanding the functional $\Omega(\Sigma)$ around $\Omega(\Sigma = \bar{\Sigma})$ in powers of Σ_{osc}, one finds [17]

$$\Omega = \frac{-1}{\beta}\,\mathrm{Tr}[\ln \bar{\mathcal{G}}^{-1} - \bar{\Sigma}\bar{\mathcal{G}}] + \Phi(\bar{\Sigma}) + O(\Sigma^2_{\mathrm{osc}}), \quad (1)$$

where $\beta = 1/k_B T$. In three dimensions,

$$\frac{1}{\beta}\,\mathrm{Tr}[\ln(\bar{\mathcal{G}}^{-1})] \sim \left(\frac{\omega_c}{\mu}\right)^{5/2}, \quad (2)$$

$$\frac{\Sigma_{\mathrm{osc}}}{\bar{\Sigma}} \sim \left(\frac{\omega_c}{\mu}\right)^{3/2}, \quad (3)$$

and also $\Phi(\bar{\Sigma}) + \beta^{-1}\,\mathrm{Tr}(\bar{\Sigma}\bar{\mathcal{G}}) = 0$ at least to $\sim O((\omega_c/\mu)^3)$. Thus writing $\Omega = \Omega_0 + \Omega_{\mathrm{osc}}$, we have that the leading oscillatory contribution up to $O((\omega_c/\mu)^{5/2})$ is contained in

$$\Omega \sim \frac{-1}{\beta}\,\mathrm{Tr}[\ln \bar{\mathcal{G}}^{-1}] \quad (4)$$

$$= \frac{-1}{\beta}\sum_{i\omega_m, n, \sigma, k_z} \ln[i\omega_m - \epsilon_{\sigma n k_z} + \bar{\Sigma}(i\omega_m, \epsilon_{\sigma n k_z})], \quad (5)$$

with

$$\epsilon_{\sigma n k_z} = \epsilon_\sigma + \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m} - \mu, \quad (6)$$

where μ is defined as the zero of the energy, n labels the Landau levels, and σ is a spin index. Equation (5), which contains the *nonoscillatory* self-energy $\bar{\Sigma}$, provides the fundamental justification for extracting the *zero field*, many-body interaction-renormalized band structure from QO experiments [1].

In this paper we show that (a) Luttinger's expansion *fails* in any interacting 2D electronic system; however, (b) an alternative expansion can be found under certain circumstances (see below), in which now the full self-energy (including the highly singular Σ_{osc}) must be used. (c) This new expansion can give results sharply different from the previous ones [1,10,15–17].

 0031-9007/98/80(15)/3312(4)$15.00

To show the practical importance of these results, we will apply them to CF's; however, they are relevant in principle to any 2D electronic system [19].

(i) Failure of Luttinger's expansion.—We first repeat the analysis which yields Eqs. (2) and (3), but now in two dimensions. We shall find quite generally that

$$\frac{1}{\beta}\,\mathrm{Tr}[\ln \bar{\mathcal{G}}^{-1}] \sim \left(\frac{\omega_c}{\mu}\right)^2, \tag{7}$$

$$\frac{\Sigma_{\mathrm{osc}}}{\bar{\Sigma}} \sim \frac{\omega_c}{\mu}. \tag{8}$$

Thus the term $\sim O(\Sigma_{\mathrm{osc}}^2)$ is as important as the "leading" term, and the whole expansion must be reexamined.

Equation (7) is easily verified. To justify (8) we first repeat, in two dimensions, Luttinger's three-dimensional calculation of the graph in Fig. 1(a); we then extend the argument to higher graphs. The graph in Fig. 1(a) has the real-space form

$$\Sigma_R(\vec{r}) = \int d^3r'\, V(\vec{r} - r') \times [g(0) - \hat{P}_{rr'} g(r - r') e^{i\phi(r,r')}], \tag{9}$$

where $\hat{P}_{rr'}$ is the exchange operator, $\phi(r, r')$ is a gauge-dependent phase factor [17], and $g(r) = \bar{g}(r) + g_{\mathrm{osc}}(r)$, where

$$\bar{g}(r) = \oint_{\bar{C}} dt\, M(r,t), \tag{10}$$

$$g_{\mathrm{osc}}(r) = \sum_l \oint_{C_l} dt\, M(r,t), \tag{11}$$

$$M(r,t) = \frac{-ie^{\beta\mu t}}{2\sin \pi t} F_{2\mathrm{D}}(r,t), \tag{12}$$

$$F_{2\mathrm{D}}(r,t) = \frac{\omega_c}{4\pi \sinh(\beta\omega_c t/2)} \times \exp\!\left[\frac{-\omega_c}{4}\coth\!\left(\frac{\beta\omega_c t}{2}\, r^2\right)\right]. \tag{13}$$

The contour $\bar{C}$ encircles the negative real axis counterclockwise, the contours C_l likewise encircle the points $T_l = 2\pi i l/\beta\omega_c$, with $l = \pm1, \pm2, \ldots$. The 3D function $F_{3\mathrm{D}}(r,t)$ differs from (13) by the factor $(2\pi\beta t)^{-1/2} \times \exp[-z^2/2\beta t]$, where z is the third dimension, perpendicular to r [cf. Ref. [16], Eq. (A.16)]. It is this difference which yields (8), instead of (3), upon integrating over t in (10) and (11).

Consider now graph 1(b), assuming that the internal boson line represents either (i) a phonon or a conventional "Fermi liquid" electronic fluctuation or (ii) a singular gauge fluctuation [11–14]. Using the known results for Σ_{osc} for these cases [14], one easily verifies (7) and (8) again. In fact, the *scaling property* (8) of $\Sigma_{\mathrm{osc}}/\bar{\Sigma}$ as a function of ω_c/μ depends only on the dimensionality of the graph (as well as the presence of at least one internal fermion line [17]), and is true of all higher graphs.

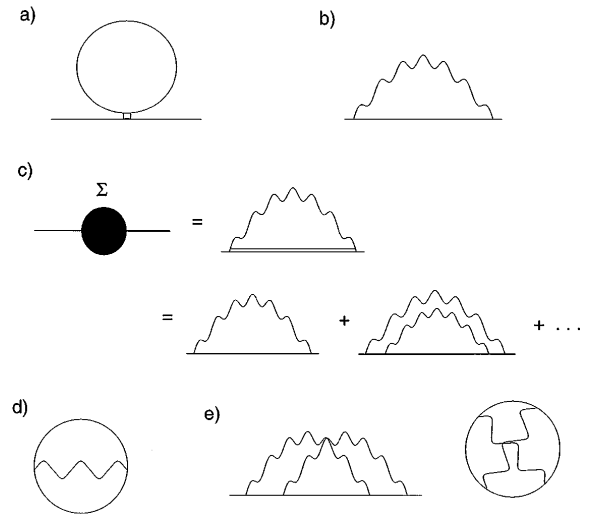

FIG. 1. The Feynman graphs discussed in the text. The self-energy graphs include (a) the Hartree-Fock term, (b) the lowest order "RPA" term, (c) a self-consistent "rainbow graph" sum of such terms. In (d) we have the lowest order (second-order) contribution to Ω, and (e) shows the lowest (fourth-order) "crossed graph" contributions to Σ and Ω.

(ii) Alternative expansions.—There are two cases for which a simple alternative to (5) can be found for Ω_{osc}.

The first is where vertex corrections to the usual Schwinger-Dyson–Nambu-Eliashberg self-energy [Fig. 1(c)] can be neglected. Then $\Sigma = \lambda^2 \int \mathcal{G}\mathcal{D}$, where $\mathcal{G}$ and $\mathcal{D}$ are given self-consistently in terms of Σ, thus summing over all "rainbow graphs." The relevant skeleton graph Φ_2 [Fig. 1(d)] then exactly cancels $\beta^{-1}\,\mathrm{Tr}[\Sigma\mathcal{G}]$ in (1), and $\Omega = \bar{\Omega} + \Omega_{\mathrm{osc}}$ is given, *to all orders in* ω_c/μ, by

$$\Omega = \frac{-1}{\beta}\,\mathrm{Tr}[\ln \mathcal{G}^{-1}] \tag{14}$$

$$= \frac{-1}{\beta} \sum_{i\omega_m, n, \sigma} \ln[i\omega_m - \epsilon_{\sigma n} - \Sigma(i\omega_m, \epsilon_{\sigma n})]. \tag{15}$$

The crucial difference from (5) (apart from the suppression of k_z) is that Σ now includes Σ_{osc}. Deviations from (15) arise from "crossed" graphs [Fig. 1(e)], and there are many physical cases in which these are unimportant. In the case of composite fermions the corrections from crossed "gauge fluctuation" graphs are not small, but at low energy their main effect both in zero field [20] and in finite field [14] is simply to renormalize the vertices in the rainbow graph sum, without changing the functional form of Σ. Thus, this approximation actually works well even beyond the "Migdal limit" in which crossed graphs are small. The difference between $\mathrm{Tr}[\ln \bar{\mathcal{G}}^{-1}]$ and $\mathrm{Tr}[\ln \mathcal{G}^{-1}]$ depends crucially on how big is $\Sigma_{\mathrm{osc}}/\bar{\Sigma}$; even though formally this is $\sim O(\omega_c/\mu)$ for all 2D systems, its actual value, for a given ω_c/μ, varies enormously between different systems.

The second case is of more academic interest; it arises when we may write Ω in terms of a set of "statistical quasiparticle" (SQP) energies [21] $\varepsilon_{\sigma\nu}$ as

$$\Omega = \frac{-1}{\beta} \sum_{\nu,\sigma} \ln[1 + e^{\beta(\mu - \varepsilon_{\sigma\nu})}], \tag{16}$$

where the $\varepsilon_{\sigma\nu}$ are conventionally defined by a Landau expansion, are *real*, and are *not* equal to the energies $E_{\sigma\nu}$ defined from the one-particle Green function by $E_{\sigma\nu} - \mathrm{Re}\,\Sigma_{\sigma\nu}(E_{\sigma\nu}) = 0$. The problem with (16) is that it relies on the usual assumption that switching on interactions, in a system already in Landau level states, does not reclassify the energy levels. This is definitely not true for CF's, once the gauge interactions are switched on.

We now consider the new result (15) in more detail. Suppressing the sum over spins, we rewrite (15) as an integral,

$$\Omega = \frac{m\omega_c}{\Phi_0} \sum_{n=0} \int \frac{dx}{\pi} n_f(x) \tan^{-1}[\phi(x,n)], \tag{17}$$

where $\phi(x,n) = \mathrm{Im}\,G/\mathrm{Re}\,G$. The prefactor $\frac{m\omega_c}{\Phi_0}$ is the Landau level degeneracy. Defining $\epsilon = n\omega_c - \mu$, the Poisson sum formula is used to separate the oscillatory components of Ω,

$$\Omega = \frac{m\omega_c}{\Phi_0} \int \frac{dx}{\pi} n_f(x) \left[\int_{-\mu}^{\infty} d\epsilon \, \tan^{-1}[\phi(x,\epsilon)] - 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{2\pi k} \int_{-\infty}^{\infty} d\epsilon \, \mathrm{Im}\mathcal{G}(\epsilon,x) \sin\left(\frac{2\pi k(\epsilon + \mu)}{\omega_c}\right)\right], \tag{18}$$

where in the oscillatory (i.e., $k > 0$) terms in the Poisson sum we have extended the limits of the ϵ integral to $\pm\infty$ and integrated by parts.

As noted above, the important difference from previous expressions for Ω here is the inclusion of oscillatory terms in $\mathcal{G}$; in Fig. 2 we show the effect of this oscillatory structure in $\mathcal{G}$, for the particular case of CF excitations.

(iii) Magnetization oscillations.—The classic dHvA effect is in the magnetization oscillations; recently experiments have succeeded in seeing these in *single layer* systems [22]. Taking the derivative of Ω_{osc}, we get

$$M_{\mathrm{osc}} = \frac{-\partial\Omega}{\partial B} = M_1 + M_2, \tag{19}$$

$$M_1 = \frac{-2\pi\mu}{\Phi_0\omega_c} \sum_{k=1}^{\infty} (-1)^k \int \frac{dx}{\pi} n_f(x) \times \mathrm{Re}\left[\exp\left(\frac{2\pi i k}{\omega_c}[x - \Sigma(x) + \mu]\right)\right], \tag{20}$$

$$M_2 = \frac{-2\pi m}{\Phi_0} \sum_{k=1}^{\infty} (-1)^k \int \frac{dx}{\pi} n_f(x) \times \mathrm{Re}\left[\frac{\partial\Sigma(x)}{\partial B} \exp\left(\frac{2\pi i k}{\omega_c}[x - \Sigma(x) + \mu]\right)\right] - \frac{m}{\Phi_0} \int_{-\mu}^{\infty} d\epsilon \int \frac{dx}{\pi} n_f(x) \,\mathrm{Im}\, G(\epsilon, x) \frac{\partial\Sigma}{\partial B}. \tag{21}$$

At first glance the first term M_1 resembles the results of Fowler and Prange [15] and Engelsberg and Simpson [16]; however, it now involves the *full* Σ (including Σ_{osc}). The second term M_2 is formally of the same order in ω_c/μ as M_1, and quite new. Typically the term in $\frac{\partial\Sigma}{\partial B}$ dominates M_2, and we shall see below that in two dimensions it can be much larger than M_1.

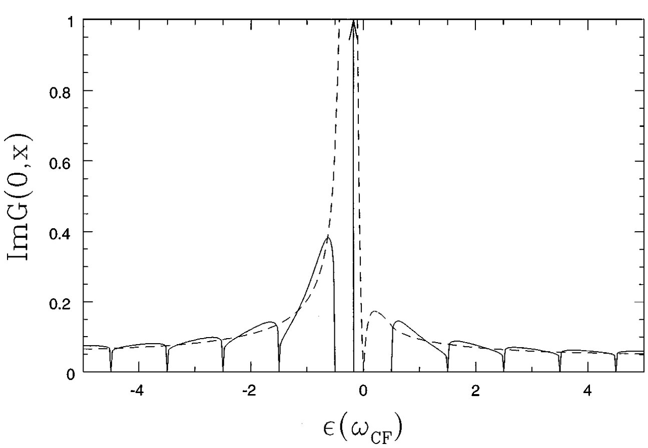

FIG. 2. Plot of the quasiparticle spectral function $\mathrm{Im}\,\mathcal{G}(0,x)$ at zero temperature for the case of composite fermions (with Σ calculated to second order in the gauge coupling). The solid line shows the result using the full $\mathcal{G}$ (including all oscillatory terms), whereas the dashed line uses only the nonoscillatory $\bar{\mathcal{G}}$. Note the presence of a gap, as well as an isolated pole, in the full spectral function. These results are for $k_B T = 0$.

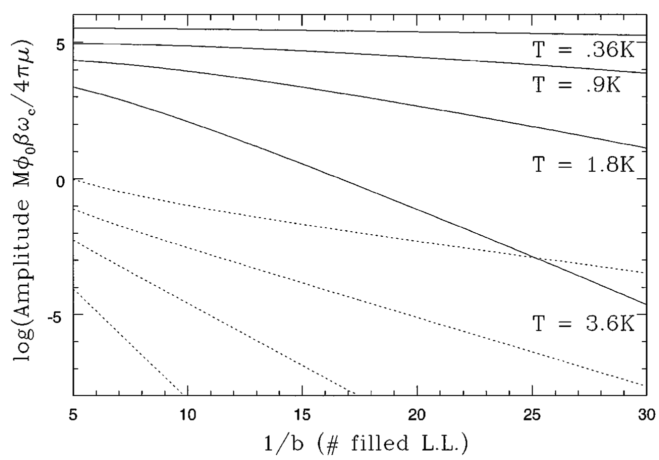

FIG. 3. A numerical evaluation of $\ln(A)$, where A is the amplitude of the dHvA oscillations in M, as a function of $1/B$ for various fixed temperatures, for a system of CF quasiparticles (we assume an unscreened Coulomb interaction, and use the second-order result for Σ derived earlier [14]. We assume a chemical potential of 180 K, and a coupling constant $\kappa_2 = 0.8$. The dashed lines show $\ln(A_1)$, and the solid lines show $\ln(A_1 + A_2)$, where A_1 and A_2 are the amplitudes of oscillations of M_1 and M_2.

Equations (19)–(21) are valid for any two-dimensional charged system for which the full self-energy (including oscillatory contributions) can be written down.

(iv) Application to the composite fermion system.— We now wish to demonstrate on a particular example that the deviations from orthodox behavior can be rather large. We choose the CF gauge theory, for which the oscillatory self-energy for composite fermions interacting with gauge fluctuations was previously calculated [13,14]. Here we assume unscreened Coulomb interactions (i.e., the dynamical exponent [4,11–14] is $s = 2$). For numerical work it is convenient to use a Matsubara sum over $\Sigma(x)$ evaluated at $x = i\omega_l = i\pi(2l + 1)/\beta$; writing $\mathrm{Im}\,\Sigma(i\omega_l) \equiv \xi(i\omega_l)$, we have

$$\xi(i\omega_l) = 2\kappa_2\left[\frac{\omega_l}{\pi}\ln\left(\frac{\omega_l}{\mu}\right) + \frac{4}{\beta}\sum_{k'=1}^{\infty}(-1)^{k'}\sum_{\omega_m>0}\exp\left(\frac{-2\pi k'\omega_m}{\omega_{\mathrm{CF}}}\right)\ln\left(\frac{\omega_l+\omega_m}{\omega_{\mathrm{CF}}}\right)\cos\left(\frac{2\pi k'\mu}{\omega_{\mathrm{CF}}}\right)\right], \tag{22}$$

where the coupling κ_2 is usually slightly less than one [14]. In QO experiments one examines $\ln(A)$; in LK theory $\ln(A)$ should be a linear function of $1/B$ (the "Dingle plot"), as well as of T (the "mass plot"). Figure 3 shows (for the example of CF fermions) the importance of M_2, as well as the considerable nonlinearity shown in Dingle plots (which we also find in the mass plots, not shown here). Thus, in this example a conventional analysis of QO phenomena, using either the LK formula or its generalizations [15,16], clearly fails. We do not believe this example to be untypical (in fact, if we choose *screened* short-range interactions between the CF's [4, 13,14], with dynamical exponent $s = 3$, we get much worse deviations). We thus believe that where strong violations of conventional behavior are observed [19] or where interaction effects are known to be strong [22], one should reanalyze the data using the results herein. In the context of the FQHE near half-filling, fits of QO results to LK theory should clearly be treated with caution.

In summary, we have shown that the LK formalism (or its many-body generalizations [15,16]) for describing quantum oscillations *breaks down* in two dimensions. To remedy this, we have derived new results that can be applied when crossed diagrams may be neglected. We have applied these results to a problem of current interest [5], i.e., composite fermions interacting with gauge fluctuations (believed to give a good description of the fractional quantum Hall states, at least near half-filling). The results show radical departures from LK behavior. Such departures should also exist in other strongly interacting two-dimensional electronic systems, whether or not they behave as Fermi liquids in zero field.

P. C. E. S. thanks G. Martinez, I. D. Vagner, and P. Wyder, for hospitality and support in Grenoble, as well as the CIAR and NSERC of Canada. S. C. acknowledges support from the Feinberg Graduate School of the Weizmann Institute.

*Present address.

[1] D. Shoenberg, *Magnetic Oscillations in Metals* (Cambridge University Press, Cambridge, 1984).
[2] N.W. Ashcroft and N.D. Mermin, *Solid State Physics* (Saunders College, Fort Worth, 1976), Chap. 14.
[3] J.K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[4] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[5] See the review of R.L. Willett, Adv. Phys. **46**, 447 (1997).
[6] R.R. Du *et al.*, Phys. Rev. Lett. **73**, 3274 (1994); H.C. Manoharan, M. Shayegan, and J.S. Klepper, Phys. Rev. Lett. **73**, 3270 (1994); P.T. Coleridge *et al.*, Phys. Rev. B **52**, R11 603 (1995).
[7] D.R. Leadley *et al.*, Phys. Rev. B **53**, 2057 (1996).
[8] R.L. Willett *et al.*, Phys. Rev. Lett. **71**, 3846 (1993).
[9] J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B **50**, 1760 (1994).
[10] I.M. Lifshitz and A.M. Kosevich, Sov. Phys. JETP **2**, 636 (1956).
[11] A. Stern and B.I. Halperin, Phys. Rev. B **52**, 5890 (1995).
[12] Y.B. Kim, X.G. Wen, P.A. Lee, and P.C.E. Stamp, Phys. Rev. B **51**, 10 779 (1995).
[13] S. Curnoe and P.C.E. Stamp, J. Phys. Condens. Matter **8**, 5890 (1996).
[14] S. Curnoe and P.C.E. Stamp, Int. J. Mod. Phys. B **11**, 1477 (1997).
[15] M. Fowler and R.E. Prange, Physics **1**, 315 (1965).
[16] S. Engelsberg and G. Simpson, Phys. Rev. B **2**, 1657 (1970); A. Wasserman and M. Springford, Adv. Phys. **45**, 471 (1996).
[17] J.M. Luttinger, Phys. Rev. **121**, 1251 (1961).
[18] It is sometimes objected that the self-energy, not being gauge invariant, should not be used in formulas for physical quantities like Ω or M (which are). We simply note here that the gauge dependence of Σ is irrelevant in the formulas in this paper since one *integrates over* Σ; moreover, typically only the pole structure in the Green function is relevant, and this is also gauge invariant (cf. Stern and Halperin [11]).
[19] Problems with the fitting of LK theory to two-dimensional QO phenomena have been noted on several occasions (see, e.g., E. Balthes *et al.*, Z. Phys. B **99**, 163 (1996), for a recent example). Explanations of this have centered recently on the possible breakdown of Fermi liquid theory (FLT) in two dimensions. We stress that the strong deviations from orthodox behavior we discuss here exist *even in the absence of a breakdown of FLT.*
[20] J. Gan and E. Wong, Phys. Rev. Lett. **71**, 4226 (1993).
[21] For SQP theory, see, e.g., R. Balian and C. de Dominicis, Ann. Phys. (N.Y.) **62**, 229 (1971); J.M. Luttinger, Phys. Rev. **174**, 263 (1968); C.J. Pethick and G.M. Carneiro, Phys. Rev. A **7**, 304 (1973); Phys. Rev. B **11**, 1106 (1975); P.C.E. Stamp, Europhys. Lett. **4**, 453 (1987).
[22] S.A.J. Weigers *et al.*, Phys. Rev. Lett. **79**, 3238 (1997).