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# Bound values for Hall conductivity and percolation under quantum Hall effect conditions

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**Abstract**

The percolation under Quantum Hall Effect conditions in inhomogeneous medium has been studied. The lower and upper bound possible values for effective Hall conductivity values have been established. It has been shown that these bound values for Hall conductivity differ from bound values for metal conductivity. It comes from unusual character of current percolation under Quantum Hall Effect conditions. The physical sense of obtained results has been discussed.


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## 1. Introduction

In the study of the current percolation in inhomogeneous medium the problem of the effective conductivity of the medium $\sigma_e$ is appeared. The effective conductivity has been determined as a coefficient of proportionality between average electric current $\vec{J} = \langle \vec{j} \rangle$ and average electric field $\vec{E} = \langle \vec{e} \rangle$

$$\vec{J} = \sigma_e \vec{E}. \tag{1}$$

The effective conductivity has been easily established from boundary conditions for the simple case of layered system, consisting of two alternating layers with different conductivities $\sigma_1$ and $\sigma_2$ and equal widths $d_1 = d_2$. In the case, when electric current flows perpendicular to the interface of layers, the normal component of electric current is conserved:

$$j_{1n} = j_{2n} = J/2 \tag{2}$$

and the resistance has been averaged:

$$\rho \perp = \langle \rho \rangle = \frac{1}{2}(\rho_1 + \rho_2). \tag{3}$$

In the other case, when electric current flows along layers, the longitude component of electric field is conserved:

$$e_{1t} = e_{2t} = E/2 \tag{4}$$

and the conductivity has been averaged:

$$\sigma^{||} = \frac{1}{2}(\sigma_1 + \sigma_2). \tag{5}$$

In two-dimensional heterogeneous medium some exact results for effective conductivity of the random medium have been obtained due to the dual symmetry. Firstly it was the Keller theorem [1] and secondly a general approach, which has been independently put forward in Dykhne works [2]. It has been shown, that the effective conductivity of two-phase medium at the percolation threshold (at equal phase concentrations) equals to:

$$\sigma_e = \sqrt{\sigma_1 \sigma_2}. \tag{6}$$

The duality relation for effective conductivity has been established at arbitrary phase concentrations:

$$\sigma_e(\varepsilon)\sigma_e(-\varepsilon) = \sigma_1 \sigma_2, \tag{7}$$

where $\varepsilon = X - X_c$ is the deviation from percolation threshold $X_c = 1/2$. (We call the system as dual system

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relative to the initial one if it differs from initial only by the replacement of phases and if it has the same geometric phase placement). This relation has been obtained due to the additional symmetry of the two-dimensional equations for the constant current and Ohm's law relative to rotational transformations (see below Section 3). It has been shown that formula (9) has a sense of stationary point for Dykhne's transformations in Ref. [3]. In the works of [4,5] the local distributions of currents (fields) have been founded for "chess-board" square structures by the method of conformal map additionally.

In the general case the bound values for effective conductivity have been obtained:

$$\left\langle \frac{1}{\sigma} \right\rangle^{-1} \leqslant \sigma_{\mathrm{e}} \leqslant \langle \sigma \rangle. \tag{8}$$

Here $\langle a \rangle$ means the average value of the quantity $a$. Briefly remind how these results have been obtained [6]. For this aim the expression for Joulle dissipation energy has been used.

$$Q = \frac{1}{V} \int (\vec{j}, \vec{e})\, \mathrm{d}V = \vec{J}\vec{E} = \sigma_{\mathrm{e}} \vec{E}^2. \tag{9}$$

If we insert the value for average current $\langle \vec{j} \rangle = \vec{J}$ in the integral of formula (9), so the lower bound value of effective conductivity in the formula (8) has been obtained . If the value for average electric field $\langle \vec{e} \rangle = \vec{E}$ has been inserted, so the upper bound value of formula (8) has been followed. One can see estimations for bound values of effective conductivity in more details in review [7].

In this paper the percolation under quantum Hall effect (QHE) conditions ($\sigma_{xx} = 0, \sigma_{xy} = \mathrm{const}$) has been studied. Feature of such a percolation under QHE conditions consists of that the Hall current is always directed perpendicular to the electric field:

$$\vec{j} = \sigma_{xy} [\vec{n}, \vec{e}]. \tag{10}$$

Here $\vec{n}$ is an unit vector, which directed along magnetic field and perpendicular to the considered plane. So the dissipation always equals zero under QHE regime: $Q = 0$, that is, the Hall phases are non-dissipative phases. It means also that the we cannot apply above reasonings to obtain the bound values for effective Hall conductivity under QHE conditions and cannot estimate possible values for Hall conductivity of heterogeneous medium by usual way.

Also from standard boundary conditions (2), (4) and the expression for Hall current (10) the new boundary conditions have been obtained:

$$j_{1n} = 0, \quad j_{2n} = 0. \tag{11}$$

The another problem comes from these new boundary conditions, because it seems that there is no a transfer (current) through interface of phases accordingly (11), for exception, only few singular points. Consequently, it seems too that the effective Hall conductivity must be equal to zero everywhere: $\sigma_{xy}^{e} = 0$.

But as it will be shown below it is not so due to percolation through singular points. In this paper we find the bound values for effective Hall conductivity and explain the physical sense of percolation in composite systems under Quantum Hall Effect conditions. The paper is constructed as follows. In Section 2 the usual layered systems under QHE conditions have been considered. The effective Hall conductivity tensor has been obtained for layered systems. In Section 3 the effective Hall conductivity has been calculated for random two-phase systems. The Dykhne's method of rotational transformations has been used to solve this problem. It was shown that the Hall conductivity bound values have been determined by the connectivity of systems. In Section 4 the simple model of circular metal inclusion has been considered to clarify the physical sense of current percolation under QHE conditions. In Section 5 short discussion of obtained results has been given.

## 2. Percolation in layered systems under QHE conditions

To understand the features of current percolation under QHE conditions let us consider the simple model of layered media, consisting of two alternating layers with equal widths and different Hall conductivities $\sigma_{xy}^{(1)}$ and $\sigma_{xy}^{(2)}$. Let us assume all layers are directed along $y$-direction.

In the case when electric current flows perpendicular to the phase interfaces, the electric field directs along layers according (10) for Hall current and it equals to the average value. It has been followed from boundary conditions (4). Correspondingly, we obtain the following formulae for effective Hall conductivity in this case:

$$\sigma_{xy}^{e} = \langle \sigma_{xy} \rangle = \frac{1}{2} (\sigma_{xy}^{(1)} + \sigma_{xy}^{(2)}). \tag{12}$$

To check this result we calculate the distributions of electric fields and currents at every phase, using the definitions for averaged electric field and averaged electric current:

$$\sigma_{xy}^{(1)} \langle [\vec{n}, \vec{e}] \rangle_1 + \sigma_{xy}^{(2)} \langle [\vec{n}, \vec{e}] \rangle_2 = \sigma_{xy}^{(e)} [\vec{n}, \vec{E}],$$
$$\langle \vec{e} \rangle_1 + \langle \vec{e} \rangle_2 = \vec{E}. \tag{13}$$

After simple calculations we obtain the formula for electric fields (currents) at every phase:

$$\langle \vec{e} \rangle_1 = \vec{E}\, \frac{\sigma_{xy}^{(e)} - \sigma_{xy}^{(2)}}{\sigma_{xy}^{(1)} - \sigma_{xy}^{(2)}}, \quad \langle \vec{e} \rangle_2 = \vec{E}\, \frac{\sigma_{xy}^{(1)} - \sigma_{xy}^{(e)}}{\sigma_{xy}^{(1)} - \sigma_{xy}^{(2)}}. \tag{14}$$

Inserting the formula (12) into expressions (14) it is easy to see that

$$\langle \vec{e} \rangle_{1y} = \langle \vec{e} \rangle_{2y} = E_y / 2. \tag{15}$$

So we find the solution with constant electric field.

It is necessary to clear the physical sense of obtained result for effective conductivity (12). Because according the boundary conditions in the case of QHE conditions $j_{1n} = 0$, $j_{2n} = 0$ the Hall edge current cannot cross interfaces of conducting media, for exception, only few singular points. In the studied case of layered medium this singular point has been placed at infinity. The considered Hall edge currents, which flow along interfaces, were crossed at this singular point of infinity and the non-zero value of Hall conductivity has been appeared as result of it. In the case of "chess-board" systems these singular points have been appeared at the corners of structures—in more details—see for example [8]. And the effective Hall conductivity has been formed due to percolation through these singular points. So the effective Hall conductivity does not equal to zero.

Let us obtain another solution with constant electric current. For this aim we study the case, when electric current flows along layers. Let suppose that in this case the electric current is constant:

$$\langle\vec{j}\rangle_{1y} = \langle\vec{j}\rangle_{2y} = J_y/2. \tag{16}$$

(Namely, its $y$-component of current, which is directed along layers). It would be stressed that this supposition is not obvious and we need check it further.

After calculations for averaging quantities we obtain the formula for Hall conductivity in the form:

$$\sigma^{e}_{yx} = \frac{2\sigma^{(1)}_{xy}\sigma^{(2)}_{xy}}{\sigma^{(1)}_{xy} + \sigma^{(2)}_{xy}}. \tag{17}$$

Inserting this formula (17) for expression for electric fields and for Hall currents at every of phases (14) we exactly confirm our supposition (16):

$$\langle\vec{j}\rangle_{1y} = \langle\vec{j}\rangle_{2y} = J_y/2 = \frac{\sigma^{(1)}_{xy}\sigma^{(2)}_{xy}}{\sigma^{(1)}_{xy} + \sigma^{(2)}_{xy}} E_x. \tag{18}$$

Consequently, the effective Hall conductivity tensor for layered system under quantum Hall effect conditions has a following form:

$$\hat{\sigma}^{e}_{xy} = \begin{pmatrix} 0 & \frac{1}{2}(\sigma^{(1)}_{xy} + \sigma^{(2)}_{xy}) \\ \frac{2\sigma^{(1)}_{xy}\sigma^{(2)}_{xy}}{\sigma^{(1)}_{xy} + \sigma^{(2)}_{xy}} & 0 \end{pmatrix}.$$

So above consideration shows that the problem of calculation of effective Hall conductivity under QHE conditions is not obvious.

## 3. Dykhne approach, based on rotational transformation

To solve the bound values problems for Hall conductivity in general case of composite materials let us consider the two-dimensional two-phase conducting medium in more details. (The "check-board" structure is a simple model of such a medium.) It has been described by the constant current (dc) equations and Ohm's law:

$$\mathrm{div}\,\vec{j} = 0, \quad \mathrm{curl}\,\vec{e} = 0, \quad \vec{j} = \sigma\vec{e}. \tag{19}$$

Here $\vec{j}$ and $\vec{e}$ are the electric field and current, $\sigma$ is a medium conductivity. In two-dimensional case, which only will be considered below, these equations are invariant relative to rotational transformations [1,2,9]:

$$\vec{j} = b[\vec{n}, \vec{e}'], \quad \vec{e} = d[\vec{n}, \vec{j}'], \tag{20}$$

where $\vec{n}$ is an unit vector, normal to the "check-board" plane.

Due to the its linearity the Ohm's law likewise holds in the new (primed) system:

$$\vec{j}' = \sigma'\vec{e}' \tag{21}$$

and the following expression has been obtained for the conductivity of the primed system:

$$\sigma' = \frac{b}{d\sigma}. \tag{22}$$

The analogous relation has been obtained for effective conductivities also

$$\sigma'_{\mathrm{e}} = \frac{b}{d\sigma_{\mathrm{e}}}. \tag{23}$$

Choosing the coefficients $b, d$ as following coefficients:

$$b = \frac{1}{d} = \sqrt{\sigma_1\sigma_2}, \tag{24}$$

we obtain the primed system, which differs from initial one by the replacement of phases only:

$$\sigma'_1 = \sigma_2, \quad \sigma'_2 = \sigma_1. \tag{25}$$

The new primed system is dual to the initial one, that is

$$\sigma'_{\mathrm{e}}(\varepsilon) = \sigma_{\mathrm{e}}(-\varepsilon). \tag{26}$$

And the above described duality relation has been followed—formula (7).

In a magnetic field $\vec{B}$, directed perpendicular to the plane, the Ohm's law has a tensor form:

$$\vec{j} = \hat{\sigma}\vec{e}, \tag{27}$$

where $\hat{\sigma}$ is a conductivity tensor in a magnetic field

$$\hat{\sigma}^{e}_{xy} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

with components $\sigma_{xx} = \sigma/(1+\beta^2)$, and $\sigma_{xy} = -\sigma_{yx} = \sigma\beta/(1+\beta^2)$, where $\beta = \mu B/c$ is a Hall factor, $\mu$ is a particle mobility, $c$ is a light velocity.

In this case general linear rotational transformations have been used:

$$\vec{j} = a\vec{j}' + b[n, \vec{e}'], \quad \vec{e} = c\vec{e}' + d[n, \vec{j}']. \tag{28}$$

For the primed system we again obtain the Ohm's law in a tensor form:

$$\vec{j}' = \hat{\sigma}'\vec{e}' \tag{29}$$

and the following expressions for components:

$$\sigma'_{xx} = \frac{\sigma_{xx}(ac+bd)}{(\sigma_{xx}d)^2 + (\sigma_{xy}d+a)^2},$$
$$\sigma'_{xy} = \frac{\sigma^2_{xx}cd + (\sigma_{xy}c-b)(\sigma_{xy}d+a)}{(\sigma_{xx}d)^2 + (\sigma_{xy}d+a)^2} \quad (30)$$

## 4. Symmetry transformations and Plateau of Hall conductivity

Below we consider the random mixture of the two Hall phases and we show that the effective Hall conductivity of this medium has constant value (plateau), which is equal to the value of first or second Hall phases:

$$\sigma^e_{xy} = \sigma^{(i)}_{xy}, \quad i = 1, 2. \quad (31)$$

Here the Hall phase is the phase under quantum Hall effect conditions ($\sigma_{xx} = 0, \sigma_{xy} = \text{const}$). To solve this problem we consider a general case and then we make the transition to QHE regime. The case of random mixture of metal phase and Hall phase has been considered at [10].

Firstly we find all possible symmetry transformations. There are three symmetrical transformations at least—see [8,11]. The initial two-phase system in a magnetic field has been transformed by the rotational transformations to the dual system, which differ by replacement of the phases:

$$\hat{\sigma}'_1 = \hat{\sigma}_2, \quad \hat{\sigma}'_2 = \hat{\sigma}_1. \quad (32)$$

The coefficients $a, b, c, d$ have been determined by the conditions (32). Consequently, the effective conductivity tensor of the primed system equals to the conductivity of dual system:

$$\hat{\sigma}'_e(\varepsilon) = \hat{\sigma}_e(-\varepsilon) \quad (33)$$

and the following duality relation for effective characteristics has been obtained:

$$\frac{\sigma^e_{xx}(\varepsilon)\sigma^e_{xy}(-\varepsilon) + \sigma^e_{xx}(-\varepsilon)\sigma^e_{xy}(\varepsilon)}{\sigma_{xx}(\varepsilon) + \sigma_{xx}(-\varepsilon)} = \frac{\sigma^{(1)}_{xx}\sigma^{(2)}_{xy} + \sigma^{(2)}_{xx}\sigma^{(1)}_{xy}}{\sigma^{(1)}_{xx} + \sigma^{(2)}_{xx}}. \quad (34)$$

At second transformation the primed system differs from initial only by inversion of the magnetic field direction:

$$\sigma'_{xx} = \sigma_{xx}, \quad \sigma'_{xy} = -\sigma_{xy}. \quad (35)$$

The general Dykhne's relation, connected the components of the effective conductivity tensor at arbitrary phase concentrations, has been obtained:

$$[(\sigma^e_{xx}(\varepsilon))^2 + (\sigma^e_{xy}(\varepsilon))^2]cd + \sigma^e_{xy}(\varepsilon)(ac-bd) - ab = 0. \quad (36)$$

At the third transformation the primed system differs from initial one by the replacement of the phases and by inversion of the direction of magnetic field:

$$\sigma'^{(1)}_{xx} = \sigma^{(2)}_{xx}, \quad \sigma'^{(2)}_{xx} = \sigma^{(1)}_{xx}, \quad \sigma'_{xy} = -\sigma_{xy}. \quad (37)$$

In this case the components of the two systems have been connected by the following relations:

$$\sigma^{e\prime}_{xx}(\varepsilon) = \sigma^e_{xx}(-\varepsilon), \quad \sigma^{e\prime}_{xy}(-\varepsilon) = -\sigma^e_{xy}(\varepsilon). \quad (38)$$

For effective characteristics we obtain:

$$\frac{\sigma^e_{xx}(\varepsilon)\sigma^e_{xy}(-\varepsilon) - \sigma^e_{xx}(-\varepsilon)\sigma^e_{xy}(\varepsilon)}{\sigma^e_{xx}(\varepsilon) - \sigma^e_{xx}(-\varepsilon)} = \frac{\sigma^{(1)}_{xx}\sigma^{(2)}_{xy} - \sigma^{(2)}_{xx}\sigma^{(1)}_{xy}}{\sigma^{(1)}_{xx} - \sigma^{(2)}_{xx}}. \quad (39)$$

Let us make the transition to the quantum Hall effect regime for first phase: $\sigma^{(1)}_{xx} = 0, \sigma^{(1)}_{xy} = \text{const}$. We suppose too that infinite cluster from the first Hall phase has been formed. It means that the concentration of first Hall phase is more than percolation threshold $X_c$ and that diagonal component of conductivity tensor is equal to zero: $\sigma^e_{xx}(\varepsilon) = 0$. At first step we consider second phase as metal phase. And after this we make the second step—transition for quantum Hall Effect regime ($\sigma_{xx} = 0, \sigma_{xy} = \text{const}$) for second phase. We need make this transition step by step because the uncertain relation $0:0$ has been appeared in general case.

As it is easy to see from the formulae (34) and (39) that until there is a percolation on the first Hall phase the effective non-diagonal component has constant value and equals to the value of first Hall phase.

$$\sigma^e_{xy} = \sigma^{(1)}_{xy}. \quad (40)$$

When the concentration of first Hall phase is below than the percolation threshold, the percolation is going through second phase. Repeating the same reasons we obtain that the effective Hall conductivity equals to the value of the second Hall conductivity:

$$\sigma^e_{xy} = \sigma^{(2)}_{xy}. \quad (41)$$

These results were in accordance with general Dykhne's relation, which connects the components of the effective conductivity tensor in a magnetic field at arbitrary concentrations—formula (36). It is easy to see that accordingly this relation in the heterogeneous Hall medium the effective Hall conductivity has a constant value, which did not dependent on phase concentrations.

We can obtain these both results from above results (14). In the case of percolation on second Hall phase (the infinite cluster forms from second Hall phase only) it means that if $\sigma^e_{xy} = \sigma^{(2)}_{xy}$ that electric field at first phase is equal zero: $\langle e \rangle_1 = 0$. And vice verse if $\sigma^e_{xy} = \sigma^{(1)}_{xy}$ that electric field at second phase is equal zero: $\langle e \rangle_2 = 0$.

It is necessary to stress that as follows from results (40) and (41) the values of the effective Hall conductivity do not restricted by the bound values (8), which have been correct for usual diagonal conductivity. In the case of quantum Hall effect the bound values for effective Hall conductivity have been determined by the connectivity of the system (existence of percolation cluster) and are equal to:

$$\min(\sigma_{xy}^{(1)}, \sigma_{xy}^{(2)}) \leqslant \sigma_{xy}^{e} \leqslant \max(\sigma_{xy}^{(1)}, \sigma_{xy}^{(2)}). \quad (42)$$

## 5. Distribution of electric fields

To clarify the physical sense of above results let us consider the metal circular planar inclusion of radius $R$, which placed in the Hall medium. To solve problem let us use the two-dimensional character of problem and use complex variable function methods. Let us introduce the complex variable $z = x + \mathrm{i}y$ and complex analytic functions of electric current and electric fields:

$$j(z) = j_x(z) + \mathrm{i}j_y(z), \quad e(z) = e_x(z) + \mathrm{i}e_y(z), \quad (43)$$

which have been connected by Ohm's law:

$$j(z) = \frac{\sigma}{1 - \mathrm{i}\beta} e(z). \quad (44)$$

In the complex variable plane under quantum Hall effect conditions Ohm's law has the form:

$$j(z) = \mathrm{i}\frac{\sigma}{\beta} e(z). \quad (45)$$

So usual boundary conditions have been written as follows:

$$\mathrm{Re}[tj(t)]_1 = \mathrm{Re}[tj(t)]_2, \quad \mathrm{Im}\left[t\frac{\mathrm{i}\sigma}{\beta} j(t)\right]_1 = \mathrm{Im}\left[t\frac{1 + i\beta}{\sigma} j(t)\right]_2. \quad (46)$$

Here $t = \cos(\theta) + \mathrm{i}\sin(\theta)$ is the unit normal, the notation 1 is described the Hall medium, the notation 2 is described the circular inclusion. Accordingly (46) for case of the boundary conditions between metal inclusion and Hall medium we obtain that the electric field must be equal to zero inside of metal inclusion:

$$E_2(z) = 0, \quad |z| < R. \quad (47)$$

It is easy to understand this result. Accordingly Prigogine's theorem for dissipation of energy the percolation of current has a minimal dissipation. The Hall medium is the non-dissipative medium, so current prefers to percolate on Hall phase only without any dissipation. But the electric current in metal inclusion will be zero only if the electric field will be zero in metal inclusion. Of course the same result has been obtained when we consider this problem exactly [12]. In this case the transition for Hall medium has been corresponded to the limiting cases:

$$\sigma_1 \to \infty, \quad \beta_1 \to \infty, \quad \frac{\sigma_1}{\beta_1} = \mathrm{const}. \quad (48)$$

From analyticity of the functions $j_1(z)$ and $j_2(z)$ it follows that these functions are represented by the converging set expansions in its areas of definition:

$$j_1(z) = A_0 + \frac{A_1}{z} + \frac{A_2}{z^2} + \cdots, \quad |z| > R$$

and

$$j_2(z) = B_0 + B_1 z + B_2 z^2 + \cdots, \quad |z| < R. \quad (49)$$

It is necessary to take account that the current is homogeneous at infinity $j_1(\infty) = \langle J \rangle$, and at the inclusion there are no singularities. So we need to satisfy the boundary conditions (46). After calculations the following results have been obtained:

$$j_1(z) = \langle J \rangle - \langle \bar{J} \rangle \frac{R^2}{z^2}, \quad j_2(z) = 0. \quad (50)$$

Here $\langle \vec{J} \rangle$ is average current and $\langle \bar{J} \rangle$ is the complex conjugated current (50). This result means that the negative dipole moment of the circular inclusion has been appeared when the current flows around metal circular inclusion. So the electric field from dipole moment and external electric field $\langle \vec{E} \rangle$ have been exactly compensate each other, and resulting electric field has been equal zero—see formula (47).

This result has a clear physical sense. The current percolation in inhomogeneous medium goes with a minimal dissipation. In our case there is such a possibility—the percolation on Hall phase with zero dissipation and it has been realized.

## 6. Conclusion

These results for bound values of effective Hall conductivity have been connected with unusual character of current percolation in the quantum Hall effect regime. In this case the current always is perpendicular to an electric field:

$$\vec{j} = \sigma_{xy}[\vec{n}, \vec{e}], \quad (51)$$

where $n$ is a unit normal to the plane. Then from the equation $\mathrm{div}\,\vec{j} = 0$ with taking account of the potentiality of an electric field $\mathrm{curcl}\,\vec{e} = 0$ we obtain:

$$\vec{e} \times \nabla\sigma_{xy} = 0 \quad (52)$$

that is the current lines are not intersected the lines of constant values of the quantity $\sigma_{xy}$ [13]. In other words, the Hall current does not percolate from one phase to another and was "frozen" in each of phases. The constant value of the plateau $\sigma_{xy}$ has been explained by this fact. And the value of the plateau has been determined by the value of the percolating phase, when infinite cluster of certain first or second phase has been appeared. This solution has a sense of stable point for Dykhne's transformations too [14].

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