Cherenkov radiation by an electron bunch that moves in a vacuum above a left-handed material

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Cherenkov radiation by a nonrelativistic electron bunch that moves above an interface of a vacuum-left-handed material has been investigated theoretically. The electron density of the bunch is described by a Gauss distribution. Cherenkov radiation for the frequency range where the refractive index is negative is shown to lead to simultaneous excitation of both bulk and surface electromagnetic waves over one and the same frequency range. In this case the wave vector magnitude in the plane of the interface of surface electromagnetic waves is larger than the corresponding wave vector magnitude of bulk electromagnetic waves. The energy flows in a left-handed material have been calculated. The spectral density and the radiation pattern have been investigated.

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I. INTRODUCTION

In recent years a good deal of attention has been given to studying the electromagnetic properties of left-handed materials (LHM's). These materials came to be known by this particular name because in these media the directions of electric and magnetic field vectors as well as the direction of a wave vector form a left-handed triplet. The unusual properties of the LHM electrodynamics were originally suggested in the paper by Veselago in 1967.[1] In Ref. 1 it was shown that the LHM, which possesses negative permittivity ε and negative permeability μ simultaneously, would exhibit unusual properties such as the negative index of refraction, antiparallel wave vector $\vec{k}$ and Poynting vector $\vec{S}$, antiparallel phase and group velocities, and the time-averaged energy flux opposite to the time-averaged momentum density. The latter signifies that the wave has a negative group velocity. In the plane-wave refraction at the interface of right- and left-handed materials the incident and refractive waves (i.e., their group velocities) propagate on one side of the normal to the interface. This implies that the plane-parallel plate made of a LHM can bring to a focus an image of a point source that is located away from the plate at a distance which is less than its thickness. Besides, as indicated in Ref. 1, opposite directions of vectors $\vec{S}$ and $\vec{k}$ in the LHM result in a reverse Doppler shift and Cherenkov radiation and the other phenomena of interest. The mere fact is that the negative group velocity is not fundamentally new *per se*. As long ago as 1945 Mandelstam had stated this particular fact.[2] A considerably great interest in the LHM's has been evoked after they had been practically implemented in Refs. 3–7 in the form of alternating layers with negative ε and positive μ and layers with positive ε and negative μ. The permeability frequency dispersion of complex composites is provided by a periodic structure of nonmagnetic circular conducting units such as the split-ring resonators, spirals, etc. The permittivity frequency dispersion is provided by a periodic grating of thin conducting wires. If a wavelength of the electromagnetic wave that propagates in such a material is much larger than the period of composite structure, the composite for this particular wave is similar to a continuous one. The parameters of structural elements are selected in such a way that ε and μ become negative over the GHz frequency range. The negative sign of the refractive index of LHM's needs specifying the fundamental principle such as Fermat's principle.[8] For instance, in Ref. 8 it was shown that the light propagation path in a medium corresponds to a local extremum of the optical path length. The term "local" means that a light propagation problem can have several possible paths for which the Snell law is obeyed and the optical path variation is equal to zero.

It should be pointed out that the refractive index can become negative not only in the above-described composite materials, but also in the so-called photonic crystals (see, for example, Refs. 9–11). In Ref. 9 a GaAs semiconductor specimen with a periodic set of cylindrical holes that form a two-dimensional grating was examined and it was shown that the electromagnetic-wave dispersion in such a crystal has a band structure similar to that of the electronic band in semiconductors. The refractive index in these structures may become negative in the vicinity of the photonic band gap. In metallic photonic crystals the negative refraction can be observed over a wider frequency range as compared to that range for dielectric photonic crystals.[11] In photonic crystals the refractive index is found to be negative at wavelengths comparable to a period of a crystal structure. At the same time the requirement for homogeneity of composite media in which both ε and μ may become negative restricts the wavelength from below.

The Cherenkov energy loss by an electron bunch that moves through the photonic crystal along one of its pores is experimentally observed in Ref. 12. In particular, it has been established that the dependence of the loss probability on the energy loss qualitatively varies with the number of surrounding pores. This fact is explained by excitation of guided modes of Cherenkov radiation that bounces back and forth between the pore walls. The wavelengths are of the order of the distance between the pores. So the change in the pore number qualitatively varies the shape of the above-mentioned dependence. It has been shown that fast electrons can be used for directly probing photonic crystal band structures. In this connection Ref. 13 should be pointed out.

In the present paper Cherenkov radiation[14] of bulk and surface electromagnetic waves by an electron bunch that moves in a vacuum above a composite medium has been theoretically investigated. This medium may have negative values of ε and μ over a certain frequency range. Cherenkov radiation is shown to give rise to simultaneous excitation of bulk and surface electromagnetic waves over one and the same frequency range. The excited bulk electromagnetic waves comprise all electric and magnetic field components. The excited surface electromagnetic waves can be of two different types: namely, the electric and magnetic ones.[15] It has been shown that these surface waves can be excited and propagate along an interface provided that the characteristic frequencies of the composite medium and the velocity of the bunch satisfy certain conditions. Besides, the widths of the frequency ranges, at which surface waves exist, depend on the electron bunch velocity. These circumstances are regarded to be new as compared to the known results obtained from the analysis of surface electromagnetic waves propagation at the interface of right- and left-handed materials.[16–18] The problem considered in the present paper is not only of theoretical, but also of practical interest in terms of excitation of delayed surface waves of electric type. These waves are widely used in the investigation of surfaces. It will be shown that the surface waves of electric type can be excited much more easily at the interface of the vacuum-left-handed material as compared to that of the vacuum-right-handed one. Besides, the results obtained can be used to determine typical frequencies of LHM's such as the resonance frequency of permeability and plasma frequency.

It should be stressed that the Cherenkov radiation by a point charge in LHM's with incorporation of small amount of loss has been theoretically studied in Ref. 19. It has been shown that when losses exist, the directions of power propagation differ from those of phase propagation. Besides, it has been noted that the radiation pattern of the Cherenkov radiation presents lobes at very large angles close to 90° with respect to the particle motion. This fact, in the authors' view, allows one to improve the Cherenkov detectors based on the use of LHM's. It is worthwhile to emphasis that in spite of substantial progress in theoretical investigations into the electrodynamics of LHM's the problem of the influence of small losses on the negative refraction is still a relevant one.

II. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Let us initially consider Cherenkov radiation by a single electron that moves above a left-handed material and generalize the obtained results for an electron bunch. Let the interface of the vacuum-left-handed material be located in the xy plane. An electron moves in a vacuum parallel to the interface at a distance of h from it at a velocity $v \ll c$ (where c is the speed of light in a vacuum). We consider the direction of the electron velocity as a positive one of axis ox. The electron charge density is determined by the formula

$$\rho(\vec{r},t) = e\,\delta(x - vt)\,\delta(y)\,\delta(z - h), \tag{1}$$

where $\delta(x)$ is the Dirac delta function. The electromagnetic fields of the electron are expressed in terms of Fourier integrals:

$$\vec{E}^e(\vec{r},t) = \int \vec{E}^e(\vec{k},\omega)\exp[i(\vec{k}\vec{r} - \omega t)]d\vec{k}d\omega. \tag{2}$$

The wave equation for the Fourier component $\vec{E}^e(\vec{r},\omega)$ has the form

$$\Delta\vec{E}^e(\vec{r},\omega) + \frac{\omega^2}{c^2}\varepsilon_1\mu_1\vec{E}^e(\vec{r},\omega) = \frac{4\pi}{\varepsilon_1}\left\{\mathrm{grad}[\rho(\vec{r},\omega)] - i\frac{\omega}{c^2}\vec{v}\varepsilon_1\mu_1\rho(\vec{r},\omega)\right\}, \tag{3}$$

where $\beta = v/c$. The solutions of the wave equation for a single electron are[20]

$$\vec{E}^e(\vec{k}) = \frac{4\pi i}{\varepsilon_1}\frac{(\omega/c^2)\varepsilon_1\mu_1\vec{v} - \vec{k}}{k^2 - (\omega/c)^2\varepsilon_1\mu_1}\rho(\vec{k},\omega), \tag{4}$$

$$\vec{H}^e(\vec{k},\omega) = \frac{\varepsilon_1}{c}[\vec{v},\vec{E}^e(\vec{k},\omega)], \tag{5}$$

where

$$\rho(\vec{k},\omega) = \frac{e}{(2\pi)^3}\exp(-ik_z h)\,\delta(k_x v - \omega). \tag{6}$$

Hereafter, we shall hold symbols ε_1 and μ_1 for reasons of generality. Upon integrating solutions (4) and (5) with respect to k_x and k_z we obtain the following expressions for the Fourier components of the electron field:

$$E_x^e(k_y,\omega) = -\frac{ie\omega(1 - \beta^2\varepsilon_1\mu_1)}{2\pi\varepsilon_1 v^2\xi_1}\exp(-|z - h|\xi_1), \tag{7}$$

$$E_y^e(k_y,\omega) = -\frac{iek_y}{2\pi\varepsilon_1 v\xi_1}\exp(-|z - h|\xi_1), \tag{8}$$

$$E_z^e(k_y,\omega) = \frac{\mathrm{sgn}(z - h)e}{2\pi\varepsilon_1 v}\exp(-|z - h|\xi_1), \tag{9}$$

$$H_y^e(k_y,\omega) = -\frac{\mathrm{sgn}(z - h)e}{2\pi c}\exp(-|z - h|\xi_1), \tag{10}$$

$$H_z^e(k_y,\omega) = -\frac{iek_y}{2\pi c\xi_1}\exp(-|z - h|\xi_1), \tag{11}$$

where

$$\xi_1(\vec{k}_\perp,\omega) = \sqrt{k_\perp^2 - \frac{\omega^2}{c^2}\varepsilon_1\mu_1}, \tag{12}$$

$\vec{k}_\perp = (k_x, k_y)$, and $k_x = \omega/v$. We define the electromagnetic fields of radiation in terms of the vector potential $\vec{A}_l^r(\vec{r},t)$ in the following way:[21]

$$\vec{E}_l^r(\vec{r},t) = -\frac{1}{c}\frac{\partial\vec{A}_l^r(\vec{r},t)}{\partial t}, \tag{13}$$

$$\vec{B}_l^r(\vec{r},t) = \mathrm{rot}\vec{A}_l^r(\vec{r},t), \tag{14}$$

$$\vec{A}_l^r(\vec{r},t) = \int \vec{A}_l^r(\vec{k},\omega) \times \exp\{i[\vec{k}_\perp \vec{r}_\perp + k_{z,l}(\vec{k}_\perp,\omega)z - \omega t]\} d\vec{k}_\perp d\omega, \quad (15)$$

where $\vec{r}_\perp = (x,y)$, $l=1$ for vacuum and $l=2$ for the LHM,

$$k_{z,1}(\vec{k}_\perp,\omega) = i\xi_1(\vec{k}_\perp,\omega), \quad (16)$$

$$k_{z,2}(\vec{k}_\perp,\omega) = -p\sqrt{\frac{\omega^2}{c^2}\varepsilon_2\mu_2 - k_\perp^2}, \quad (17)$$

where $p=1$ for the right-handed material and $p=-1$ for the LHM. Note that the vector potential $\vec{A}_l^r(\vec{r},t)$ satisfies the Coulomb gauge:

$$\operatorname{div} \vec{A}_l^r(\vec{r},t) = 0.$$

The choice of sign $p=-1$ in Eq. (17) corresponds to propagation of the radiation fields in the interface direction as $\varepsilon_2 < 0$ and $\mu_2 < 0$. Since the Cherenkov radiation condition in a vacuum is not valid, $k_{z,1}$ is a purely imaginary value and the electromagnetic fields in a vacuum decay exponentially away from the interface. The radiation fields in a vacuum and in the LHM are derived from the continuity conditions for tangential components of electric and magnetic fields at the interface between the two media:

$$\{E_x\}_{z=0} = 0, \quad \{E_y\}_{z=0} = 0, \quad (18)$$

$$\{H_x\}_{z=0} = 0, \quad \{H_y\}_{z=0} = 0. \quad (19)$$

Substituting the expressions for the electron fields, Eqs. (7)–(11), and the radiation fields, Eqs. (13)–(15), into boundary conditions (18) and (19), we have

$$E_{x,1}^r = \frac{e\omega \exp(-h\xi_1)}{2\pi\varepsilon_1 v^2 k_{z,1}}\left[(\beta^2\varepsilon_1\mu_1 - 1)\frac{\varepsilon_2 k_{z,1} + \varepsilon_1 k_{z,2}}{\Delta_1} - 2\beta^2\varepsilon_1\mu_1 k_y^2 \frac{\varepsilon_1\mu_1 - \varepsilon_2\mu_2}{\Delta_1\Delta_2}\right], \quad (20)$$

$$E_{y,1}^r = -\frac{ek_y \exp(-h\xi_1)}{\pi\varepsilon_1 v k_{z,1}}\left[\frac{k_{z,1}^2}{\Delta_1\Delta_2} - \frac{k_{z,1}\mu_2 + k_{z,2}\mu_1}{2\Delta_2}\right], \quad (21)$$

$$E_{z,1}^r = -\frac{e \exp(-h\xi_1)}{2\pi\varepsilon_1 v}\frac{\varepsilon_2 k_{z,1} + \varepsilon_1 k_{z,2}}{\Delta_1}, \quad (22)$$

$$H_{x,1}^r = -\frac{e\omega k_y \exp(-h\xi_1)}{\pi v c}\frac{\varepsilon_1\mu_1 - \varepsilon_2\mu_2}{\Delta_1\Delta_2}, \quad (23)$$

$$H_{y,1}^r = \frac{e \exp(-h\xi_1)}{\pi c}\left[\frac{\omega^2}{v^2}\frac{\varepsilon_1\mu_1 - \varepsilon_2\mu_2}{\Delta_1\Delta_2} - \frac{k_{z,1}\mu_2 + k_{z,2}\mu_1}{2\Delta_2}\right], \quad (24)$$

$$H_{z,1}^r = \frac{ek_y}{2\pi c k_{z,1}}\exp(-h\xi_1)\frac{k_{z,1}\mu_2 + k_{z,2}\mu_1}{\Delta_2} \quad (25)$$

and

$$E_{x,2}^r = -\frac{e\omega \exp(-h\xi_1)}{\pi v^2 k_{z,1}}\left[\frac{k_{z,2}}{\Delta_1}(1 - \beta^2\varepsilon_1\mu_1) + \beta^2 k_y^2 \mu_1 \frac{(\varepsilon_1\mu_1 - \varepsilon_2\mu_2)}{\Delta_1\Delta_2}\right], \quad (26)$$

$$E_{y,2}^r = -\frac{ek_y \exp(-h\xi_1)}{\pi v}\frac{k_{z,2}\mu_2 - k_{z,1}\mu_1}{\Delta_1\Delta_2}, \quad (27)$$

$$E_{z,2}^r = -\frac{ek_{z,1}\exp(-h\xi_1)}{\pi v \Delta_1}, \quad (28)$$

$$H_{x,2}^r = H_{x,1}^r, \quad (29)$$

$$H_{y,2}^r = \frac{e \exp(-h\xi_1)}{\pi c}\left[\frac{\varepsilon_2 k_{z,1}}{\Delta_1} - k_y^2 \frac{\varepsilon_1\mu_1 - \varepsilon_2\mu_2}{\Delta_1\Delta_2}\right], \quad (30)$$

$$H_{z,2}^r = \frac{e\mu_1 k_y \exp(-h\xi_1)}{\pi c \Delta_2}, \quad (31)$$

where

$$\Delta_1 = \varepsilon_2 k_{z,1} - \varepsilon_1 k_{z,2}, \quad (32)$$

$$\Delta_2 = \mu_2 k_{z,1} - \mu_1 k_{z,2}. \quad (33)$$

The total radiation energy losses by the electron bunch can be evaluated as the work done on the bunch per unit time per unit volume by the radiation field:

$$Q = \rho(\vec{r},t) v E_{x,1}^r(\vec{r},t), \quad \text{for } \vec{r} = \vec{v}t. \quad (34)$$

We find the energy losses by the electron bunch to radiation in a vacuum and in the LHM with the aid of the time-averaged Poynting vector:

$$\langle \vec{S} \rangle = \frac{c}{4\pi}\operatorname{Re}\left\{\int_{-\infty}^{\infty}[\vec{E}^r, \vec{H}^{r*}]dt\right\}. \quad (35)$$

From Eq. (17) it follows that for $\beta^2\varepsilon_2\mu_2 < 1$, $k_{z,2}$ is a purely imaginary value, but for $\beta^2\varepsilon_2\mu_2 > 1$ $k_{z,2}$ can be a real one for $k_y \leqslant \omega\sqrt{\beta^2\varepsilon_2\mu_2 - 1}/v$. When $k_{z,1}$ and $k_{z,2}$ are imaginary values (i.e., for $\beta^2\varepsilon_2\mu_2 < 1$) and there is no energy loss, the Fourier component of the radiation field in a vacuum $E_{x,1}^r(k_y,\omega)$ is also the imaginary value and

$$\operatorname{Re}\left\{\int_{-\infty}^{\infty} d\omega dk_y E_{x,1}^r(k_y,\omega) \times \exp(-\xi_1 z)\exp\left[i\left(\frac{\omega}{v}x + k_y y - \omega t\right)\right]\right\} = 0, \quad (36)$$

for $x = vt$ and $y = 0$ in the absence of a pole in Eq. (20). At the same time, as $k_{z,1}$ is an imaginary value and $k_{z,2}$ is a real one (for $\beta^2\varepsilon_2\mu_2 > 1$), the value of $E_{x,1}^r(k_y,\omega)$ is complex and the real part of the integrand in Eq. (36) becomes nonzero. This means that particle radiation energy losses occur. This radiation is identified as Cherenkov radiation of bulk electromagnetic waves [in the absence of a pole in Eq. (20)], since the condition $\beta^2\varepsilon_2\mu_2 > 1$ holds. If the integrand in Eq. (36) has a pole, the surface electromagnetic waves will be excited. As follows from Eqs. (20)–(31), the surface electromagnetic waves, which are described by the dispersion equation $\Delta_1=0$, have no perpendicular components of the magnetic field—i.e., $H^{sw}_{z,1}=H^{sw}_{z,2}=0$. At the same time, the surface electromagnetic waves, which are described by the dispersion equation $\Delta_2=0$, have no perpendicular components of the electric field—i.e., $E^{sw}_{z,1}=E^{sw}_{z,2}=0$. If we use the terminology in Ref. 15, we will term the surface electromagnetic waves with $\Delta_1=0$ as the electric surface waves (or E-type surface waves), whereas the surface electromagnetic waves with $\Delta_2=0$ will be represented as the magnetic surface waves (or H-type surface waves).

A. Regimes of electromagnetic wave excitation: Qualitative evaluations

1. E-type surface waves

Let us consider E-type surface waves and assume that

$$\varepsilon_1=1,\quad \mu_1=1,\quad \varepsilon_2=\varepsilon<0,\quad \mu_2=\mu<0. \tag{37}$$

In this case the components of wave vectors $k_{z,1}$ and

$$k_{z,2}=-i\xi_2=-i\sqrt{k_y^2-\frac{\omega^2}{v^2}(\beta^2\varepsilon\mu-1)}$$

are imaginary values and the equation $\Delta_1=i(\varepsilon\xi_1+\xi_2)=0$ has the following solution:

$$k_y^2=-\frac{\omega^2}{v^2}\left[1+\beta^2\varepsilon\frac{\mu-\varepsilon}{\varepsilon^2-1}\right]. \tag{38}$$

The solution (38) will be a real one for $k_y^2>0$. Physically this implies that the phase velocities of surface electromagnetic waves are less than the bunch velocity:

$$v_{ph}^{sw}=\frac{\omega}{\sqrt{k_x^2+k_y^2}}<v. \tag{39}$$

Substituting $k_x=\omega/v$ and k_y from Eq. (38) into Eq. (39), we obtain the following condition of Cherenkov excitation of E-type surface waves:

$$\frac{c}{\sqrt{1+\dfrac{\varepsilon\mu-1}{1-\varepsilon^2}}}<v. \tag{40}$$

From (40) it is seen that the excitation of surface E waves is possible when either of the two following conditions is fulfilled:

$$\varepsilon\mu>1 \quad \text{and } \varepsilon^2<1 \tag{41}$$

or

$$\varepsilon\mu<1 \quad \text{and } \varepsilon^2>1. \tag{42}$$

Note that the E-type surface-wave excitation is also possible when

$$\varepsilon\mu<0,\quad \varepsilon<0,\quad \text{and } \varepsilon^2>1. \tag{43}$$

However, in this case the composite medium does not behave like a left-handed one. From (41) and the following condition of bulk electromagnetic wave excitation,

$$\varepsilon\mu>\beta^{-2}, \tag{44}$$

it follows that, in principle, these conditions can be compatible with each other. This means that the Cherenkov effect causes the simultaneous excitation of both bulk and surface electromagnetic waves over one and the same frequency range.

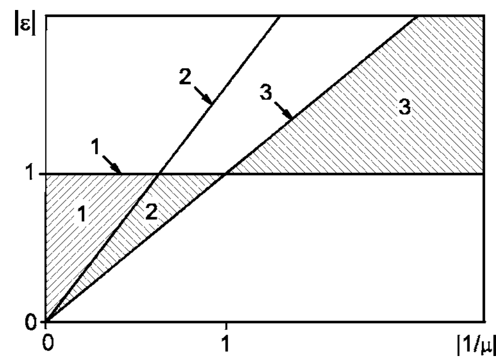

FIG. 1. The regimes of E-type surface-wave excitation along the interface of the vacuum-left-handed material. Region 1 corresponds to simultaneous excitation of bulk and surface electromagnetic waves over the same frequency range. Regions 2 and 3 correspond to excitation of E-type surface waves only. Curve 1 is for to the dependence for $|\varepsilon|=1$, curve 2 is for $|\varepsilon|=\beta^{-2}/|\mu|$, and curve 3 is for $|\varepsilon|=1/|\mu|$.

The above-considered regimes of E-type surface-wave excitation are sketched in Fig. 1. In Fig. 1 curve 1 corresponds to the dependence for $|\varepsilon|=1$, curve 2 is for $|\varepsilon|=\beta^{-2}/|\mu|$, and curve 3 is for $|\varepsilon|=1/|\mu|$. Region 1 corresponds to the frequency range at which Eqs. (41) and (44) are simultaneously valid. Both bulk and surface electromagnetic waves are simultaneously excited over this frequency range. In the regions which are to the right of curve 2 the bulk electromagnetic waves are not excited. Region 2 corresponds to condition (41) and region 3 to (42). In regions 2 and 3 the surface electromagnetic waves are only excited.

It should be noted that, as follows from (40), the phase velocities of E-type surface waves can have indefinitely small values in the vicinity of ω_0. This implies that the surface waves of this type can be excited much more easily at the interface of the vacuum-left-handed material as compared to that of the vacuum-right-handed material.

2. H-type surface waves

Let us examine the H-type surface waves and assume that

$$\varepsilon_1=1,\quad \mu_1=1,\quad \varepsilon_2=\varepsilon<0,\quad \mu_2=\mu<0. \tag{45}$$

The solution of the dispersion equation $\Delta_2=i(\mu\xi_1+\xi_2)=0$ is

$$k_y^2=-\frac{\omega^2}{v^2}\left[1+\beta^2\mu\frac{\varepsilon-\mu}{\mu^2-1}\right]. \tag{46}$$

Substituting $k_x=\omega/v$ and Eq. (46) into Eq. (39), we can write the following condition of Cherenkov excitation of H-type surface waves:

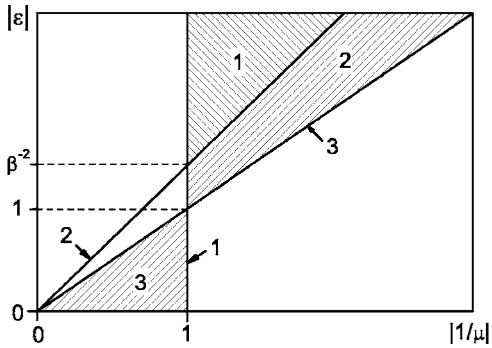

FIG. 2. The regimes of H-type surface-wave excitation along the interface of the vacuum-left-handed material. Region 1 corresponds to simultaneous excitation of bulk and surface electromagnetic waves over the same frequency range. Regions 2 and 3 correspond to excitation of H-type surface waves only. Curve 1 corresponds to the dependence for $|\mu|=1$, curve 2 is for $|\varepsilon|=\beta^{-2}/|\mu|$, and curve 3 is for $|\varepsilon|=1/|\mu|$.

$$\frac{c}{\sqrt{1+\frac{\varepsilon\mu-1}{1-\mu^2}}}<v. \quad (47)$$

From (47) it follows that the excitation of H-type surface waves is possible when either of the two following conditions is satisfied:

$$\varepsilon\mu>1 \quad \text{and } \mu^2<1 \quad (48)$$

or

$$\varepsilon\mu<1 \quad \text{and } \mu^2>1. \quad (49)$$

Note that the excitation of H-type surface waves is also possible for

$$\varepsilon\mu<0, \quad \mu<0, \quad \text{and } \mu^2>1. \quad (50)$$

However, in this case the composite medium does not behave like a left-handed one. As shown by analysis of (44) and (48) the simultaneous excitation of bulk and surface electromagnetic waves is made possible over a certain frequency range.

The regimes of excitation of H-type surface waves are qualitatively represented as diagrams in Fig. 2. In Fig. 2 curve 1 is for $|\mu|=1$, curve 2 is for $|\varepsilon|=\beta^{-2}/|\mu|$, and curve 3 is for $|\varepsilon|=1/|\mu|$. Region 1 corresponds to the frequency range in which (44) and (48) are simultaneously met. In this range the bulk and surface electromagnetic waves are simultaneously excited. In the regions lying to the right of curve 2 the bulk waves are not excited. Region 2 is appropriate to (48), and region 3 is for (49). In regions 2 and 3 the surface electromagnetic waves are only excited.

B. Regimes of electromagnetic wave excitation: Numerical evaluations

Hereafter, we will make use of the following expressions for permittivity ε and permeability μ of the (Refs. 3 and 5):

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2}, \quad (51)$$

$$\mu(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2}, \quad (52)$$

where ω_p is the effective plasma frequency, ω_0 is the resonance frequency, and $F<1$.

1. E-type surface waves

Now consider E-type surface-wave excitation. As indicated, E-type surface waves can get excited in one and the same frequency interval with bulk waves with conditions (41) and (44) being simultaneously satisfied. Upon substituting the corresponding dispersion dependences for $\varepsilon(\omega)$ and $\mu(\omega)$ from Eqs. (51) and (52) into (41) and (44), we obtain the following condition for typical frequencies ω_p and ω_0:

$$\left(\frac{\omega_p}{\omega_0}\right)^2=1+\chi, \quad (53)$$

where $0<\chi\ll 1$. The surface electromagnetic waves exist along with the bulk waves over the narrow range of frequencies (with $\beta^2\ll 1$)

$$\omega_0<\omega<\omega_0+\beta^2\frac{\chi F\omega_0}{2}. \quad (54)$$

For example, with ω_0=4 GHz, F=0.56,[17] β=0.1, and χ=0.01, the width of the frequency range (54) is $\Delta\omega\approx 2\times 10^{-4}$ GHz and the plasma frequency must be equal to ω_p=4.04 GHz. In the subsequent numerical calculations we will assume the values of ω_0=4 GHz and F=0.56 to be the same as in Ref. 17 whereas the plasma frequency ω_p will be fitted so that the required conditions are met. From the solution (38) for k_y it follows that the magnitude of the component k_y^s of a surface wave is larger than the corresponding component k_y^b for the bulk wave—i.e.,

$$|k_y^s|>\frac{\omega}{v}\sqrt{\beta^2\varepsilon\mu-1}\geqslant|k_y^b|. \quad (55)$$

From (41) and (42) it follows that the Cherenkov excitation of only surface waves will occur. For example, after substituting into (41) the dispersion dependences for $\varepsilon(\omega)$ and $\mu(\omega)$, we arrive at the following frequency interval of surface wave existence:

$$0<\frac{\omega}{\omega_0}-1\ll 1, \quad \frac{\omega_p}{\sqrt{2}}<\omega_0<\omega_p. \quad (56)$$

For example, at ω_0=4 GHz, F=0.56, β=0.1, and ω_p=5 GHz the frequency range width at which $\beta^2\varepsilon\mu<1$ and $v>v_{ph}^s$ is equal to $\Delta\omega\approx 6\times 10^{-3}$ GHz.

The analysis of (43) together with Eqs. (38), (51), and (52) shows that surface waves can only be excited at $\varepsilon<0$ and $\mu>0$ over the frequency interval

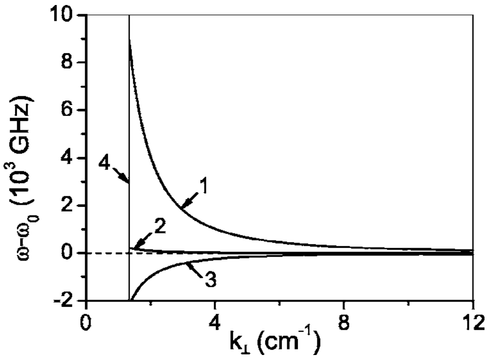

FIG. 3. The dispersion curves of E-type surface waves. Curves 1 and 2 refer to surface waves being excited over the frequency range where $\varepsilon\mu>0$. Curve 3 corresponds to the surface wave over the frequency range where $\varepsilon\mu<0$. Curve 4 is for $k_\perp=\omega/v$.

$$\omega<\omega_0<\frac{\omega_p}{\sqrt{2}},\quad 0<1-\frac{\omega}{\omega_0}\ll 1. \tag{57}$$

As follows from (57) the surface electromagnetic waves get excited in the sufficiently small vicinity of ω_0 in which the charge velocity exceeds the phase velocity of a surface wave $v>v_{ph}^s$. For instance, at $\omega_0=4$ GHz, $\omega_p=10$ GHz, $F=0.56$,[17] and $\beta=0.1$ we have $\omega\in[3.998;4)$ GHz.

Figure 3 presents the dispersion dependencies of the E-type surface waves that fit the above excitation regimes. Dispersion curve 1 refers to surface waves being excited over the frequency interval (54). Dispersion curve $k_{z,2}=0$ over the same frequency interval is practically coincident with curve 1; however, it lies slightly below curve 1. Dispersion curve 2 corresponds to the surface waves that are excited over frequency interval (56). As it takes place, the bulk waves are not excited. Dispersion curve 3 depicts surface waves which are excited over frequency interval (57), when the composite medium is a right-handed one. All the curves originate at curve 4 described by equation $k_\perp=\omega/v$. The region of the bulk waves existence is bounded by curve $k_{z,2}=0$, line $\omega=\omega_0$, and curve 4. From Fig. 3 it follows that over the frequency range where the composite medium behaves like the LHM excited bulk and surface waves are reverse (curves 1 and 2). For the frequency region where the composite medium is the right-handed one the dispersion of excited surface wave is positive (curve 3).

2. H-type surface waves

Let us consider H-type surface-wave excitation. As the analysis of (44) and (48) and Eqs. (51) and (52) suggests the excitation of bulk and surface waves can be made possible in the same frequency interval under the following condition:

$$\left(\frac{\omega_p}{\omega_0}\right)^2\gg\beta^{-2}\quad\text{for }\beta^2\ll 1. \tag{58}$$

The frequency range at which this surface waves tend to emerge is approximately described by the following inequality:

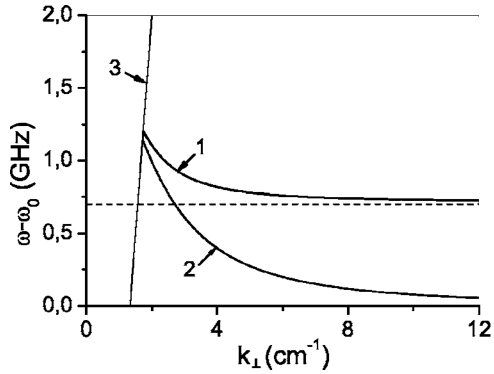

FIG. 4. The dispersion curves of H-type surface waves (curves 1) and $k_{z,2}=0$ (curve 2). Curve 3 is for $k_\perp=\omega/v$.

$$\omega^*<\omega<\frac{\omega_0}{\sqrt{1-F}}\left(1-\beta^{-2}\frac{F\omega_0^2}{2\omega_p^2}\right), \tag{59}$$

where $\omega^*=\sqrt{2/(2-F)}\omega_0$ is found from $\mu(\omega)=-1$ [because $\mu^2<1$ in accordance with (48)]. For $\omega_0=4$ GHz, $F=0.56$, $\beta=0.1$, and $\omega_p=80$ GHz the frequency interval (59) is $\Delta\omega\in[4.7;5.2]$ GHz. The excited surface wave is a reverse one. In that part of the frequency range (59) where conditions (48) are satisfied while (44) is not satisfied, the surface waves are only excited.

The above-mentioned case (50) is found to be unrealizable for $\varepsilon(\omega)$ and $\mu(\omega)$, which are specified by Eqs. (51) and (52). Indeed, as (50) is satisfied and from the requirement $k_y^2>0$, it follows that ε must be greater than unity. At the same time, as evident from Eq. (51), ε cannot be a positive value which is greater than unity.

Figure 4 presents the dispersion curves for bulk and H-type surface waves which correspond to the above-mentioned excitation regimes. Dispersion curve 1 corresponds to surface waves being excited over the frequency interval (59). In Fig. 4 this frequency interval is appropriate to the straight-line segment along the frequency axis from the dashed-line curve up to intersection point of curves 1 and 3. Dispersion curve 2 corresponds to $k_{z,2}=0$. Dispersion curve 3 corresponds to $k_x=\omega/v$. The region of the bulk waves existence is bounded by curve 2, line $\omega=\omega_0$, and curve 3. The bulk and surface waves are reverse. As seen from Fig. 4, over the frequency range where the bulk and surface waves exist simultaneously, the wave vector of the surface wave is greater than that of the bulk one.

III. ENERGY FLOWS

We generalize the above-derived expressions for the radiation fields to the case of an electron bunch. The charge density of the electron bunch is determined by the formula

$$\rho(\vec{r},t)=\rho_0\varphi_0\left(\frac{x-vt}{a_x}\right)\varphi_0\left(\frac{y}{a_y}\right)\varphi_0\left(\frac{z-h}{a_z}\right), \tag{60}$$

where ρ_0 is the effective electron density being determined from the charge conversion law,

$$\rho_0 = \frac{eN}{2^{3/2}\pi^{3/4}a_x a_y a_z}, \quad (61)$$

N is the number of electrons in a bunch, a_j are the typical dimensions of the bunch along the coordinate axes ($j=x,y,z$), and $\varphi_0(x)$ is the Hermit function with index 0. By representing the bunch field as the Fourier integral (2) we find the solutions from the Maxwell equation, which are similar to Eqs. (4) and (5), where the Fourier component of charge density $\rho(\vec{k},\omega)$ is specified in the following way:

$$\rho(\vec{k},\omega) = \frac{eN}{(2\pi)^3}\exp\left[-ik_z h - \frac{1}{2}\sum_j a_j^2 k_j^2\right]\delta(k_x v - \omega). \quad (62)$$

In the long run, the introduction of a bunch gives rise to the emergence of a complementary multiplier in the expression for the spectral energy density of the radiation of a single electron, the so-called geometrical factor of bunch f_b. For the Gaussian bunch, f_b is given as

$$f_b = N^2 \exp(-k_x^2 a_x^2 - k_y^2 a_y^2 + \xi_1^2 a_z^2). \quad (63)$$

A. Bulk waves

In the absence of losses in the LHM the vector $k_{z,1}$ is a purely imaginary value and the bulk waves are irradiated into the LHM alone. The energy losses by the bunch for bulk wave radiation in the LHM will be calculated as follows:

$$W_{j,2}^{bw} = \int_{-\infty}^{\infty} dy \langle S_{j,2}(\vec{r},t)\rangle, \quad (64)$$

where $j=x,y,z$. The quantities $W_{j,2}^{bw}$ have the meaning of time-averaged energy radiation losses by the bunch per unit length. Integration over y is performed due to the symmetry of the problem with respect to plane xz. Consequently, $W_{x,2}^{bw}$ has the time-averaged meaning of energy flow along axis ox per unit length in the direction of the normal to the interface. The quantity $W_{z,2}^{bw}$ has the time-averaged meaning of energy flow along axis oz per unit length in the direction of bunch motion. The quantity $W_{y,2}^{bw}=0$ in view of the symmetry of the problem relative to plane xz. In mathematical terms the integrand in $W_{y,2}^{bw}$ is a odd function of k_y. Introduce the azimuthal angle φ between the projection of the wave vector to plane yz and positive axis direction oy. This angle obviously ranges between 0 and π. Then, in Eq. (64) one can go over from integration over y to that over angle φ by means of the following formula:

$$k_y = p\frac{\omega}{v}\sqrt{\beta^2\varepsilon\mu - 1}\,\cos(\varphi). \quad (65)$$

In this case the integrand in Eq. (64) will describe the radiation intensity distribution from the generatrix lines of the Cherenkov cone which is defined as follows:[1]

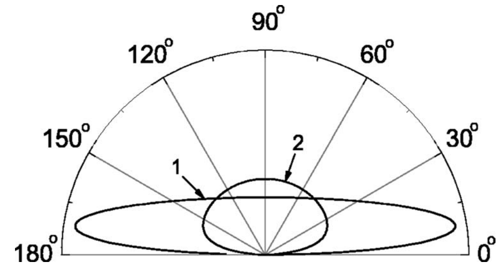

FIG. 5. The distribution of bulk waves radiation intensity in the composite medium. Curve 1 is for the energy flow in the direction of axis ox, and curve 2 in the direction of axis oz.

$$\cos(\vartheta) = \frac{1}{p|\sqrt{\varepsilon\mu}|\beta}. \quad (66)$$

Since the equation can be satisfied in the composite medium only, the cone will be semicircular. Besides, for $p=-1$ the Cherenkov angle will be obtuse ($\vartheta > \pi/2$). From Eqs. (26)–(31) it follows that the wave polarization in the second medium will be elliptical. Because the analytical expressions are cumbersome for the bulk-wave radiation energy losses, we only present numerical estimates of these quantities.

Now consider the spherically shaped electron bunch with radius a. Substituting $k_x=\omega/v$ and Eq. (65) into Eq. (63) we derive the following compact formula for f_b:

$$f_b = N^2 \exp\left(-\frac{\omega^2}{c^2}a^2\right). \quad (67)$$

From Eq. (67) it follows that the coherence length of bunch radiation is equal to $l_{coh}=c/\omega$. If the bunch radius is less than this length, then $f_b \approx N^2$ and the bunch radiates as a single whole.

Here and in what follows, we will examine the bunch having these parameters: $a=10^{-3}$ m, $N\approx 10^9$, $h=1.5\times 10^{-3}$ m, and $\beta=0.1$. Numerical estimates of energy flow densities of bulk waves excited in regime (41) in the LHM with $\omega_p=4.04$ GHz, $\omega_0=4$ GHz, and $F=0.56$ yield the following values: $W_{x,2}^{bw}\approx -6\times 10^{-12}$ J/cm and $W_{z,2}^{bw}\approx -1.5\times 10^{-12}$ J/cm.

The radiation intensity distribution in a composite medium (i.e., dependence $d^2W_{j,2}^{bw}/d\omega d\varphi$ on φ) for the above-mentioned parameters of the bunch and medium for $\omega=4.0002$ GHz in regime (41) is shown in Fig. 5. Numerical evaluations of the densities of energy flows of bulk waves excited in regime (48) and $\omega_p=80$ GHz, $\omega_0=4$ GHz, and $F=0.56$ yield the following values: $W_{x,2}^{bw}\approx -4.5\times 10^{-10}$ J/cm and $W_{z,2}^{bw}\approx -3\times 10^{-10}$ J/cm. The radiation intensity distribution in a composite medium for this regime is qualitatively consistent with the distribution in Fig. 5. Curve 1 corresponds to the φ dependence for $d^2W_{x,2}^{bw}/d\omega d\varphi$, while curve 2 is for $d^2W_{z,2}^{bw}/d\omega d\varphi$. As evident from Fig. 5, the radiation intensity distribution in the LHM is nonuniform for angle φ as compared with the case of a continuous medium. Indeed, as indicated by the Frank-Tamm formula[20] for energy radiated by an electron in a continuous medium, the radiation intensity distribution for an azimuthal angle φ in the plane which is perpendicular to the particle path has a circular form.

B. Surface waves

The surface-wave radiation fields are obtained by using a common approach—i.e., by substituting Eqs. (20)–(31) into Fourier integral definitions and by taking into account the corresponding poles in integrands while integrating over k_y.[20] In the absence of energy losses the time-averaged Poynting vector component in the direction of the normal to the interface will equal to zero $\langle S_{z,l}^{sw}\rangle=0$. Using the direct calculations one can make sure that the Poynting vector component $\langle S_{y,l}^{sw}\rangle$ is an odd function of coordinate y—i.e., $\langle S_{y,l}^{sw}\rangle \propto \text{sgn}(y)$ [where $\text{sgn}(y)=1$ for $y>0$ and $\text{sgn}(y)=-1$ for $y<0$]. Therefore the sum of components $\langle S_{y,l}\rangle^{sw}$ for $y>0$ and $y<0$ is equal to zero. The flow component $\langle S_{x,l}^{sw}\rangle$ is an even function relative to coordinate y. Hence, the resultant flow of surface waves energy in the absence of losses will be directed along axis ox. The bunch energy losses of surface waves are estimated as follows:

$$W_{x,1}^{sw}=2\int_0^{\infty}dz\langle S_{x,1}(\vec{r},t)\rangle \tag{68}$$

and

$$W_{x,2}^{sw}=2\int_{-\infty}^{0}dz\langle S_{x,2}(\vec{r},t)\rangle, \tag{69}$$

where the multiplier 2 before the integrals occurs due to summation of corresponding flows for $y<0$ and $y>0$. The quantities $W_{x,l}^{sw}$ have the meaning of time-averaged energy flows of surface waves in the direction of axis ox related to the unit length along axis oy.

1. E-type surface waves

Consider the bunch energy radiation losses of E-type surface waves. The analytical calculation indicates that the spectral densities of energy flows, $dW_{x,l}^{sw,E}/d\omega$, in the absence of losses tend to diverge at $k_y(\omega)\to 0$ when the frequency tends to the upper boundary of the frequency range in which the surface waves are existent [see (54), (56), and (57)]. In the vicinity of these frequencies one can obtain the following approximate expressions for these quantities:

$$dW_{x,1}^{sw,E}/d\omega \approx \frac{4e^2\omega^3\varepsilon^4}{v^4k_y^2(\omega)}f_b\exp\left(-2h\frac{\omega}{v}\right) \quad \text{for } k_y\to 0, \tag{70}$$

$$dW_{x,2}^{sw,E}/d\omega \approx -\frac{4e^2\omega^3\varepsilon^2}{v^4k_y^2(\omega)}f_b\exp\left(-2h\frac{\omega}{v}\right) \quad \text{for } k_y\to 0. \tag{71}$$

From (70) and (71) it follows that $dW_{x,1}^{sw,E}/d\omega>0$, $dW_{x,2}^{sw,E}/d\omega<0$, and

$$\frac{dW_{x,1}^{sw,E}/d\omega}{dW_{x,2}^{sw,E}/d\omega}\approx -\varepsilon^2 \quad \text{for } k_y\to 0. \tag{72}$$

Over the frequency range where the composite medium behaves like a LHM the frequencies of surface waves exceed the frequency of the surface plasmon, $\omega_{sp}=\omega_p/\sqrt{2}$ [at which $\varepsilon(\omega_{sp})=-1$]. This can be seen from (54) and (56). Therefore, over the frequency range where $\varepsilon<0$, $\mu<0$, and $\varepsilon\mu>0$ we have $\varepsilon^2<1$ and the sum of E-type surface-wave energy flows in a vacuum and in the LHM is a negative one:

$$W_{x,1}^{sw,E}+W_{x,2}^{sw,E}<0. \tag{73}$$

For the frequency range (57) in which the composite medium is a right-handed material the inequalities $\omega<\omega_{sp}$ and $\varepsilon^2>1$ are satisfied and the sum of these energy flows in a vacuum and in the LHM is positive:

$$W_{x,1}^{sw,E}+W_{x,2}^{sw,E}>0. \tag{74}$$

The expressions for ε and μ with existing small losses are

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\Gamma_\varepsilon)}, \tag{75}$$

$$\mu(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2+i\omega\Gamma_\mu}, \tag{76}$$

where Γ_ε and Γ_μ are the dissipation factors.

The quantities $W_{x,1}^{sw,E}$ and $W_{x,2}^{sw,E}$ are numerically calculated for the above-mentioned parameters of the bunch with due account of low losses $\Gamma_\varepsilon/\omega_p\propto 10^{-5}$ and $\Gamma_\mu/\omega_p\propto 10^{-5}$. The low losses have to be allowed for to eliminate the divergencies of frequency integrals in expressions for $W_{x,1}^{sw,E}$ and $W_{x,2}^{sw,E}$ at $k_y(\omega)\to 0$. Note that with no small losses it is impossible to indicate an adequate way of the pole detour in integrals over k_y in calculating the field of the E-type surface-wave radiation.[22] To illustrate the above statement we rewrite the $E_{x,1}^r$ Fourier component as follows:

$$E_{x,1}^r=\frac{e\omega\exp(-h\xi_1)(\varepsilon k_{z,1}+k_{z,2})}{2\pi v^2k_{z,1}(\varepsilon^2-1)}\left[(\beta^2-1)(\varepsilon k_{z,1}+k_{z,2}) - 2\beta^2k_y^2\frac{(1-\varepsilon\mu)}{\Delta_2}\right]\frac{1}{k_y^2-k_{y,0}^2}, \tag{77}$$

where $k_{y,0}^2$ is given as

$$k_{y,0}^2=-\frac{\omega^2}{v^2}\left[1+\beta^2\varepsilon\frac{\mu-\varepsilon}{\varepsilon^2-1}\right].$$

It follows from Eq. (77) that, as the small losses do not exist at $k_y=k_{y,0}$, it is impossible to point out an integration path over k_y, which makes this particular integral devoid of physical meaning. This suggests that the method for calculating the integral over k_y is mathematically incorrect, because it does not take into account the small losses in a medium, which always tend to occur. Henceforth it is necessary that the value $k_{y,0}^2$ should be complex. The pole detour direction (for $k_y=k_{y,0}$) is chosen according to the rule in Ref. 22. Besides, the small losses should be allowed for to satisfy the conditions of surface wave propagation $\text{Re}\{k_{y,0}^2\}$

$\gg \mathrm{Im}\{k_{y,0}^2\}$.[23,24] Consequently formulas (70) and (71) for spectral densities of energy flows of the E-type surface wave have physical meaning solely with regard to the small losses in a medium. As the losses increase (even at $\Gamma_\epsilon \propto \Gamma_\mu \propto 10^{-3}$) the E-type surface waves cease to be surface ones, because the aforementioned conditions for $\mathrm{Re}\{k_{y,0}^2\}$ and $\mathrm{Im}\{k_{y,0}^2\}$ are violated. For the surface waves corresponding to curve 1 in Fig. 3 we get $W_{x,1}^{sw,E} \approx 4\times 10^{-17}$ J/cm and $W_{x,2}^{sw,E} \approx -10^{-13}$ J/cm. For surface waves described by curves 2 and 3 in Fig. 3 we obtain $W_{x,1}^{sw,E} \approx 8\times 10^{-9}$ J/cm, $W_{x,2}^{sw,E} \approx -3\times 10^{-8}$ J/cm and $W_{x,1}^{sw,E} \approx 5\times 10^{-9}$ J/cm, $W_{x,2}^{sw,E} \approx -2\times 10^{-10}$ J/cm, respectively. It is evident that the above-cited qualitatively estimates of $W_{x,1}^{sw,E}$ and $W_{x,2}^{sw,E}$ hold true.

2. *H*-type surface waves

Let us examine H-type surface waves. Numerical estimates indicate that for the above-discussed excitation regimes of these waves the inequality $\varepsilon\mu \gg 1$ is satisfied. In this approximation the expressions for energy flow densities in a vacuum and in the LHM will be quite straightforward:

$$dW_{x,1}^{sw,H}/d\omega \approx \frac{2e^2}{vc}\frac{\omega}{\sqrt{|\varepsilon|}} f_b \exp\left(-2h\frac{\omega}{c}\sqrt{\frac{\varepsilon\mu}{1-\mu^2}}\right) \times \frac{|\mu|^{7/2}}{(1-\mu^2)^{3/2}}, \tag{78}$$

$$dW_{x,2}^{sw,H}/d\omega \approx -\frac{2e^2}{vc}\frac{\omega}{\sqrt{|\varepsilon|}} f_b \exp\left(-2h\frac{\omega}{c}\sqrt{\frac{\varepsilon\mu}{1-\mu^2}}\right) \times \frac{|\mu|^{3/2}}{(1-\mu^2)^{3/2}}. \tag{79}$$

From (78) and (79) it follows that these densities do not have divergencies at any of the points of the frequency interval where the H-type surface waves exist [see (59)]. Moreover, from (78) and (79) it also follows that $dW_{x,1}^{sw,H}/d\omega > 0$, $dW_{x,2}^{sw,H}/d\omega < 0$ and

$$\frac{dW_{x,1}^{sw,H}/d\omega}{dW_{x,2}^{sw,H}/d\omega} \approx -|\mu|^2. \tag{80}$$

For the regime of the H-type surface-wave excitation which was previously discussed (curve 1 in Fig. 4) the inequality $|\mu| < 1$ is satisfied and the sum of surface wave energy flows in a vacuum and in the LHM is found to be negative:

$$W_{x,1}^{sw,H} + W_{x,2}^{sw,H} < 0. \tag{81}$$

For the spherical bunch with the above-mentioned parameters and including low losses ($\Gamma_\varepsilon/\omega_p \propto 10^{-5}$, $\Gamma_\mu/\omega_p \propto 10^{-5}$), we obtain the following numerical estimates for the energy flow densities: $W_{x,1}^{sw,H} \approx 10^{-9}$ J/cm and $W_{x,2}^{sw,H} \approx -1.5\times 10^{-9}$ J/cm. We can see that the numerical calculations bear out the correctness of the above qualitative estimates of $W_{x,1}^{sw,H}$ and $W_{x,2}^{sw,H}$. Here the low losses have been taken into account so that one could make a comparison between the above numerical estimates and the analogous ones for E-type surface waves.

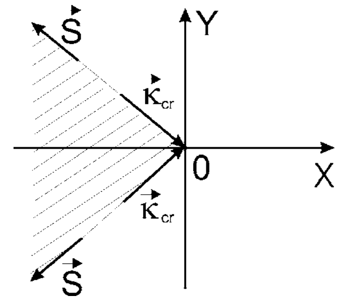

FIG. 6. The directions of wave vectors and Poynting vector of both bulk and surface waves at the interface.

C. Comparison of bulk and surface flow characteristics

From the above-mentioned calculations it follows that for the regime of simultaneous excitation of bulk and E-type surface electromagnetic waves over one and the same frequency range the time-averaged energy flow densities of the bulk waves are much greater than the similar ones for the E-type surface waves. For the regime of simultaneous excitation of bulk and H-type surface electromagnetic waves their time-averaged energy flow densities are comparable ones on the order of a magnitude. For the regimes under which the surface electromagnetic waves are only excited the time-averaged energy flow densities of these waves can be considerably greater than the similar ones for the bulk wave excitation regimes.

The wave vector and Poynting vector directions of both bulk and surface electromagnetic waves at the interface are sketched in Fig. 6. The crosshatched region corresponds to the positions of wave vectors of bulk electromagnetic waves $\vec{\kappa}^{bw}$. The wave vectors of the surface waves $\vec{\kappa}^{sw}$ are disposed to the left of axis oy outside the crosshatched region. These regions are separated by critical value κ_{cr}, which is defined from $k_{z,2}^2 = 0$:

$$\kappa_{cr} = \frac{\omega}{c}|\sqrt{\varepsilon\mu}|. \tag{82}$$

Indeed, the quantities κ^{bw}, κ^{sw}, and κ_{cr} are related to each other in the following way:

$$\kappa^{bw} < \kappa_{cr} < \kappa^{sw}. \tag{83}$$

The angle ψ_{cr} between $\vec{\kappa}_{cr}$ and the positive direction of axis ox is defined as

$$\psi_{cr} = \pi - \arctan(\sqrt{\beta^2\varepsilon\mu - 1}). \tag{84}$$

Figure 6 actually shows the field distributions behind the moving bunch in the interface plane.

IV. CONCLUSIONS

The problem of Cherenkov radiation by a moving electron bunch above a left-handed material has been theoretically examined. The electron bunch density is described by a Gaussian distribution. The Cherenkov effect is shown to lead to the excitation of both bulk and surface electromagnetic waves along the interface of vacuum-left-handed material. It is found that the surface electromagnetic waves can be excited over one and the same frequency range with bulk electromagnetic waves. However, as compared to the latter, the surface electromagnetic waves have a larger value of the wave vector in the interface plane. In addition, the possibility for Cherenkov excitation of the surface electromagnetic waves alone is demonstrated. The excited surface waves may be of *E* and *H* types and exist over different frequency ranges. It has been shown that the *E*-type surface waves with very low phase velocities can be excited in the near vicinity of the resonance frequency of the magnetic permeability. The time-averaged energy flow densities of bulk and surface waves are investigated. It has been revealed that the Cherenkov angle between the bunch movement direction and the generatrix of the Cherenkov's half of the cone in the LHM is obtuse. The excited bulk electromagnetic waves are featured by negative dispersion and comprise all electric and magnetic field components. The distribution of the bulk electromagnetic waves intensity in the LHM is a nonsymmetric one with respect to the azimuthal angle in the plane, which is perpendicular to the bunch velocity. It has been established that under the regime of simultaneous excitation of both bulk and surface electromagnetic waves over one and the same frequency range the time-averaged energy flow density of the bulk waves is much greater than the similar one for the *E*-type surface waves. At the same time, the time-averaged energy flow density of the bulk waves is of the order of the similar one for the *H*-type surface waves. For the regimes in which the surface electromagnetic waves are only excited the time-averaged energy flow densities of these waves can be much greater than the similar ones for the bulk waves for the regimes under which the bulk waves are excited. The time-averaged energy flow densities of the bulk and the surface waves are directed at different angles with respect to the bunch path in the interface plane. The time-averaged energy flow densities of the surface waves in a vacuum are positive and the similar ones in the composite medium are negative. Over the frequency range where the refraction index of the composite medium is negative the surface waves have negative dispersion and their total time-averaged energy flow density is also negative. Over the frequency region where the refraction index of the composite medium is positive the surface waves have positive dispersion and their total time-averaged energy flow density is positive too. The choice of the electron bunch to excite the electromagnetic waves is governed by the possibility of coherent radiation of its electrons for certain bunch dimensions. The results obtained allow one to solve the problem of excitation of delayed *E*-type surface electromagnetic waves with high intensity by the beam instability effect. Besides, the results obtained can be used to determine typical frequencies of the LHM such as the resonance frequency of permeability and plasma frequency.

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