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Two-dimensional impurity bound magnetic excitons in a spatially separated electron-hole system

M.A. Liberman, A.V. Petrov

Department of Physics, Uppsala University, Box 530, S-75121 Uppsala, Sweden
P. Kapitsa Institute for Physical Problems, Russian Academy of Sciences, 117334 Moscow, Russian Federation

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Abstract

The interaction of two-dimensional magnetic excitons with charged impurities in a system with spatially separated electron-hole layers is considered in the strong magnetic field limit. The energies of the lowest bound states are calculated for different projections of the total angular momentum M. The ground state of the impurity bound magnetic exciton in such a system depends on the interlayer separation. With increasing the interlayer separation the ground state changes from the $M = 1$ state to states corresponding to a higher momentum. © 1997 Published by Elsevier Science B.V.

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1. Introduction

A two-dimensional (2D) electron-hole (e-h) system in a strong perpendicular magnetic field is one of the systems with a zero-dimensional spectrum of electrons and holes and presents a quite rare example of an exactly soluble many-body system. The exact ground state of such a system corresponds to the Bose condensate of noninteracting 2D magnetic excitons with zero momentum [1,2]. Virtual transitions to the excited Landau levels cause repulsive interaction and the ground state becomes a Bose condensate of weakly repulsive excitons [1,2].

Recently the properties of 2D systems with spatially separated electrons and holes in a strong magnetic field have received much attention. At small layer separations, where the interlayer Coulomb interaction is strong, electrons and holes form excitons. The excitonically condensed state is then the preferable ground state [3,4]. With increasing the layer separation two independent Laughlin states [5] or charge-density-wave state [6,7] will be the ground state of the system, depending on the filling factor. In real coupled double quantum well systems it is possible to rearrange the exciton ground state by applying an external perpendicular electric field. Under such conditions the interwell exciton becomes the ground state at sufficiently large values of the electric field [8]. Due to the long recombination lifetime, the condensation of interwell excitons is expected to occur in coupled double quantum well systems. The suggestive results for the Bose-Einstein condensation have been reported in photoluminescence experiments [9]. The intrawell and interwell magnetoexcitons in the

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$In_xGa_{1-x}As/GaAs$ coupled double quantum well system were investigated both experimentally and theoretically in Refs. [10,11].

The interaction of 2D magnetoexcitons (MX) with impurities is of great interest for the determination of the ground state in the presence of impurities and for the spectroscopy of centers. The case of 2D intrawell magnetoexcitons bound to the impurities lying in the well was investigated in detail by Dzyubenko [12] (for theoretical studies of 2D electron complexes bound to the impurity in a strong magnetic field see Refs. [13,14], and references therein). The bound states of intrawell excitons on the impurities located in the barrier were studied experimentally in Ref. [15]. The results indicate that the binding energies of such states depend very strongly on the doping location.

In this paper the bound states of interwell excitons on the impurities located in the barrier are considered for different interlayer separations and for different impurity positions within the barrier. We will neglect the layer width as well as the tunneling through the barrier. It is assumed that the spectrum of the e-h system is a simple two-band spectrum and the electron (hole) wave functions correspond to the motion of free 2D particles in a magnetic field. The magnetic field has to be strong enough, so that virtual transitions between Landau levels can be neglected. For simplicity only particles at the zero Landau level will be considered.

2. Bound states of two-dimensional magnetic excitons in the spatially separated electron-hole system

We shall consider a model where the electron layer and hole layer are separated by the distance d. The layer width as well as the tunneling between the two layers will be neglected throughout the paper. This means that the spread of the wave functions of electrons and holes in the direction perpendicular to the layers is negligibly small. Such a situation can be achieved by applying a strong perpendicular electric field to the coupled double quantum well systems [8]. The magnetic field direction is perpendicular to the layers and the magnetic field H is assumed to be sufficiently strong enough that the following inequality holds,

$$r_H \ll a_{e,h}, \tag{1}$$

where $a_{e,h} = \varepsilon/m_{e,h}e^2$ are the effective Bohr radii of an electron and a hole, $m_{e,h}$ and ε are the effective masses at $H = 0$ and the dielectric constant, respectively. In such a magnetic field regime the virtual transitions of particles between the Landau levels both in the impurity field and due to interparticle interactions are unimportant. For simplicity, it will be assumed below that electrons and holes occupy only the zeroth Landau levels. We will consider bound states of interwell excitons on the Coulomb impurities located in the barrier and in the layers. To find the binding energies of such states the technique developed in Ref. [12] will be used.

The Hamiltonian of the system can be represented in the following form,

$$H = H_{0e} + H_{0h} + V_1(|\boldsymbol{r}_1|) + V_2(|\boldsymbol{r}_2|) + U(|\boldsymbol{r}_1 - \boldsymbol{r}_2|), \tag{2}$$

where $\boldsymbol{r}_1$ ($\boldsymbol{r}_2$) denotes 2D coordinates of e (h). $H_{0e} + H_{0h}$ is the Hamiltonian of two noninteracting two-dimensional particles with opposite charges in a magnetic field and $U, V_{1,2}$ are the Coulomb potentials of the interparticle interaction and the interaction with the impurity. We suppose that the positive Coulomb impurity is located within the barrier at the distance d_l from the electron layer. We have the following expressions for $U, V_{1,2}$,

$$U(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) = -e^2/\varepsilon\sqrt{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2 + d^2},$$

$$V_1(|\boldsymbol{r}_1|) = -e^2/\varepsilon\sqrt{(\boldsymbol{r}_1)^2 + d_l^2},$$

$$V_2(|\boldsymbol{r}_2|) = e^2/\varepsilon\sqrt{(\boldsymbol{r}_2)^2 + (d - d_l)^2}.$$

In what follows we will use the cylindrical coordinate system (ρ, z, ϕ) with the z axis directed along the magnetic field and the symmetric gauge,

$$\boldsymbol{A} = \tfrac{1}{2}\boldsymbol{H} \times \boldsymbol{r}. \tag{3}$$

The eigenfunctions of H_{0e} with definite projection of the angular momentum $-m$ on the magnetic field direction can be written as (at the zeroth Landau level),

$$\Phi_m(\rho, \varphi) = \frac{1}{\sqrt{2\pi r_H^2 2^m m! r_H^m}} \mathrm{e}^{-im\varphi} \exp\left(-\frac{\rho^2}{4r_H^2}\right). \tag{4}$$

The corresponding eigenfunction for the hole (with the angular momentum projection m) can be obtained by the complex conjugate of (4).

The impurity bound states of a 2D exciton in a magnetic field can be described by the exact quantum number - the total generalized momentum M. Since e and h rotate in opposite directions in the magnetic field and therefore possess angular momenta of opposite signs, for given M there are a number of states for which $m_h - m_e = M$. The wave function of the interwell exciton bound to the positive Coulomb impurity with $M \geqslant 0$ can be represented in the form

$$\Psi_M(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_{m=0}^{\infty} A_m(m) \Phi^{(e)}_{-m}(\boldsymbol{r}_1) \Phi^{(h)}_{M+m}(\boldsymbol{r}_2). \quad (5)$$

The coefficients A_m and the eigenvalues should be determined by solving the corresponding secular equation. Here we introduce the useful notation

$$|m;\ n\rangle \equiv \Phi^{(e)}_m(\boldsymbol{r}_1) \Phi^{(h)}_n(\boldsymbol{r}_2). \quad (6)$$

The matrix elements of the Hamiltonian (2) with respect to the wave functions (6) have the form

$$\langle n'; m'|H|m; n\rangle = \delta_{n'-n, m'-m} U_{mn}(|n'-n|) + \delta_{n',n}\delta_{m',m}(V_{1m} + V_{2n}), \quad (7)$$

where

$$V_{il} = \int V_i(|\boldsymbol{r}|) |\Phi_l(\boldsymbol{r})|^2 \, d^2\boldsymbol{r}, \quad (8)$$

and $i = 1, 2$. The pair interaction matrix elements for $l \geqslant 0$ take the form

$$U_{mn}(l) = \left(\frac{m!}{(m+l)!} \frac{n!}{(n+l)!} \right)^{1/2} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} U(\boldsymbol{q}) \times \exp(-q^2 r_H^2) \left(\frac{q^2 r_H^2}{2} \right)^l L^l_m \left(\frac{q^2 r_H^2}{2} \right) L^l_n \left(\frac{q^2 r_H^2}{2} \right). \quad (9)$$

Here L^l_n is the Laguerre polynomial [16] and $U(\boldsymbol{q})$ is the Fourier transform of the e-h interaction potential. For $U(\boldsymbol{q})$ one obtains

$$U(\boldsymbol{q}) = -\frac{2\pi e^2}{\varepsilon} \frac{1}{q} e^{-qd}. \quad (10)$$

We shall further use as unit of length the cyclotron radius r_H and express all energies in units of the energy of the free exciton with zero momentum. The latter can be written in the following form,

$$E_d = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} U(\boldsymbol{q}) \exp(-q^2/2). \quad (11)$$

Note that this energy depends on the interlayer separation d.

As was shown in Ref. [12] for the e-h system in a quantum well in a strong perpendicular magnetic field all matrix elements of Hamiltonian (2) decrease very fast with increasing m, n. The origin of such behavior is that with increasing m, n the separations from the impurity and between the particles also increase. In particular, the radius of the cyclotron orbit is

$$\langle r^2 \rangle_m = 2m + 1 \quad (12)$$

and for the 2D exciton bound to the positive impurity we have

$$\langle r_h^2 - r_e^2 \rangle_M = 2M. \quad (13)$$

So with increasing M the hole tends to be more in the outer orbitals. The limit $M = \infty$ corresponds to the electron bound to the impurity with $m = 0$ and the hole going to infinity and becoming free. It is interesting to note that in the case of a quantum well in a strong magnetic field the energy of the impurity bound electron with $m = 0$ is equal to the binding energy of the zero-momentum exciton [17]. Also, as one can see from expression (13) the states with $M < 0$ correspond to a hole closer to the impurity than the electron when the impurity is located in the middle of the barrier (here we consider only this case). The energy of such states depends strongly on the doping location and lies above the energy of the $M = 0$ state.

The decreasing behavior of the matrix elements allows us to consider only the N first terms in the sum (5) for sufficiently large N. In our calculations we take into account the first 20 terms in (5). The energies of the bound states are obtained by the numerical diagonalization of 20×20 matrices corresponding to the secular equation where $|0; M\rangle, \ldots, |19; M+19\rangle$ states are taken into account. The accuracy of these calculations is better than $10^{-3} E_d$ for the lowest level at fixed M and for $d < 10 r_H$. Note that the binding

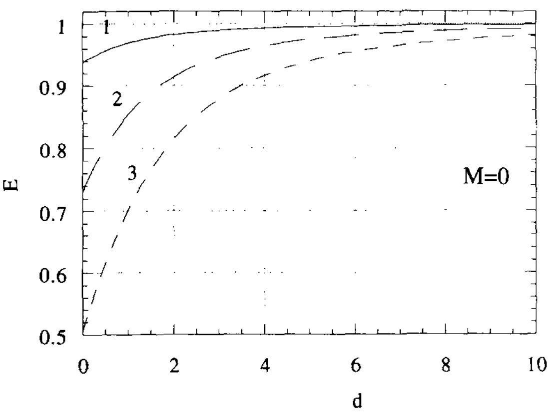

Fig. 1. The evolution of the three lowest levels for $M = 0$ with the increasing of the interlayer separation d for the case $d_l = d/2$. The binding energy and the interlayer separation are expressed in units of E_d and r_H, respectively.

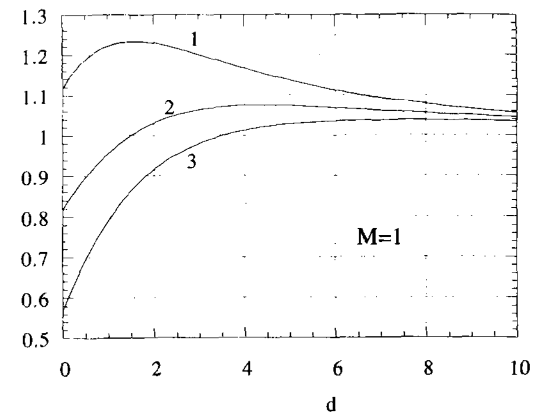

Fig. 2. The evolution of the three lowest levels for $M = 1$ with the increasing of the interlayer separation d for the case of $d_l = d/2$. The binding energy and the interlayer separation are expressed in units of E_d and r_H, respectively.

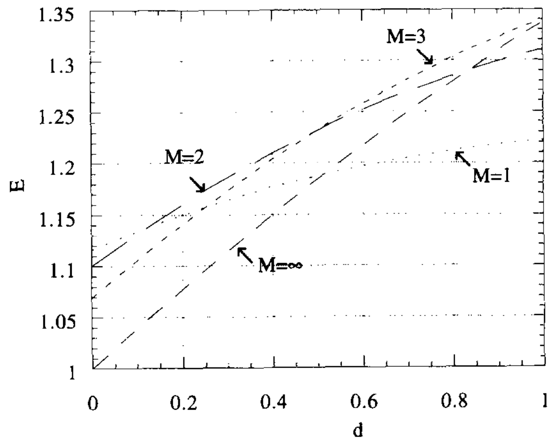

Fig. 3. The dependence of the binding energies of lowest levels for $M = 1, 2, 3, \infty$ states on the interlayer separation d for the case of $d_l = d/2$. The binding energy and the interlayer separation are expressed in units of E_d and r_H, respectively.

energies will only increase within the accepted accuracy with allowance of more orbital mixing in (5).

The evolution of the $M = 0$ and $M = 1$ states (the energies with opposite sign) with increasing interlayer separation d are shown in Fig. 1 and Fig. 2, respectively, for the case of an impurity located in the middle of barrier ($d_l = d/2$). The numbering 1, 2, 3 on the figures correspond to the three lowest levels (0, 1, 2 in Ref. [12]). The impurity bound exciton energies for $d = 0$ coincide with the results obtained in Ref. [12]. As in the case of a 2D MX in a quantum well the $M = 0$ state lies within the free magnetoexciton band. The energy of this state strongly depends on the position of the impurity in the barrier. This is due to the fact that the electron- and hole-impurity interactions are canceled in all matrix elements (7) only for the case $d_l = d/2$ when $V_{1m} = -V_{2m}$ (in Ref. [12] the interaction of the $M = 0$ MX with any axially symmetric external field is absent).

As it was shown by Dzyubenko [12] the ground state of the impurity bound 2D magnetoexciton corresponds to $M = 1$. This situation dramatically changes in a system with spatially separated e–h layers. With increasing the interlayer separation the bound states with higher momenta become more favorable. The dependence of the binding energy of the lowest levels at fixed M on the interlayer separation d is shown in Fig. 3 for $M = 1, 2, 3, \infty$ and $d_l = d/2$. As one can see from Fig. 3 the state $M = 1$ is the ground state only at the very small interlayer separation $d \leqslant 0.13$. For example, at $0.13 \leqslant d \leqslant 0.51$ the ground state is $M = 2$ and at $0.51 \leqslant d \leqslant 0.77$ the ground state is $M = 3$. With further increasing d the states with the larger M become ground states. This picture does not depend qualitatively on the position of the impurity in the barrier if the interlayer separation is not too large. Fig. 4 shows the behavior of $M = 1, 2, \infty$ states with changing the impurity position within the barrier at $d = 0.5$. One can see that the $M = 2$ state remains the ground state for all d_l.

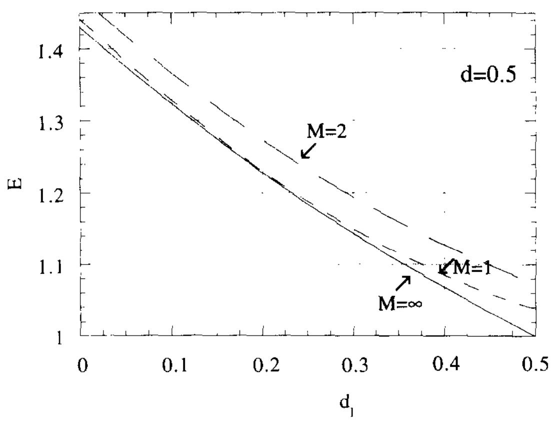

Fig. 4. The dependence of the lowest state binding energies on the position of impurity within the barrier for $M = 1, 2, \infty$ states at $d = 0.5$. The binding energy and the interlayer separation are expressed in units of $E_{0.5}$ and r_H, respectively.

All the ground states (at different interlayer separation) are mostly superpositions of the wave functions $|0; M\rangle, |1; M+1\rangle, |2; M+2\rangle$, in agreement with Ref. [12]. For example, the wave function for the ground state $M = 2$ at $d = 0.3$ ($d_l = d/2$) can be represented in the form

$$\Psi_2 \approx 0.954|0;2\rangle + 0.293|1;3\rangle + 0.17|2;4\rangle + 0.112|3;5\rangle + \ldots \qquad (14)$$

(the sum of the squares of coefficients is 1.037) and the ground state wave function for $d = 1$ ($d_l = d/2$) is (the sum of the squares of coefficients is 1.009)

$$\Psi_5 \approx 0.98|0;5\rangle + 0.2|1;6\rangle + 0.084|2;7\rangle + 0.043|3;8\rangle + \ldots . \qquad (15)$$

As one can see from (14), (15) with increasing the interlayer separation the contribution of the functions $|0; M\rangle, |1; M+1\rangle, |2; M+2\rangle$ to Ψ increases too.

3. Conclusion

In conclusion, we have found the energies of low-lying states of the interwell magnetoexciton bound to the impurities in the strong magnetic field limit for different positions of the impurity and different interlayer separations d. Contrary to the case considered in Ref. [12] the ground state of the interwell magnetoexciton bound to the impurity is no longer the $M = 1$ state and it depends on the interlayer separation. The $M = 1$ state is the ground state only at very small d ($d \leqslant 0.13$). With increasing the interlayer separation the bound states with higher momentum become more favorable. The dependence of the energy of bound states on the position of the impurity between two layers is strong, but the ground state remains the same (with the same M) for all impurity locations, if $d \leqslant 1$. For $d > 1$ the dependence of the ground state on the impurity position is very weak.

The results obtained in this paper can be used for the ground state identification of the spatially separated 2D electron–hole system in a strong magnetic field with the impurities invariably present in the quantum wells and in the barriers. The ground state of such a system may correspond to the "freezing out" of excitons at the centers and free excitons which can interact with the latter creating many-particle complexes or droplets. The calculation of the energies of such complexes is now in progress. Note that the existence of biexcitons and many-particle complexes in such a system was predicted earlier [6] in the absence of impurities.

Another subject of interest is the magneto-optics of shallow-impurity states in semiconductor quantum wells. Although the results of this paper are not directly applicable to the coupled double quantum well systems in strong perpendicular electric and magnetic fields, they can be used for the qualitative description of impurity bound magnetic excitons in such systems. For a quantitative description the mixing between the Landau levels, the finite size of the wells and the motion perpendicular to the quantum wells must be taken into account. This problem will be considered in a separate paper.

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