Gauge-invariant Green functions of Dirac fermions coupled to gauge fields

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(Received 12 December 2001; published 7 June 2002)

We develop a unified approach to both infrared and ultraviolet asymptotics of the fermion Green functions in the condensed-matter systems that allow for an effective description in the framework of quantum electrodynamics. By applying a path-integral representation to the previously suggested form of the physical electron propagator we demonstrate that in the massless case this gauge-invariant function features a "stronger-than-a-pole" branch-cut singularity instead of the conjectured Luttinger-like behavior. The obtained results alert one to the possibility that construction of physically relevant amplitudes in the effective gauge theories might prove more complex than previously thought.

DOI: 10.1103/PhysRevB.65.235111 PACS number(s): 71.27.+a

I. INTRODUCTION

In a generic many-body fermion system, a repulsive electron-electron interaction is normally expected to result in a suppression of any amplitude which describes propagation of fermionic quasiparticles. For instance, in the phenomenological Fermi-liquid theory, the residue of the electron Green function $G(\epsilon,\boldsymbol{p})=Z(\epsilon)/[\epsilon-E(\boldsymbol{p})+\mu]$ gets reduced compared to the noninteracting value $[Z(\epsilon)=1]$, thus exhibiting a partial $[0<Z(0)<1]$ suppression of the simple pole which corresponds to the bare fermionic quasiparticles.

The question as to whether or not the repulsive fermion interactions can result in an even more severe, complete, destruction of the pole $[Z(0)=0]$ remains the subject of an ongoing debate. Such a behavior is well known to occur in the one-dimensional (1D) Luttinger and related models with short-ranged interactions, in which case the residue of the fermion Green function exhibits a characteristic algebraic decay $Z(p)\sim p^{\eta}$ as a function of the Lorentz-invariant momentum $p=\sqrt{-p^2}=(p^2-\omega^2)^{1/2}$ and is controlled by an anomalous dimension $\eta>0$.

In the 1D coordinate space, this behavior corresponds to the suppression of the electron propagator $G(t,x) \sim\Sigma_{\pm}\exp(\pm ik_F x)/|x\pm t|^{1+\eta}$, which at long times and distances decays faster than the noninteracting one $(\eta=0)$. In the absence of spin, the above Green function is Lorentz invariant, apart from the oscillating factors $\exp(\pm ik_F x)$ that stem from a finite $(2k_F)$ separation between the two 1D Fermi points, in accordance with the fact that the low-energy excitations $\psi_{R,L}$ confined to the vicinity of the Fermi points constitute one Dirac fermion $\Psi=(\psi_R,\psi_L)$.

The marked difference between this, so-called Luttinger, behavior and the Fermi-liquid one prompts fundamentally important questions pertaining to the possibility of a similar behavior in $D>1$ and/or the presence of long-ranged electron-electron interactions. While in the case of the short-ranged interactions the possibility of the $D>1$ Luttinger-like behavior is likely to be limited to the infinitely strong coupling limit, the long-ranged forces appear to be capable of destroying the Fermi liquid even at finite couplings. As the best studied example of this kind, the model of degenerate nonrelativistic massive fermions $(T\ll\mu\ll mc^2)$ which are minimally coupled to an Abelian gauge field was found to have a distinctly non-Fermi-liquid behavior,[1] although the latter appears to be quite different from the Luttinger one.[2]

More recently, there has been an upsurge of interest in the relativistic counterpart of this model, which is a zero-density $(\mu=0)$ system of the N-flavored relativistic Dirac fermions coupled to an Abelian gauge field which is described by the standard action of quantum electrodynamics (QED),

$$S[\Psi,\overline{\Psi},\mathbf{A}]=\int d\mathbf{x}\Bigg[\sum_{f=1}^{N}\overline{\Psi}_f(i\gamma_\mu\partial_\mu+\gamma_\mu A_\mu-m)\Psi_f + \frac{1}{4g^2}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2\Bigg], \qquad (1)$$

where, for the sake of completeness, we also included a finite fermion mass m.

Among the previously discussed examples of the 2D condensed-matter systems that support the Dirac-like low-energy excitations and allow for such an effective description are the so-called flux phase in the planar quantum disordered magnets[3,4] and the layered disordered d-wave superconductors with strong phase fluctuations proposed as an explanation of the pseudogap[5,6] and insulating[7] (spin-density wave) phases of high-T_c cuprates. Also, the non-Lorentz-invariant version of QED_{2+1} was shown to provide a convenient description of the normal semimetalic state of highly oriented pyrolytic graphite.[8,9]

The number of the fermion flavors N depends on the problem in question, although it is not necessarily equal to the number of different conical Dirac points in the bare electron dispersion of a lattice system. In all of the previously discussed 2D examples,[3–9] $N=2$ is a number of the electron-spin components, while the number of conical points turns out to be either two[8,9] or four,[4–6] which merely forces one to use the four-component Dirac fermions and the corresponding (reducible) representation of the γ matrices $\gamma_\mu=\sigma_\mu\otimes\sigma_3$ constructed from the triplet σ_μ of the Pauli matrices.

In the above-mentioned condensed-matter-related applications, the effective gauge fields serve as a somewhat exotic, yet often more convenient, representation of such bosonic collective excitations as spin or pairing fluctuations, while

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the Dirac fermions correspond to the auxiliary fermionic excitations such as, e.g., spinons,[3,4] "topological" fermions,[5–7] and so forth. Generically, the quantum-mechanical amplitudes describing such degrees of freedom turn out to be gauge dependent, while all the physical observables which experimental probes can only couple to must be manifestly gauge invariant.

Among such gauge-invariant amplitudes is the one containing a phase factor (sometimes referred to as a "gauge connector" or a "parallel transporter")

$$G_{inv}^{\Gamma}(x,y)=\left\langle \Psi(x)\exp\left(i\int_{\Gamma}A_{\mu}dz_{\mu}\right)\overline{\Psi}(y)\right\rangle, \qquad (2)$$

whose suggestive form makes it tempting to identify Eq. (2) with the physical electron Green function (in spite of its being gauge independent, the function G_{inv}^{Γ} explicitly depends on the choice of the contour Γ).

To this end, it was conjectured[4] that by analogy with the problem of the compressible quantum Hall effect described by yet another kind of 2D auxiliary (this time, nonrelativistic) fermionic quasiparticles, the so-called composite fermions, interacting with the statistical Chern-Simons field,[12] the electron Green function is given by Eq. (2) with the contour Γ chosen as a straight line from the end point **x** to **y**.

Furthermore, it was argued in Ref. 4 that in the case $m=0$ and at energies and momenta which are small as compared to the bandwidth and the inverse lattice spacing, respectively, the gauge-invariant amplitude (2) features the Luttinger-like behavior with a positive exponent η (hereafter we use notations $\mathbf{q}\cdot\mathbf{p}=q_{\mu}p_{\mu}$ and $\hat{p}=\gamma_{\mu}p_{\mu}$),

$$G_{inv}^{|}(p)\sim\hat{p}/p^{2-\eta}, \qquad (3)$$

which was also invoked in Refs. 4 and 6 to explain the experimental data on angular-resolved photoemission spectra (ARPES) in high-T_c cuprates.[13]

In the general case of a D-dimensional condensed-matter system which possesses a number of isolated Fermi points located at $\mathbf{k}_{Fi}$, the conjectured behavior (3) corresponds to the algebraic suppression of the electron propagator at long times and distances,

$$G_{inv}^{|}(x)\sim\sum_{i} e^{i\mathbf{k}_{Fi}\cdot\mathbf{x}}\frac{\hat{x}}{x^{D+1+\eta}}, \qquad (4)$$

where the sum is taken over all the Fermi points.

In the present paper, we employ a functional-integral technique to compute the function (3) and discern the true nature of its singular behavior (if any). This approach, which had been pioneered by Schwinger and later advanced by a number of other authors (see, e.g., Refs. 10 and 11 and references therein), exploits a functional-integral representation of the exact solution of the equation for $G_{inv}^{|}(x,y|\mathbf{A})$ as a functional of an arbitrary configuration of the gauge field $\mathbf{A}(z)$. Subsequently, by averaging over the gauge field, one obtains a sum of all the multiloop diagrams with no couplings between the fermion polarization insertions into the gauge-field propagators, and the open fermion line corresponding to the fermion's propagation between the space-time points **x** and **y**. Likewise, in the case of a generic multifermion amplitude, the allowed graphs can only contain open fermion lines which connect the incoming and outgoing asymptotical fermionic states, provided that the fermion polarization has already been absorbed into the gauge-field propagator.

This approach can be viewed as a systematic improvement of the celebrated Bloch-Nordsieck model, where all the spin-related effects are ignored, which makes this model exactly solvable but restricts its applicability to the infrared (IR) regime $|p^2-m^2|\ll m^2$ near the fermion's mass shell.

We emphasize that the IR regime can only exist if the fermions are massive, while in the massless case the entire region below the upper cutoff Λ (which is set by the conditions of the applicability of the effective QED-like description itself) falls into the opposite, ultraviolet (UV), regime which, in the case of a finite fermion mass, is defined as $|p^2-m^2|\gg m^2$.

The rest of the paper is organized as follows. We first describe Schwinger's functional technique and investigate both the IR and UV asymptotics of the ordinary (gauge-dependent) fermion Green function in the general D-dimensional case. Then, after having compared our general formulas with the well-known 3D results as well as with the partially known 2D ones, we proceed with the gauge-invariant fermion amplitude proposed in Ref. 4 and ascertain its true behavior. We conclude our analysis with a discussion of the alternatives to the previously suggested form of the physical electron propagator as well as to the fits to the ARPES data[13] exploiting the QED_{2+1}-related scenarios.

II. FUNCTIONAL-INTEGRAL REPRESENTATION OF FERMION AMPLITUDES

The conventional fermion Green function is given by the (properly normalized) functional integral over the fermion and gauge-field configurations,

$$G(x,y)=\langle\Psi(x)\overline{\Psi}(y)\rangle =\int D[\overline{\Psi}]D[\Psi]D[\mathbf{A}]\Psi(x)\overline{\Psi}(y)\exp(iS[\Psi,\overline{\Psi},\mathbf{A}]). \qquad (5)$$

Upon integrating the fermions out, one arrives at the expression

$$G(x,y)=\int D[\mathbf{A}]G(x,y|\mathbf{A})\exp(iS_{eff}[\mathbf{A}]), \qquad (6)$$

where the effective action of the gauge field includes the fermion polarization

$$S_{eff}[\mathbf{A}]=\frac{1}{4g^2}\int d\mathbf{x}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+\ln\frac{\det[i\hat{\partial}+\hat{A}-m]}{\det[i\hat{\partial}-m]}$$

$$=\frac{1}{2}\int d\mathbf{x}\int d\mathbf{y}A_\mu(x)\mathcal{D}^{-1}_{\mu\nu}(x-y)A_\nu(y)+\cdots. \quad (7)$$

By neglecting all but the Gaussian term in Eq. (7) one excludes from consideration any processes of "light-light scattering" and alike. Thus far, none of the aforementioned effective QED-like descriptions of the condensed-matter systems has gone anywhere beyond this common approximation.

Nonetheless, the gauge-field is not completely quenched, as one still accounts for the quadratic polarization $\Pi(q)$, resulting in the gauge-field propagator, which, in the covariant λ gauge, assumes the form

$$\mathcal{D}_{\mu\nu}(q)=\frac{g^2}{q^2+\Pi(q)}\left[\delta_{\mu\nu}+(\lambda-1)\frac{q_\mu q_\nu}{q^2}\right]. \quad (8)$$

In turn, the fermion Green function $G(x,y|\mathbf{A})$ computed for a given gauge-field configuration obeys the equation

$$[i\hat{\partial}+\hat{A}(x)-m]G(x,y|\mathbf{A})=\delta(\mathbf{x}-\mathbf{y}). \quad (9)$$

Its formal solution can be written in the form of a quantum-mechanical (i.e., *single-particle*) path integral[10]

$$G(x,y|\mathbf{A})=-i\int_0^\infty ds e^{is(-m^2+i\delta)}[i\hat{\partial}+\hat{A}(x)+m]$$
$$\times\int D[\mathbf{a}]\delta\left(\mathbf{x}-\mathbf{y}-2\int_0^s\mathbf{a}(\tau_2)d\tau_2\right)$$
$$\times\exp\left[-i\int_0^s d\tau\left\{\left(\mathbf{a}^2(\tau)-[2a_\mu(\tau)+\sigma_{\mu\nu}i\partial_\nu]\right.\right.\right.$$
$$\left.\left.\left.\times A_\mu\left(\mathbf{x}-2\int_\tau^s\mathbf{a}(\tau_1)d\tau_1\right)\right)\right\}\right], \quad (10)$$

where $\sigma_{\mu\nu}=[\gamma_\mu,\gamma_\nu]/2$ and $\delta\to 0^+$. The integral over the fermion's momentum $\mathbf{a}(s)$ as a function of the proper time s parametrizing its space-time trajectory is normalized in such a way that

$$\int D[\mathbf{a}]\exp\left[-i\int_0^s\mathbf{a}^2(\tau)\right]d\tau=1.$$

Next, we perform functional averaging over different gauge-field configurations with the use of Eq. (7), then Fourier transform Eq. (10) to the momentum representation, and finally switch to the integration over the fluctuating part of the total fermion momentum $\mathbf{v}(s)=\mathbf{a}(s)-\mathbf{p}$, thus obtaining

$$G(p)=-i\int_0^\infty ds e^{is(p^2-m^2+i\delta)}\int D[\mathbf{v}]\exp\left[-i\int_0^s\mathbf{v}^2(\tau)d\tau\right]$$
$$\times[\hat{p}+m+M(s|\mathbf{v})]\exp[i\Phi(s|\mathbf{v})]. \quad (11)$$

In this expression, the terms which are odd in $\mathbf{A}(z)$ contribute to the gauge-invariant (see below) part of the mass operator

$$M(s|\mathbf{v})=\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\int_0^s d\tau\gamma_\mu[2v_\nu(\tau)+2p_\nu$$
$$-\sigma_{\nu\lambda}q_\lambda]\exp\left[2i\mathbf{p}\cdot\mathbf{q}(s-\tau)+2i\int_\tau^s\mathbf{q}\cdot\mathbf{v}(\tau_1)d\tau_1\right], \quad (12)$$

while the even ones stem from the exponential of the (gauge-dependent) "phase factor,"

$$\Phi(s|\mathbf{v})=\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\int_0^s d\tau_1\int_0^{\tau_1}d\tau_2[2v_\mu(\tau_1)+2p_\mu$$
$$+\sigma_{\mu\alpha}q_\alpha][2v_\nu(\tau_2)+2p_\nu-\sigma_{\nu\beta}q_\beta]$$
$$\times\exp\left[2i\mathbf{p}\cdot\mathbf{q}(\tau_1-\tau_2)+2i\int_{\tau_2}^{\tau_1}\mathbf{q}\cdot\mathbf{v}(\tau_3)d\tau_3\right]. \quad (13)$$

In the above expressions, the integrations over the proper time parameters τ_i are ordered according to the order of their appearance in the products of the noncommutative factors $[2v_\mu(\tau_i)+2p_\mu\pm\sigma_{\mu\nu}q_\nu]$.

III. INFRARED BEHAVIOR

By using Eqs. (11)–(13) one can readily determine the IR behavior of the fermion Green function. With its momentum satisfying the condition $|p^2-m^2|\ll m^2$ a fermion behaves as a heavy particle whose velocity remains essentially unchanged after emitting and absorbing an arbitrary number of gauge-field quanta. Therefore, the Green function receives its main contribution from the fermion trajectories close to the straight-line path [which only coincides with the semiclassical trajectory in the case of a timelike separation between the end points $(x-y)^2>0$].

This allows one to neglect the fluctuations of the total fermion's momentum with respect to its average value $\mathbf{p}$, in which case the mass operator introduces only a small correction

$$M_{IR}(s|\mathbf{v})=i\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\gamma_\mu p_\nu\frac{1-e^{2i\mathbf{q}\cdot\mathbf{p}s}}{\mathbf{q}\cdot\mathbf{p}}$$
$$=\hat{p}O\left(\frac{1}{sp^2}\right)$$
$$\sim\hat{p}\frac{|p^2-m^2|}{m^2}\ll\hat{p}. \quad (14)$$

In deriving Eq. (14) we took into account that a characteristic value of the parameter $s\sim|p^2-m^2|^{-1}$ is determined by Eq. (11) and the fact that the integral (14) receives its main contribution from small transferred momenta $q\lesssim 1/sp\sim|p^2-m^2|/p\ll p$.

In contrast, the integrals over τ_i in the gauge-dependent IR phase factor are formally divergent. They must be tackled by first computing the momentum integral and then applying the so-called "ribbon" regularization[11] $\mathbf{p}(\tau_1-\tau_2)\rightarrow\mathbf{p}(\tau_1-\tau_2)+\mathbf{l}$ with $(\mathbf{p}\cdot\mathbf{l})=0$ and $|\mathbf{l}|=1/\Lambda$, which yields the expression

$$\Phi_{IR}(s)=4\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\int_0^s d\tau_1\times\int_0^{\tau_1}d\tau_2 p_\mu p_\nu e^{2i\mathbf{p}\cdot\mathbf{q}(\tau_1-\tau_2)}$$
$$=ig^2 I_D\int_0^s d\tau_1\int_0^{\tau_1}d\tau_2\left[(D-2+\lambda)\frac{p^2}{|\mathbf{p}(\tau_1-\tau_2)+\mathbf{l}|^2}-2(\lambda-1)\frac{p^4(\tau_1-\tau_2)^2}{|\mathbf{p}(\tau_1-\tau_2)+\mathbf{l}|^4}\right]$$
$$=ig^2 I_D\left[(D-2+\lambda)\left(\frac{\pi}{2}(sp\Lambda)-\ln(sp\Lambda)\right)-2(\lambda-1)\left(\frac{\pi}{4}(sp\Lambda)-\ln(sp\Lambda)\right)\right]. \tag{15}$$

In the massive case, the linear divergence of $\Phi(s)$ would be routinely attributed to the renormalization of the bare mass $m\rightarrow m+O(\Lambda)$. After having separated this linear divergence, we observe that the subleading logarithmic terms conspire to give rise to the nonperturbative formula

$$G_{IR}(p)=-i(\hat{p}+m)\int_0^\infty ds e^{is(p^2-m^2+i\delta)}(sp\Lambda)^{-\eta_{IR}/2}\sim\frac{\hat{p}+m}{(p^2-m^2+i\delta)^{1-\eta_{IR}/2}}, \tag{16}$$

which, near the mass shell, exhibits the anticipated algebraic behavior (3) with the IR anomalous dimension

$$\eta_{IR}=2g^2 I_D(\lambda-D), \tag{17}$$

where

$$I_D=[2^D\pi^{(D+1)/2}\Gamma(\{D+1\}/2)]^{-1}. \tag{18}$$

Thus, in the 3D case of the conventional weakly coupled QED_{3+1}, we recover the well-known IR exponent (see, e.g., Ref. 14)

$$\eta_{IR}^{3D}=\frac{e^2}{4\pi^2}(\lambda-3), \tag{19}$$

which vanishes in the so-called Yennie's gauge $\lambda=3$ (η_{IR} is also known to be zero in some noncovariant gauge, such as the Coulomb gauge $\mathbf{q}\cdot\mathbf{A}=0$).

In the (parity-even) 2D case, which is of a particular interest in view of its condensed-matter-related applications,[3–9] the weak-coupling regime turns out to be intrinsically unstable against the effects of the fermion polarization. In fact, for $q\lesssim Ng^2$ the gauge propagator is totally dominated by the fermion polarization, which, for $N\gg1$, is given by the one-loop term

$$\Pi(q)=\frac{Ng^2}{8}\sqrt{-q^2}, \tag{20}$$

and the gauge-field propagator reads as

$$\mathcal{D}^{2D}_{\mu\nu}(q)=\frac{8}{N\sqrt{-q^2}}\left(\delta_{\mu\nu}+(\lambda-1)\frac{q_\mu q_\nu}{q^2}\right). \tag{21}$$

Instead of the bare coupling g, it is $1/N$ that now becomes a parameter of the perturbative expansion. We note that above the momentum scale Ng^2 no further logarithmic corrections are generated, so that the latter is now playing the role of the UV cutoff. Nonetheless, for the sake of uniformity of our presentation, in the following discussion we will continue using the notation Λ and the label UV for the range of momenta $m\ll q\lesssim\Lambda=Ng^2$.

It is also worth mentioning that, owing to the parity conserving structure of the reducible four-fermion representation, the radiative corrections generate no Chern-Simons terms.

Using Eq. (21) we obtain a coupling-independent anomalous exponent

$$\eta_{IR}^{2D}=\frac{8}{\pi^2 N}(\lambda-2), \tag{22}$$

thus discovering the 2D analog $(\lambda=2)$ of the 3D Yennie's gauge.

Notably, the IR wave-function renormalization assumes the anticipated power-law form, in full accord with the physical origin of the IR singularity. The latter is known to stem from the processes involving independent emission and absorption of an arbitrary large number of soft gauge quanta. Due to their uncorrelated nature, these multiple "bremsstrahlung" events obey a Poisson distribution formula, hence the appearance of the factorials in the statistical weights, resulting in the natural exponentiation of the lowest-order ($\sim g^2\ln\Lambda$) correction.

IV. ULTRAVIOLET BEHAVIOR

Schwinger's functional technique is also capable of exploring the UV regime ($|p^2-m^2|\gg m^2$), which is the only regime of interest present in the massless case. Despite the fact that the procedure is straightforward, there seems to have been no such systematic attempt made in the past.

Technically, the UV behavior is more difficult to analyze, because the path integral (11) is no longer saturated by the trajectories close to the semiclassical straight line. In fact, the relevant paths can strongly deviate from the straight-line one, for they suffer no exponential suppression, unlike in the IR regime.

Despite the fact that the functional integration over $\mathbf{v}(s)$

can no longer be carried out exactly, one can instead resort to the formula

$$\int D[\mathbf{v}]\exp\left(-i\int \mathbf{v}^2 d\tau + F[\mathbf{v}]\right) = e^{\langle F\rangle}\int D[\mathbf{v}]\exp\left(-i\int \mathbf{v}^2 d\tau\right)\sum_{n=0}^{\infty}\frac{(F[\mathbf{v}]-\langle F\rangle)^2}{n!}, \tag{23}$$

where $\langle F\rangle = \int D[\mathbf{v}]\exp(-i\int \mathbf{v}^2 d\tau)F[\mathbf{v}]$.

Equation (23) has been extensively used, e.g., in implementing Feynman's variational principle in the polaron and related problems. Expanding Eq. (11) to the first order in $\mathcal{D}_{\mu\nu}(q)$ we obtain

$$\delta_1 G_{UV}(p) = -i\int_0^\infty ds e^{is(p^2+i\delta)}[\langle M_{UV}(s)\rangle + i\hat{p}\langle \Phi_{UV}(s)\rangle]. \tag{24}$$

The functionally averaged mass operator (12) is now determined by the transferred momenta $q \gg p \sim 1/\sqrt{s}$ and it needs to be computed only to the first order in $\mathbf{p}$,

$$\langle M_{UV}(s)\rangle = i\int \frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}}{q^2+2\mathbf{q}\cdot\mathbf{p}} \times \gamma_\mu(q_\nu + 2p_\nu - \sigma_{\mu\lambda}q_\lambda) = 2g^2\hat{p}I_D\frac{D}{D+1}\ln(s\Lambda^2)+\cdots. \tag{25}$$

Notably, Eq. (25) is independent of the gauge parameter. In contrast, the averaged phase factor (13), which can be calculated in the $\mathbf{p}\to 0$ limit,

$$\langle \Phi_{UV}(s)\rangle = \int \frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q) \times\left[\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}+is(q^2+2\mathbf{q}\cdot\mathbf{p})}{(q^2+2\mathbf{q}\cdot\mathbf{p})^2}(q_\mu+2p_\mu + \sigma_{\mu\alpha}q_\alpha)(q_\nu+2p_\nu-\sigma_{\nu\beta}q_\beta) - is\,\delta_{\mu\nu}\right] = \frac{i}{2}g^2 I_D(D+\lambda)\ln(s\Lambda^2)+\cdots, \tag{26}$$

does manifest a dependence on the gauge parameter. Combining Eqs. (25) and (26) together, we obtain the total correction to the Green function,

$$\delta_1 G_{UV}(p) = \int_0^\infty ds e^{is(p^2+i\delta)}\int \frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q) \times\left[\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}}{q^2+2\mathbf{q}\cdot\mathbf{p}}\gamma_\mu(q_\nu+2p_\nu-\sigma_{\nu\lambda}q_\lambda) + \hat{p}\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}+is(q^2+2\mathbf{q}\cdot\mathbf{p})}{(q^2+2\mathbf{q}\cdot\mathbf{p})^2}(q_\mu+2p_\mu + \sigma_{\mu\alpha}q_\alpha)(q_\nu+2p_\nu-\sigma_{\nu\beta}q_\beta) - i\hat{p}s\,\delta_{\mu\nu}\right] = \frac{g^2}{2}\frac{\hat{p}}{p^2}I_D\left[\frac{D(3-D)}{D+1}-\lambda\right]\ln\left(\frac{\Lambda^2}{p^2}\right)+\cdots. \tag{27}$$

By using the identity

$$\hat{p}\gamma_\mu(\hat{p}+\hat{q})\gamma_\nu\hat{p} = \hat{p}(q_\mu+2p_\mu+\sigma_{\mu\alpha}q_\alpha)(q_\nu+2p_\nu-\sigma_{\nu\beta}q_\beta) - \gamma_\mu p^2(q_\nu+2p_\nu-\sigma_{\nu\lambda}q_\lambda) - \delta_{\mu\nu}\hat{p}(p+q)^2$$

and integrating in Eq. (27) over the proper time s prior to the momentum integration, one can also check that the correction given by Eq. (27) exactly reproduces the one-loop result of the conventional diagrammatic expansion

$$\delta_1 G_{UV}(p) = -i\int \frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{\mathcal{D}_{\mu\nu}(q)}{p^4(p+q)^2}\hat{p}\gamma_\mu(\hat{p}+\hat{q})\gamma_\nu\hat{p}. \tag{28}$$

Instead of expanding Eq. (11) to higher orders in $\mathcal{D}_{\mu\nu}(q)$ one can perform a summation of the leading $(g^2\ln\Lambda)^n$ terms by virtue of the standard renormalization-group equation, which reflects the scaling properties of a generic two-point amplitude (gauge invariant and noninvariant alike) under the change of the upper cutoff,[14]

$$\left[\Lambda\frac{\partial}{\partial\Lambda} - \beta(\tilde{g})\frac{\partial}{\partial\tilde{g}} + \eta(\tilde{g})\right]\hat{p}G_{UV}(p;\Lambda;\tilde{g}) = 0, \tag{29}$$

where the leading-order dependence of the anomalous dimension of the fermion Green function on the renormalized coupling strength $\tilde{g}$ is given by the explicit form of the first-order correction (27),

$$\eta(g) = -\Lambda\frac{\partial}{\partial\Lambda}\hat{p}\,\delta_1 G_{UV}(p;\Lambda;g)\big|_{p=\Lambda}, \tag{30}$$

while $\beta(g) = \Lambda\partial\tilde{g}/\partial\Lambda|_{p=\Lambda} = 0$, and, therefore, the coupling strength retains it bare value $\tilde{g} = g$, for as long as the dynamics of the gauge field is considered quenched.

The solution of Eq. (29) suggests that the first logarithmic correction (27) merely gets exponentiated, thus yielding the algebraic behavior controlled by the UV exponent,

$$\eta_{UV} = g^2 I_D\left(\lambda + \frac{D(D-3)}{D+1}\right). \tag{31}$$

Further corrections to Eq. (31) require one to not only extract the subleading corrections of order $g^{2n}\ln\Lambda$ from the nth-order terms in the expansion of Eq. (11) in powers of $\mathbf{v}(s)$ and account for the improved fermion polarization $\Pi(q)$, but also to proceed beyond the quenched approximation (7) for the effective action of the gauge field.

In the weakly coupled 3D case, Eq. (31) reproduces the well-known result[14]

$$\eta_{UV}^{3D}=\frac{e^2}{8\pi^2}\lambda, \tag{32}$$

while in the 2D case it yields the coupling-independent UV exponent

$$\eta_{UV}^{2D}=\frac{4}{3\pi^2 N}(3\lambda-2), \tag{33}$$

in agreement with the result obtained in Ref. 15.

V. GAUGE-INVARIANT FERMION AMPLITUDE

After having tested our formalism against the known examples, we turn to the proposed candidate for the physical electron propagator, which is given by Eq. (2) with the straight-line contour Γ,

$$G_{inv}^{|}(x,y)=\int D[\mathbf{A}]G(x,y|A)\exp\left(-i\int_y^x dz^\mu A_\mu(z)\right)\times\exp(iS_{eff}[A]). \tag{34}$$

Proceeding by analogy with the derivation presented in Sec. II, one readily obtains Eq. (11), where Eqs. (12) and (13) are replaced, respectively, with

$$\begin{aligned}M_{inv}(s|\mathbf{v})=&\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\int_0^s d\tau\gamma_\mu\Bigg[\{2v_\nu(\tau)+2p_\nu\\&-\sigma_{\nu\lambda}q_\lambda\}\exp\left(2i\mathbf{p}\cdot\mathbf{q}(s-\tau)\right.\\&\left.+2i\int_\tau^s\mathbf{q}\cdot\mathbf{v}(\tau_1)d\tau_1\right)-[2v_\nu(\tau_2)+2p_\nu-\sigma_{\nu\lambda}q_\lambda]\\&\times\exp\left(2i\mathbf{p}\cdot\mathbf{q}\tau+2i\tau/s\int_0^s\mathbf{q}\cdot\mathbf{v}(\tau_1)d\tau_1\right)\int_0^s\frac{d\tau_2}{s}\Bigg]\end{aligned} \tag{35}$$

and

$$\begin{aligned}\Phi_{inv}(s|\mathbf{v})=&\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\int_0^s d\tau_1\int_0^{\tau_1}d\tau_2[2v_\mu(\tau_1)\\&+2p_\mu+\sigma_{\mu\alpha}q_\alpha][2v_\nu(\tau_2)+2p_\nu-\sigma_{\nu\beta}q_\beta]\\&\times\Bigg[\exp\left(2i\mathbf{p}\cdot\mathbf{q}(\tau_1-\tau_2)+2i\int_{\tau_2}^{\tau_1}\mathbf{q}\cdot\mathbf{v}(\tau_3)d\tau_3\right)\\&+2\int_0^s\frac{d\tau_3}{s}\int_0^{\tau_3}\frac{d\tau_4}{s}\exp\left(2i\mathbf{p}\cdot\mathbf{q}(\tau_3-\tau_4)\right.\\&\left.+2i(\tau_3-\tau_4)/s\int_0^s\mathbf{q}\cdot\mathbf{v}(\tau_5)d\tau_5\right)-2\int_0^s\frac{d\tau_3}{s}\\&\times\exp\left(2i\mathbf{p}\cdot\mathbf{q}(s-\tau_1)+2i\int_{\tau_1}^s\mathbf{q}\cdot\mathbf{v}(\tau_4)d\tau_4\right.\\&\left.-2i\mathbf{p}\cdot\mathbf{q}\tau_3-2i\tau_3/s\int_0^s\mathbf{q}\cdot\mathbf{v}(\tau_5)d\tau_5\right)\Bigg].\end{aligned} \tag{36}$$

In the IR regime the path integration can still be carried out exactly by simply neglecting $\mathbf{v}(s)$ with respect to the average fermion momentum $\mathbf{p}$. In the same approximation as that used in Sec. III (which is only justified in the vicinity of the mass shell, provided that $m\neq 0$), one readily obtains

$$\begin{aligned}M_{inv,IR}(s)=&2\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)\\&\times\int_0^s d\tau\gamma_\mu p_\nu[e^{2i\mathbf{p}\cdot\mathbf{q}(s-\tau)}-e^{2i\mathbf{p}\cdot\mathbf{q}\tau}]\\=&0\end{aligned} \tag{37}$$

and

$$\begin{aligned}\Phi_{inv,IR}(s)=&4\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\mathcal{D}_{\mu\nu}(q)p_\mu p_\nu\int_0^s d\tau_1\int_0^{\tau_1}d\tau_2\\&\times\Bigg[e^{2i\mathbf{p}\cdot\mathbf{q}(\tau_1-\tau_2)}+\frac{2}{s^2}\int_0^s d\tau_3\int_0^{\tau_3}d\tau_4 e^{2i\mathbf{p}\cdot\mathbf{q}(\tau_3-\tau_4)}\\&-\frac{2}{s}\int_0^s d\tau_3 e^{2i\mathbf{p}\cdot\mathbf{q}(s-\tau_1-\tau_3)}\Bigg]=0.\end{aligned} \tag{38}$$

Thus, as first pointed out by the authors of Refs. 10, in the IR regime the gauge-invariant propagator (34) retains a simple pole

$$G_{inv,IR}^{|}(p)\approx\frac{\hat{p}+m}{p^2-m^2+i\delta}, \tag{39}$$

hence, $\eta_{inv,IR}=0$.

By comparing Eqs. (39) and (17) one can also deduce the IR anomalous dimension of the exponential factor $\exp(i\int dx_\mu A_\mu)$ itself,

$$\eta_{exp,IR}=2g^2 I_D(D-\lambda), \tag{40}$$

which of course vanishes in Yennie's gauge.

Next, going over to the UV regime and expanding Eqs. (35) and (36) to the first order in $\mathcal{D}_{\mu\nu}(q)$ we arrive at Eq. (24), where the functional average of the gauge-dependent phase factor,

$$
\begin{aligned}
\langle\Phi_{inv,UV}(s)\rangle=&\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{g^2}{q^2+\Pi(q)}\\
&\times\Bigg[\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}+is(q^2+2\mathbf{q}\cdot\mathbf{p})}{(q^2+2\mathbf{q}\cdot\mathbf{p})^2}\\
&\times\left(\frac{p^2q^4}{(\mathbf{q}\cdot\mathbf{p})^2}-(\sigma_{\mu\nu}q_\nu)^2-q^2\right)-is\Bigg]\\
=&\,O(p^2s)\lesssim 1, \qquad (41)
\end{aligned}
$$

now exhibits neither linear nor logarithmic divergence as a function of s, unlike in the case of the noninvariant amplitude [see Eq. (26)]. In turn, the value of the mass operator

$$
\begin{aligned}
\langle M_{inv,UV}(s)\rangle=&\,2i\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{g^2}{q^2+\Pi(q)}\frac{1-e^{is(q^2+2\mathbf{q}\cdot\mathbf{p})}}{q^2+2\mathbf{q}\cdot\mathbf{p}}\\
&\times\left[\hat{p}-\hat{q}\frac{p^2}{\mathbf{q}\cdot\mathbf{p}}-\hat{q}\frac{p_\mu\sigma_{\mu\nu}q_\nu}{q^2}+\frac{\mathbf{q}\cdot\mathbf{p}}{q^2}\gamma_\mu\sigma_{\mu\nu}q_\nu\right]\\
=&\,2g^2\hat{p}I_D\frac{D}{D+1}\ln(s\Lambda^2)+\cdots \qquad (42)
\end{aligned}
$$

appears to coincide with Eq. (25). Thus, it is Eq. (42) that solely determines the correction to the gauge-invariant Green function,

$$
\begin{aligned}
\delta_1 G_{inv,UV}(p)=&-i\int_0^\infty ds e^{is(p^2+i\delta)}[\langle M_{inv,UV}(s)\rangle\\
&+i\hat{p}\langle\Phi_{inv,UV}(s)\rangle]\\
=&\,2g^2\frac{\hat{p}}{p^2}I_D\frac{D}{D+1}\ln\left(\frac{\Lambda^2}{p^2}\right). \qquad (43)
\end{aligned}
$$

The same result can be obtained by working in the axial gauge $\mathbf{n}\cdot\mathbf{A}=0$ defined by the vector $\mathbf{n}=(\mathbf{x}-\mathbf{y})/|x-y|$. In this gauge, the exponential factor in Eq. (34) is identically equal to unity, and the first-order correction is given by Eq. (28), where one has to use the gauge-field propagator

$$
\mathcal{D}^{ax}_{\mu\nu}(q)=\frac{g^2}{q^2+\Pi(q)}\left[\delta_{\mu\nu}+n^2\frac{q_\mu q_\nu}{(\mathbf{n}\cdot\mathbf{q})^2}-\frac{n_\mu q_\nu+q_\mu n_\nu}{(\mathbf{n}\cdot\mathbf{q})}\right]. \qquad (44)
$$

Notably, the result (43) obtained with the use of Eq. (28) is independent of the direction of the vector $\mathbf{n}$, for all the terms proportional to $\hat{n}(\mathbf{n}\cdot\mathbf{p})$ cancel out and only those proportional to $\hat{p}$ remain in the final expression.

It is worth mentioning that the integrals in Eqs. (41) and (42) as well as in Eq. (28) with the gauge propagator (44) are all plagued with spurious poles, such as $1/(\mathbf{q}\cdot\mathbf{p})^{1,2}$. We handle these singular denominators by resorting to the exponential integral representation: $1/(\mathbf{q}\cdot\mathbf{n})=-i\int_0^\infty ds\,\exp[is(\mathbf{q}\cdot\mathbf{n}+i\delta)]$. Then, after having performed the Lorentz-invariant momentum integration, we carry out the remaining integrals over the auxiliary parameter s with the use of the "ribbon" regularization.[11] This procedure yields the following logarithmic integrals appearing in our calculation:

$$
\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{q_\mu}{q^{D-1}(p+q)^2(\mathbf{q}\cdot\mathbf{n})}=\frac{iI_D}{2}\frac{n_\mu}{n^2}\ln\left(\frac{\Lambda^2}{p^2}\right),
$$

$$
\begin{aligned}
&\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{q_\mu q_\nu}{q^{D-1}(p+q)^2(\mathbf{q}\cdot\mathbf{n})^2}\\
&=\frac{iI_D}{2}\frac{2n_\mu n_\nu-\delta_{\mu\nu}n^2}{n^4}\ln\left(\frac{\Lambda^2}{p^2}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&\int\frac{d\mathbf{q}}{(2\pi)^{D+1}}\frac{q_\mu q_\nu q_\lambda}{q^{D+1}(p+q)^2(\mathbf{q}\cdot\mathbf{n})}\\
&=\frac{iI_D}{2(D+1)}\left(\frac{n_\mu\delta_{\nu\lambda}+n_\nu\delta_{\mu\lambda}+n_\lambda\delta_{\mu\nu}}{n^2}\right.\\
&\left.-2\frac{n_\mu n_\nu n_\lambda}{n^4}\right)\ln\left(\frac{\Lambda^2}{p^2}\right).
\end{aligned}
$$

One can check that the above expressions are fully consistent with the standard "principal-value" prescription for spurious poles, whose advanced form is known in the field-theoretical literature as the Leibbrandt-Mandelstam rule (see Ref. 16 and references therein).

Finally, by invoking the renormalization-group equation (29) we find that the logarithmic correction (43) tends to exponentiate, thereby resulting in the UV anomalous dimension

$$
\eta_{inv,UV}=-4g^2I_D\frac{D}{D+1}, \qquad (45)
$$

which appears to be *negative*.

Although we were unable to find in the literature any result pertaining to the weakly coupled 3D Abelian gauge theory (e.g., the conventional QED_{3+1}), in which case Eq. (45) yields

$$
\eta^{3D}_{inv,UV}=-\frac{3g^2}{8\pi^2}, \qquad (46)
$$

we did find some comfort in comparing Eq. (46) with the exponent which had been previously found to control the power-law UV behavior of the non-Abelian analog of Eq. (34) in the SU(3)-symmetrical case,[17]

$$
\eta^{3D,SU(3)}_{inv,UV}=-\frac{g^2}{2\pi^2}. \qquad (47)
$$

By construction, Eq. (47) is proportional to the quadratic Casimir operator in the fundamental representation of the color group, which, in the case of SU(N), equals

$$c_F = \frac{1}{N}\sum_{a=1}^{N^2-1} \mathrm{tr}(T^a T^a) = \frac{N^2-1}{2N}. \qquad (48)$$

Evaluating Eq. (48) for SU(3) we obtain $c_F^{\mathrm{SU}(3)} = 4/3$ and, upon separating this factor out, recover the result (46) pertinent to the Abelian case (with the electric charge e substituted for g).

Likewise, by using Eq. (21) we obtain the anomalous exponent which controls the gauge-invariant propagator in QED_{2+1},

$$\eta_{inv,UV}^{2D} = -\frac{32}{3\pi^2 N}, \qquad (49)$$

which is negative, contrary to the result of Ref. 4. However, it remains to be seen whether the exponentiation of $M_{inv}^{UV}(s)$ as well as vanishing of $\Phi_{inv}^{UV}(s)$ still hold beyond the leading $1/N$ order.

Lastly, by comparing Eqs. (31) and (45) one can also deduce the UV anomalous dimension of the exponential factor $\exp(i\int A_\mu dz_\mu)$,

$$\eta_{exp,UV} = -g^2 I_D(D+\lambda). \qquad (50)$$

Interestingly enough, for $\lambda = -D$ this exponent equals zero, and the UV anomalous dimension of the noninvariant propagator coincides with Eq. (45), in agreement with the observation made in the 3D non-Abelian case.[17]

VI. DISCUSSION

Our calculation demonstrates that in the massless case the gauge-invariant Green function (34) appears to decay *slower* than the bare one, in a marked contrast with the previously conjectured Luttinger-like behavior. In this concluding section, we make an attempt to rationalize these findings, although we refrain from making any final judgement on their physical implications.

Albeit somewhat counterintuitive, the found UV behavior is not totally incomprehensible. In fact, the generic behavior of an invariant fermion amplitude is manifested by the asymptotic formula

$$G_{inv}^{\Gamma}(x) \sim \exp[-C|x|\Lambda + \eta \ln(|x|\Lambda)], \qquad (51)$$

where $C>0$, and the expression (51) decays with $|x|$ exponentially, regardless of the sign of η, because the logarithmic term in the exponent is subleading to the linear one. However, in a renormalizable gauge theory, where the gauge invariance is reinforced throughout the whole process of renormalization, the latter would be routinely canceled out by counterterms, which leaves behind the logarithmic term of (potentially) either sign.

This situation would change, however, should one choose to relax the condition of renormalizability at the expense of the gauge invariance, since the radiative corrections to the action (1) generically produce a finite mass of the vector field A_μ. Roughly speaking, the situation would then resemble that in Schwinger's QED_{1+1}, where the gauge field acquires a mass $M \sim g$, and the analog of Eq. (34) behaves as

$$G_{inv}^{|}(x) \sim \exp\left(-\frac{1}{2}[\ln(Mx) + K_0(Mx)] - Mx\right). \qquad (52)$$

It is worth noting that, should one decide to intentionally disregard the exponential factor e^{-Mx}, Eq. (52) would appear to exhibit a power-law decay $\sim 1/\sqrt{x}$ at $x \gg 1/M$, thus suggesting $\eta_{inv}^{1D} = -1/2$.

We mention, in passing, that the exponential, rather than a power-law, behavior has also been found in the problem of Dirac fermions in the presence of a static random vector potential $[\mathbf{A}(x) = (0, \boldsymbol{a}(\boldsymbol{r}))]$, which allows for an asymptotically exact solution in the ballistic regime of large fermion energies.[18]

Conceivably, in some of the above-mentioned physical applications of QED_{2+1} with $N=2$, the problem of the slow space-time decay of the gauge-invariant amplitude (34) can be thwarted by a spontaneous development of a finite fermion mass, in which case the behavior of $G_{inv}^{|}(x)$ at large x will be governed by the (free) IR asymptotic (39) instead of the UV one. However, the intrinsic propensity of the 2D Dirac fermions in QED_{2+1} towards generating a finite mass (usually referred to as the phenomenon of chiral-symmetry breaking) is believed to occur only at sufficiently small $N < N_c$.[19] While in the case of the Lorentz-invariant action (1) the critical number of flavors N_c was found to be as low as 3/2,[19] the Lorentz- (or even rotationally) noninvariant generalizations of the action (1) are still awaiting to be fully explored.

To this end, the authors of Refs. 7 conjectured that the critical value N_c in the QED-like description of the quantum disordered planar d-wave superconductor may become greater than 2 due to the lack of rotational invariance. On the other hand, in the finite-temperature counterpart of the 2D chiral-symmetry-breaking transition in the (spatially) rotationally invariant effective theory of a single layer of graphite, N_c was found to be further reduced as compared to the Lorentz-invariant case.[9]

However, should one insist on maintaining both the gauge and Lorentz invariances of the renormalized gauge-field action, the problem of the slow spatial decay of the alleged physical electron propagator (34) associated with its negative UV anomalous dimension (45) could not be resolved without reexamining the "minimal" form of this Green function. In fact, the task of constructing the proper gauge transformation which converts the auxiliary Dirac fermions into the physical electrons may not be limited to a particular choice of the contour Γ in Eq. (2), but may also require one to modify the phase factor itself.

It is worth noting that in the previous calculations of the "zero-bias anomaly" in the tunneling density of states in the compressible quantum Hall effect,[12] the construction of the electron Green function, albeit seemingly given by the same Eq. (34) with the contour Γ now chosen along the temporal axis, was, in fact, more involved. Indeed, in the semiclassical approximation employed in Ref. 12, the gauge-field dependence of the exponential factor $\exp(i\int dz_\mu A_\mu)$ would have been exactly compensated by that of the non-gauge-invariant Green function $G(t,\mathbf{0}|\mathbf{A})$, thus making the functional average of the product of the two behave essentially as in the absence of any gauge coupling.

Nevertheless, the electron density of states computed in Ref. 12 appeared to be strongly affected by the Chern-Simons gauge fluctuations, which can be traced back to the fact that, in addition to the above-mentioned factors, the electron Green function happened to contain yet another factor: the exponent of the saddle-point value of the effective action of the Chern-Simons gauge field. It was, in fact, this factor that was solely responsible for the strong suppression of the tunneling density of states, consistent with the physical interpretation of the Chern-Simons field as representing the effect of the Coulomb coupling in the presence of strong magnetic field. In light of the fact that in the problem at hand the time-reversal symmetry remains unbroken, no such an additional factor can be readily incorporated into the naive form of the electron propagator (34).

In order to further elaborate on this point, we mention yet another example demonstrating the sensitivity of a generic gauge-invariant amplitude to the details of its construction. To this end, we recall Dirac's original idea of explicitly constructing a ''dressed charge'' corresponding to a physical electron by means of the gauge transformation

$$\Psi_{phys}(x)=\exp\left(i\int d\mathbf{y}\chi_\mu(x-y)A_\mu(y)\right)\Psi(x), \quad (53)$$

where the vector function $\chi_\mu(x)$ obeys the equation $\partial_\mu\chi_\mu(x)=\delta(x)$. In the time-independent Schrödinger operator representation, the originally proposed transformation from the bare fermions to the physical electrons was implemented as a spacelike Dirac string between the location of the fermion and an infinitely remote point

$$\chi_0=0, \quad \chi_i=\langle\mathbf{x}|\frac{1}{\boldsymbol{\nabla}^2}\partial_i|\mathbf{y}\rangle.$$

The Fourier transform of the electron propagator $\langle\Psi_{phys}(x)\bar{\Psi}_{phys}(y)\rangle$ is IR finite [see Eq. (39)] and undergoes multiplicative UV renormalization at a single point $p_\mu=(m,\mathbf{0})$ on the mass shell corresponding to a static charge,[16] in agreement with the general expectation that the absence of any singularity other than a simple pole is characteristic of the propagator of an exact eigenstate with the quantum numbers of an electron.

It was shown in Ref. 16 that in the case of a dressed charge moving with a finite velocity $\mathbf{u}$ the above phase factor needs to be further modified,

$$\Psi_{phys}(x|\mathbf{u})=\exp\left(i\gamma\int d^{D-1}\mathbf{y}_\perp dy_\parallel\langle\mathbf{x}|\frac{1}{\boldsymbol{\nabla}^2}|\mathbf{y}\rangle[\gamma^{-2}\partial_\parallel A_\parallel + \boldsymbol{\partial}_\perp\mathbf{A}_\perp-\mathbf{u}\cdot\mathbf{E}]\right)\Psi(x), \quad (54)$$

where $\gamma=1/\sqrt{1-\mathbf{u}^2}$ and both the parallel and perpendicular components of $\mathbf{A}$ are determined with respect to the velocity vector. As shown in Ref. 16, Eq. (54) gives rise to the operator whose propagator is both IR finite and UV renormalizable at $p_\mu=m\gamma(1,\mathbf{u})$.

Such a strong dependence on the exact details of the construction of the phase factor appearing in the gauge transformation (53) indicates that the true electron Green function may well be quite different from Eq. (34). In particular, it remains to be seen whether one can at all find an alternate form $G^\Gamma_{inv}(x)$ which would decay faster than the bare propagator. Given the intellectual appeal of the QED_{2+1} picture, such an endeavor is definitely worth the effort, and a further investigation into this possibility is currently under way.

Should, however, the sought after Luttinger-like behavior fail to occur even in the modified prototype of the electron propagator, one can still consider an alternative approach to the quantum disordered *d*-wave systems, e.g., the one that was put forward in the context of the scenario of a second pairing transition in the 2D superconducting phase.[20] In Ref. 21, apart from fully idenifying the true nature of this transition and its critical properties (the specific predictions of Ref. 21 for the critical exponents are consistent with the recent tunneling data in Ca-doped YBaCuO, Ref. 22), it was further speculated that it might be possible to extend the effective Higgs-Yukawa theory of the nodal fermion excitations coupled to the fluctuations of the secondary order parameter of either id_{xy} or is symmetry well into the pseudogap phase. Rather than a global superconducting coherence, this would only require the presence of a local parent $d_{x^2-y^2}$-wave order. If this speculation proves valid, it can provide a viable alternative to the QED_{2+1}-based fits to the ARPES data,[4,6] since in the Higgs-Yukawa theory the anomalous dimension of the Dirac fermions is indeed *positive*.[20,21,23]

To summarize, in the present paper we applied Schwinger's functional-integral representation of the fermion amplitudes to the analysis of both the infrared and ultraviolet asymptotics of the conventional (non-gauge-invariant) fermion Green function and a particular gauge-invariant amplitude (34).

In the IR regime, this method provides a substantial improvement with regard to the spinless Bloch-Nordsieck model or the customary semiclassical (eikonal) approximation, since it preserves the exact spinor structure of the fermion amplitudes. Moreover, the intrinsic ''exponential'' form of Schwinger's integral representation facilitates truly nonperturbative calculations.

In the opposite, UV, regime, the method allows one to naturally separate between the gauge-invariant and-noninvariant contributions to the mass operator and systematically compute the higher-order contributions into both kinds of terms. For a specific class of problems, including the amplitudes given by Eq. (2), it has a significant advantage as compared to the conventional diagrammatic technique, which is not particularly well suited for such calculations, for the very rules of the diagrammatic expansion turn out to be amplitude specific and depend on a particular choice of the contour Γ.[16]

The previously suggested ''minimal'' form of the physical electron Green function (34) was found to manifest a negative anomalous dimension, contrary to the much anticipated Luttinger-liquid behavior. The implications of this observation were discussed, some of them pertaining to the applicability of Eq. (34) and others to the possible alternatives to the QED_{2+1}-like description of the ARPES data in high-T_c cuprates.

ACKNOWLEDGMENTS

The author acknowledges valuable discussions and email communications with T. Appelquist, D. Dyakonov, A. Luther, N. Mavromatos, K. Martin, J. Ng, N. Stefanis, Z. Tesanovic, and A. Yashenkin. This research was supported by the NSF under Grant No. DMR-0071362 and also by the Aspen Center for Physics, ICTP (Trieste) and Nordita (Copenhagen), where part of this work was carried out.

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