Controlling enhanced transmission through semiconductor gratings with subwavelength slits by a magnetic field: Numerical and analytical results

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We exploit the change in the permittivity tensor of a conductor by a static magnetic field as a handle to control enhanced transmission in a semiconductor grating. Numerically, results of rigorous coupled-wave analysis (RCWA) incorporating the tensorial permittivity reveal that zeroth order transmission peaks at normal incidence can be shifted by about 15% to longer wavelengths and the peak values of transmission readily doubled when a moderate magnetic field is applied. Analytically, a single-mode theory incorporating anisotropy is developed and results are in quantitative agreement with RCWA, indicating that the tunability in the transmission stems from the waveguide mode. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3173822]

The phenomenon of enhanced transmission through patterned metallic films with subwavelength holes has triggered intensive research interest since its discovery.[1] The physical mechanism is generally regarded to be related to surface plasmon (SP) excitations,[2] and the new area called plasmonics[3,4] has flourished. The advancement has opened up exciting possibilities such as enhanced spectroscopy, high-resolution microscopy and sensing, and better light sources.[3–6] Metallic gratings with subwavelength slits, in which the slits show periodicity in one dimension, show similar phenomena in transmission.[7–9] In addition to SP resonance, waveguide resonance plays a crucial role in transmission through a thick metallic grating.[8–10] In this mechanism, the slits behave as open Fabry–Pérot resonant cavities that channel the incident electromagnetic (EM) waves through.[11,12] This mechanism is particularly important in metallic gratings in that there exists propagating waveguide mode with a vanishing cutoff frequency, while such a mode is absent in metallic films patterned with an array of holes. To make use of the enhanced transmission, it will be very useful if one can control the transmission through a grating *and* the frequencies at which the transmission peaks, without having to fabricate another grating using different slit widths, thicknesses, and/or different materials. There have been many attempts, and the main idea is to try to manipulate the permittivities in the system. For example, one could put an anisotropic material, such as a liquid crystal, into the slits[13] and tune the anisotropy and thus the permittivity tensor by aligning the directors.[13,14] Enhanced transmission in the terahertz frequency range has also been observed in semiconductors patterned with an array of subwavelength holes.[15,16] The permittivity of semiconductors depends on the carrier concentration through the bulk plasma frequency and the transmission can be varied thermally.[16,17] Similarly, for metallic gratings fabricated on a substrate of *n*-type GaAs, one can change the permittivity and thus the transmission by varying the doping in the substrate[18] and/or by varying a bias voltage.[19,20]

Here, we study the transmission of TM-polarized EM wave through a semiconducting grating in the presence of a static magnetic field applied parallel to the slits. While most previous works introduce an anisotropic material into the slits, we exploit the anisotropy in the material forming the grating in the presence of a static magnetic field as a handle in controlling transmission. The physics is that the permittivity of a conductor becomes a *tensor* and thus anisotropic when a magnetic field is applied. For metals, the change in the permittivity by a magnetic field is tiny due to the dominance of the bulk plasma frequency over the cyclotron frequency. For semiconductors, the carrier concentration and the bulk plasma frequency are much lower than that in metals and a moderate magnetic field could lead to an appreciable change in the elements of the permittivity tensor. Accompanying with the use of semiconductor in the grating is that the frequency range of interest is shifted toward terahertz or lower, depending on the temperature and doping. Interestingly, this range is of great interest for civilian applications as well as those related to national security.[21] While the shift of the transmission peaks by a magnetic field in a semiconductor grating has recently been studied numerically by Hu *et al.*,[22] these authors ignored the effects of damping and did not take full account of the tensorial nature of the permittivity, and thus missed the effects of the magnetic field on the magnitude of transmission. Here, we study the transmission both numerically and analytically. Using the method of rigorous coupled-wave analysis (RCWA), generalized to treat media of tensorial permittivities, we show that both the transmission and the wavelengths at which the transmission peaks can be tuned over an appreciable range by a magnetic field of strength that is readily attainable in a laboratory. Analytically, we explain the tunability in the transmission by developing a single-mode theory based on waveguide resonance and accounted for anisotropy.

Figure 1 shows the system we propose to study. It consists of a semiconductor grating of thickness d, period L, and slit width w. The separation between adjacent slits is much larger than the skin depth in the relevant frequency range and the grating is thick in the sense that EM waves in the frequency range of interest cannot penetrate through. A static

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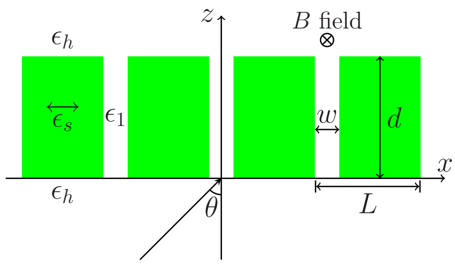

FIG. 1. (Color online) Schematic diagram showing a semiconductor grating of period L, thickness d, and slit width w. The tensorial permittivity is due to an external magnetic field $B_0\hat{y}$ applied in the direction parallel to the slits. The slits are filled with a medium with a scalar permittivity ϵ_1 and the media on both sides have scalar permittivities ϵ_h. Here, θ is the angle of incidence of a TM-polarized EM wave.

magnetic field $\mathbf{B}=B_0\hat{y}$ is applied to the grating in the direction parallel to the slits (see Fig. 1). The relative permittivities on both sides of the grating are assumed to be identical and taken to be a scalar ϵ_h and that inside the slit is a scalar ϵ_1, both independent of B_0. We are interested in the transmission of TM-polarized EM waves. For a semiconductor, the charge carriers lead to a relative permittivity of the Drude form in the absence of a magnetic field[23]

$$\epsilon_{\parallel}(\omega) = \epsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}, \tag{1}$$

where ϵ_∞ is the permittivity at high frequencies, ω_p is the plasma frequency, and Γ is a damping constant. For $B_0 \neq 0$, the semiconductor develops a tensorial relative permittivity $\overleftrightarrow{\epsilon_s}$ given by[24]

$$\overleftrightarrow{\epsilon_s}(\omega) = \begin{pmatrix} \epsilon_\infty - \frac{\omega_p^2}{\omega^2+i\Gamma\omega}\frac{1}{1-f^2} & 0 & -\frac{\omega_p^2}{\omega^2+i\Gamma\omega}\frac{if}{1-f^2} \\ 0 & \epsilon_\infty - \frac{\omega_p^2}{\omega^2+i\Gamma\omega} & 0 \\ \frac{\omega_p^2}{\omega^2+i\Gamma\omega}\frac{if}{1-f^2} & 0 & \epsilon_\infty - \frac{\omega_p^2}{\omega^2+i\Gamma\omega}\frac{1}{1-f^2} \end{pmatrix}$$

$$\equiv \begin{pmatrix} \epsilon_\perp & 0 & -i\delta \\ 0 & \epsilon_\parallel & 0 \\ i\delta & 0 & \epsilon_\perp \end{pmatrix}, \tag{2}$$

where $f=\omega_c/(\omega+i\Gamma)$ with $\omega_c \equiv eB_0/m^*$ being the cyclotron frequency. Here, $-e$ and m^* are the electron charge and carrier effective mass, respectively. For a semiconductor, $\hbar\omega_p$ is typically 10^{-4} that of a metal and its value depends on the carrier concentrations and thus on doping and the temperature. This has the important consequence that a readily attainable B_0 (e.g., <1 T) is sufficient to cause significant deviations of $\epsilon_\perp$ from $\epsilon_\parallel$ and δ from zero. This sensitivity of $\overleftrightarrow{\epsilon_s}$ on B_0 leads to a tunable transmission.

To illustrate the tunability and the enhancement in transmission in the presence of a moderate magnetic field, we calculate the transmission in an InSb grating using the exact method of RCWA generalized to treat anisotropic media.[25] As a model system, we take $L=2$ mm, $w=0.2$ mm, and thickness $d=4$ mm. The permittivity tensor of undoped InSb can be evaluated by the reported parameters:[26] $\epsilon_\infty=15.68$, $\hbar\omega_p=5.525\times10^{-3}$ eV, and $\hbar\Gamma=2.235\times10^{-4}$ eV. These values were obtained by detailed experiments at 80 K. We assume the media on the two sides of the grating and in the slit are identical with $\epsilon_h=\epsilon_1=1$. For incident wavelengths in vacuum that range from $\lambda=3$ to 20 mm, which are larger than the slit width, the corresponding frequencies are slightly

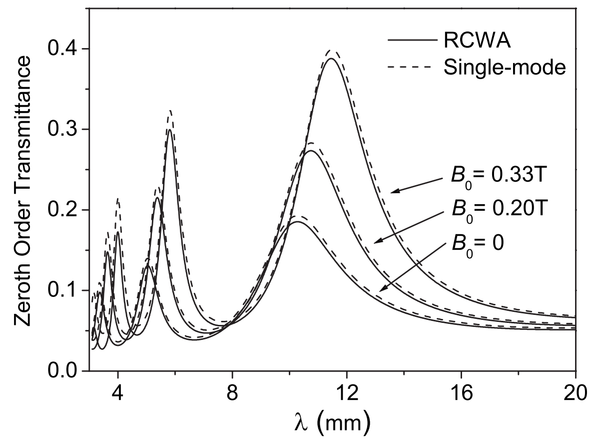

FIG. 2. The zeroth order transmission at normal incidence as a function of incident wavelength for three different values of the magnetic field $B_0=0$, 0.20 and 0.33 T. Results obtained by RCWA (solid lines) and single-mode theory [Eq. (7)] (dashed lines) are shown for comparison.

below the terahertz range. Figure 2 shows the zeroth order transmission T_0 at normal incidence as a function of the wavelength, for $B_0=0$, 0.20 and 0.33 T. The key features are: (i) There are three transmission peaks in the range of λ considered, (ii) as B_0 increases, each peak shifts to longer wavelength and the peak value of T_0 increases; (iii) the shifts and enhancements are both substantial with $\Delta\lambda \sim 11\%-19\%$ of the wavelength that T_0 peaks at $B_0=0$ and the peak values of T_0 could readily be doubled by a moderate B_0. Our system thus provides a simple way to control both the magnitude and wavelength of peak transmissions. We have checked that at room temperature, for which the permittivity tensor of InSb can be estimated,[16] these key features persist and the frequency range shifts into THz range.

The features are tractable analytically by means of a single-mode theory, as we now show. The basic assumption is that the field inside the slit is sufficiently well described by the fundamental mode. This is valid if there is only one nonevanescent propagating mode in the slit.[27] The general form of the magnetic field $H(x,z)\hat{y}$ in the slit is

$$H(x,z) = g(x)\exp(ik\gamma z), \tag{3}$$

where $k=2\pi/\lambda$ is the wavenumber in vacuum, γk is the z component of the wavevector inside the slits, and γ is the propagation constant. For the slit at $-w/2<x<w/2$, $g(x) = C\exp(ik\sqrt{\epsilon_1-\gamma^2}x)+D\exp(-ik\sqrt{\epsilon_1-\gamma^2}x)$, where $\sqrt{\epsilon_1-\gamma^2}$ is taken to give a positive imaginary part, and C and D are coefficients. For the structure under study, the slits can be treated as independent of each other since the distance between adjacent slits is much larger than the skin depth. Focusing on the period at $-L/2<x<L/2$, the field in the slit decays into the semiconductor for $|x|>w/2$ and take on the form

$$H_y(x,z) = B\exp\left[ik\alpha^-\left(x+\frac{w}{2}\right)\right]e^{ik\gamma z}, \quad x<-\frac{w}{2},$$

$$H_y(x,z) = F\exp\left[ik\alpha^+\left(x-\frac{w}{2}\right)\right]e^{ik\gamma z}, \quad x>+\frac{w}{2}, \tag{4}$$

where

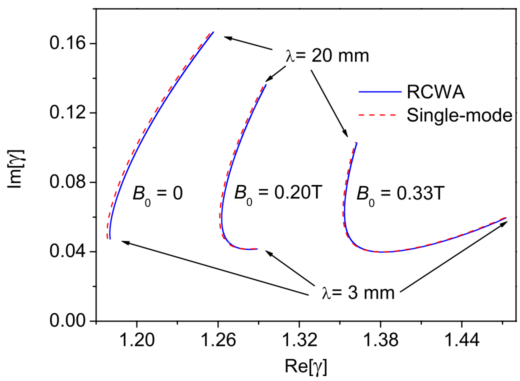

FIG. 3. (Color online) Real and imaginary parts of the propagation constant γ for incident wavelengths from 3 to 20 mm and $B_0=0$, 0.20, and 0.33 T. Results obtained by RCWA (solid lines) and single-mode theory [Eq. (6)] (dashed lines) are shown for comparison.

$$\alpha^{\pm} = \pm\left[\frac{\epsilon_\perp(\epsilon_\perp - \gamma^2) - \delta^2}{\epsilon_\perp}\right]^{1/2}. \tag{5}$$

Note that $\alpha^{+}(\alpha^{-})$ has a positive (negative) imaginary part. The corresponding **E** fields in the grating can be obtained from H_y via the Maxwell's equations. Matching boundary conditions at $|x|=w/2$ gives a transcendental equation for the propagation constant γ

$$i\tan(k\sqrt{\epsilon_1-\gamma^2}w) = \frac{2\epsilon_1\sqrt{\epsilon_1-\gamma^2}\sqrt{\epsilon_\perp[\epsilon_\perp(\epsilon_\perp-\gamma^2)-\delta^2]}}{\gamma^2(\epsilon_1^2+\epsilon_\perp^2-\delta^2)-\epsilon_1(\epsilon_\perp^2+\epsilon_1\epsilon_\perp-\delta^2)}. \tag{6}$$

Note that $\epsilon_\perp$ and δ depend on B_0 through ω_c, and γ can thus be tuned by B_0. To test our single-mode approach, we compare the complex γ obtained by numerically solving Eq. (6) and by the exact RCWA for a range of λ and three values of B_0 in Fig. 3. The results are in excellent agreement, implying that our single-mode theory captures the essential physics.

We proceed to obtain an analytic expression for the transmittance. For $w \ll \lambda$, a common treatment in single-mode theory[14,27] is to approximate $g(x)$ by a constant, since $\exp(\pm ik\sqrt{\epsilon_1-\gamma^2}x)\sim 1$. The magnetic field in the slit $(-\frac{1}{2}w < x < \frac{1}{2}w)$ then takes on the form $H(x,z)=A^{+}\exp(ik\gamma z) + A^{-}\exp[-ik\gamma(z-d)]$. The fields on both sides of the grating can be expressed as Rayleigh expansions. Matching boundary conditions and approximating the semiconductor as a perfect conductor in the matching at $z=0$ and $z=d$, which is valid for a thick grating, the zeroth order transmittance is found to be

$$T_0 = \left|\frac{4q_0\exp(ik\gamma d)}{(1+p)^2-(1-p)^2\exp(2ik\gamma d)}\right|^2, \tag{7}$$

where $p=\sum_{n=-\infty}^{\infty}q_n$ and $q_n=(w\gamma/L)\left[\mathrm{sinc}^2\{kw[\sin\theta+n(\frac{\lambda}{L})]/2\}/\sqrt{1-[\sin\theta+n(\frac{\lambda}{L})]^2}\right]$ for the case of $\epsilon_h=\epsilon_1=1$. Here, $\mathrm{sinc}(x)=\sin(x)/x$. This result is similar to that in Ref. 14, but the expression for q_n is different as the anisotropy now appears in the material forming the grating. The results of $T_0(\lambda)$ calculated from Eq. (7) are also shown in Fig. 2. They agree very well with RCWA results, except for a slight overestimation at short wavelengths. The theory captures both the shifts and enhancements in the transmittance peaks. From Eq. (7), the positions (wavelengths) of the peaks can be estimated by the Fabry–Pérot condition

$$2\mathrm{Arg}\left(\frac{1-p}{1+p}\right) + 2k\mathrm{Re}[\gamma]d = 2n\pi, \tag{8}$$

where n is an integer. As B_0 increases, p and γ vary and the peaks shift. By numerical calculations using Eq. (7) on our model system for B_0 from 0 to 0.33 T, the values of T_0 are found to enhance by 97%–132% for the three peaks and the shifts in the wavelength of the peaks $\Delta\lambda/\lambda$ range from 11% to 19%. These values agree well with RCWA results and show that the transmittance can be significantly controlled by varying B_0 over a range accessible in laboratories.

In summary, we studied a semiconductor grating with a static magnetic field applied parallel to the slits both numerically and analytically. Incorporating the tensorial permittivity of the grating material and damping in RCWA calculations, the transmission and the wavelengths at which the transmission peaks show significant changes when a moderate magnetic field is applied. A single-mode theory that incorporates the anisotropy in the grating material was developed and results on the propagation constant and transmittance were in good agreement with RCWA results, indicating that the tunability in transmission stems from the waveguide mode. The properties of tunability and analytically tractable make the system useful for designing devices that rely on controlling EM wave transmission near the terahertz range.

[1]T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, Nature (London) **391**, 667 (1998).
[2]L. Martín-Moreno, F. J. García-Vidal, H. J. Lezec, K. M. Pellerin, T. Thio, J. B. Pendry, and T. W. Ebbesen, Phys. Rev. Lett. **86**, 1114 (2001).
[3]S. A. Maier, *Plamonics: Fundamentals and Applications* (Springer, New York, 2007).
[4]E. Ozbay, Science **311**, 189 (2006).
[5]M. Righini, C. Giannini, and R. Quidant, J. Opt. A, Pure Appl. Opt. **10**, 093001 (2008).
[6]K. A. Willets and R. P. Van Duyne, Annu. Rev. Phys. Chem. **58**, 267 (2007).
[7]U. Schröter and D. Heitmann, Phys. Rev. B **58**, 15419 (1998).
[8]J. A. Porto, F. J. García-Vidal, and J. B. Pendry, Phys. Rev. Lett. **83**, 2845 (1999).
[9]E. Popov, M. Nevière, S. Enoch, and R. Reinisch, Phys. Rev. B **62**, 16100 (2000).
[10]Q. Cao and P. Lalanne, Phys. Rev. Lett. **88**, 057403 (2002).
[11]S. Astilean, P. Lalanne, and M. Palamaru, Opt. Commun. **175**, 265 (2000).
[12]K.-Y. Fong and P. M. Hui, Appl. Phys. Lett. **89**, 091101 (2006).
[13]F. Yang and J. R. Sambles, Appl. Phys. Lett. **79**, 3717 (2001).
[14]K.-Y. Fong and P. M. Hui, Appl. Phys. Lett. **91**, 171101 (2007).
[15]J. Gómez Rivas, C. Schotsch, P. Haring Bolivar, and H. Kurz, Phys. Rev. B **68**, 201306 (2003).
[16]J. Gómez Rivas, C. Janke, P. Haring Bolivar, and H. Kurz, Opt. Express **13**, 847 (2005).
[17]J. Gómez Rivas, P. Haring Bolivar, and H. Kurz, Opt. Lett. **29**, 1680 (2004).
[18]D. Wasserman, E. A. Shaner, and J. G. Cederberg, Appl. Phys. Lett. **90**, 191102 (2007).
[19]E. A. Shaner, J. G. Cederberg, and D. Wasserman, Appl. Phys. Lett. **91**, 181110 (2007).
[20]H.-T. Chen, H. Lu, A. K. Azad, R. D. Averitt, A. C. Gossard, S. A. Trugman, J. F. O'Hara, and A. J. Taylor, Opt. Express **16**, 7641 (2008).
[21]B. Ferguson and X. C. Zhang, Nature Mater. **1**, 26 (2002).
[22]B. Hu, B.-Y. Gu, B.-Z. Dong, and Y. Zhang, Opt. Commun. **281**, 6120 (2008).
[23]M. Balkanski and R. F. Wallis, *Semiconductor Physics and Applications* (Oxford University Press, Oxford, 2000), p. 247.
[24]R. F. Wallis, J. J. Brion, F. Burstein, and A. Hartstein, Phys. Rev. B **9**, 3424 (1974).
[25]L. Li, J. Mod. Opt. **45**, 1313 (1998).
[26]S. C. Howells and L. A. Schlie, Appl. Phys. Lett. **69**, 550 (1996).
[27]P. Lalanne, J. P. Hugonin, S. Astilean, M. Palamaru, and K. D. Möller, J. Opt. A, Pure Appl. Opt. **2**, 48 (2000).