ISSN 1063-780X, Plasma Physics Reports, 2007, Vol. 33, No. 12, pp. 1014–1022. © Pleiades Publishing, Ltd., 2007.
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Generation of Terahertz Radiation in the Reflection of a Laser Pulse from a Dense Plasma

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Received April 11, 2007; in final form, May 16, 2007

Abstract—The generation of low-frequency (terahertz) electromagnetic radiation in the reflection of a laser pulse from the boundary of a dense plasma is considered. Low-frequency wave electromagnetic fields in vacuum are excited by a vortex electric current that is induced at the plasma boundary by the ponderomotive force of the laser pulse. The spectral, angular, and energy parameters of the low-frequency radiation, as well as the spatiotemporal structure of the emitted waves, are investigated. It is shown that for typical parameters of present-day laser plasma experiments, the power of terahertz radiation can amount to tens of megawatts.

PACS numbers: 52.25.Os, 52.38.-r

DOI: 10.1134/S1063780X07120069

1. INTRODUCTION

Considerable current interest to terahertz radiation stems from its applications in many important areas of science and engineering. One promising method for generating terahertz waves is to use ultrashort laser pulses. The possibility of generating low-frequency (LF) waves by an electromagnetic wave packet was first pointed out by Askar'yan [1], who considered the Cherenkov and transition radiations from the packet on a qualitative level. The generation of LF radiation in a plasma was, to the best of the author's knowledge, first considered by Dysthe et al. [2], who solved a one-dimensional problem of the propagation of an amplitude-modulated electromagnetic wave through an inhomogeneous plasma (see also [3]). The emission of LF terahertz electromagnetic radiation in the irradiation of gaseous and solid-state targets by femtosecond laser pulses was revealed experimentally by Hamster et al. [4]. More recently, many different physical mechanisms have been proposed for generating terahertz radiation in a plasma in its interaction with a high-power laser pulse. One of them is associated with the conversion of a wake plasma wave excited by a short laser pulse in a low-density plasma into LF electromagnetic radiation. Such conversion of longitudinal plasma waves into a transverse electromagnetic field can occur when the plasma is inhomogeneous or when there is an external magnetic field. The theory of electromagnetic radiation generated by a short laser pulse in a periodically inhomogeneous (stratified) plasma was constructed in [5] (see also [6]). For a randomly inhomogeneous plasma, the related problem was considered in [7]. For a regularly inhomogeneous plasma, LF emission in the terahertz frequency range was investigated both numerically [8] and analytically [9, 10]. The generation of terahertz electromagnetic wave fields by a laser pulse in a low-density plasma in the presence of an external magnetic field is associated with the Cherenkov emission mechanism; this issue was studied in a number of papers [11–16]. In some recent papers [17–21], a study was made of how an electron bunch accelerated in the wake wave of a laser pulse generates transition radiation in the terahertz range as it crosses a plasma–vacuum interface. LF transition radiation generated by a laser pulse at the boundary of a low-density plasma was considered in [22, 23].

The present paper is aimed at investigating the generation of LF (terahertz) radiation by a laser pulse incident on the boundary of a plasma with an overcritical density. In this case, LF wave fields are excited in the reflection of a laser pulse from the boundary of a dense plasma, in contrast to the above-cited papers [22, 23], where the interaction of a laser pulse with a low-density plasma was considered. As the pulse is reflected from the plasma boundary, its ponderomotive force induces a surface vortex electric current, which generates terahertz waves. It is established that LF radiation propagates from the plasma into vacuum in the form of an electromagnetic pulse whose duration is determined by the time of interaction of a laser pulse with the plasma boundary and is comparable to the duration of the laser pulse. The spectral, angular, energy, and spatiotemporal parameters of the LF radiation are investigated. It is shown that a tightly focused laser pulse emits energy predominantly in a direction transverse to the normal to the plasma boundary and that the radiation spectrum has a broad peak at a frequency close to the reciprocal of the laser pulse duration. An increase in the focal spot size leads to a shift of the spectral peak toward lower frequencies and to a displacement of the directional pattern of the radiation energy toward smaller angles. A wide laser pulse whose transverse size substantially exceeds its length generates LF radiation energy nearly along the normal to the plasma boundary. The total energy of the emitted terahertz radiation and the coefficient of conversion of the laser pulse energy into LF energy are calculated. It is found that, in the interaction of femtosecond terahertz laser pulses with porous targets, the power of terahertz radiation can amount to tens of megawatts.

2. REFLECTION OF A LASER PULSE FROM THE BOUNDARY OF A DENSE PLASMA

Let a laser pulse propagating at the carrier frequency ω_0 in the positive direction of the z axis be normally incident from the vacuum onto the boundary of a dense plasma having an electron density N_{0e} above the critical density $N_c = m_e\omega_0^2/(4\pi e^2)$ for the given carrier frequency and occupying the half-space $z > 0$. We consider the problem of the reflection of a laser pulse from an overcritical plasma. We assume that the plasma boundary is sharp and that the pulse is linearly polarized and its intensity obeys a Gaussian distribution in both the longitudinal, $\xi = z - ct$, and transverse, $r_\perp = \sqrt{x^2 + y^2}$, coordinates. In vacuum ($z < 0$), the electric field of a laser pulse incident on the plasma boundary can be represented as

$$\mathbf{E}_L^{\text{inc}}(\mathbf{r}, t) = \frac{1}{2}\mathbf{e}_x E_{0L}\exp\left(-i\omega_0 t + i\frac{\omega_0}{c}z - \frac{\xi^2}{2L^2} - \frac{r_\perp^2}{2R_L^2}\right) + \text{c.c.} \tag{2.1}$$

Here, E_{0L} is electric field amplitude, $\mathbf{e}_x$ is a unit vector in the x direction, c is the speed of light, R_L and $L = c\tau$ are the transverse size of the pulse and its length, τ is the pulse duration, and e and m_e are the charge and mass of an electron.

Let us consider how a laser pulse is reflected from a semi-infinite overcritical plasma. The equation for the electric field of the pulse has the form

$$\frac{\partial^2\mathbf{E}_L}{\partial t^2} + \omega_p^2(z)\mathbf{E}_L - c^2\Delta\mathbf{E}_L = 0, \tag{2.2}$$

where $\omega_p(z) = \sqrt{4\pi e^2 N_e(z)/m_e}$ is the local plasma frequency and $N_e(z)$ is the electron density, which has a jump at the plasma boundary $z = 0$, i.e., it is zero, $N_e(z) = 0$, at $z < 0$ (in vacuum) and is equal to $N_e(z) = N_{0e}$ at $z > 0$ (in plasma). Applying the Fourier transformation in time, we find the solutions to Eq. (2.2) in vacuum and in plasma that are consistent with electric field (2.1) of the incident pulse:

$$\mathbf{E}_L^V(\mathbf{r}, \omega) = \mathbf{e}_x\tau\sqrt{2\pi}E_{0L}\exp\left(-\frac{r_\perp^2}{2R_L^2} - \frac{(\omega-\omega_0)^2\tau^2}{2}\right) \times\left\{\exp\left(i\frac{\omega}{c}z\right) + R(\omega)\exp\left(-i\frac{\omega}{c}z\right)\right\}, \tag{2.3}$$

$$\mathbf{E}_L^P(\mathbf{r}, \omega) = A(\omega)\mathbf{e}_x\tau\sqrt{2\pi}E_{0L} \times\exp\left(-\frac{r_\perp^2}{2R_L^2} - \frac{(\omega-\omega_0)^2\tau^2}{2} - \frac{\omega}{c}\chi(\omega)z\right). \tag{2.4}$$

The reflection and transmission coefficients, $R(\omega)$ and $A(\omega)$, are determined from the continuity conditions for the tangential components of the electric and magnetic fields at the plasma boundary and have the form

$$R(\omega) = \frac{1 - i\chi(\omega)}{1 + i\chi(\omega)}, \quad A(\omega) = \frac{2}{1 + i\chi(\omega)}, \tag{2.5}$$

where $\chi(\omega) = \sqrt{(\omega_p^2/\omega^2) - 1}$ and $\omega_p = \sqrt{4\pi e^2 N_{0e}/m_e}$ is the plasma frequency.

Applying the inverse Fourier transformation, we take into account relationships (2.5) to obtain from formulas (2.3) and (2.4) the electric field of the laser pulse in vacuum and in plasma, respectively:

$$\mathbf{E}_L^V(\mathbf{r}, t) = \frac{1}{2}\mathbf{e}_x\tau\sqrt{2\pi}E_{0L}\exp\left(-\frac{r_\perp^2}{2R_L^2}\right)\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi} \times\exp\left(-i\omega t - \frac{(\omega-\omega_0)^2\tau^2}{2}\right) \times\left\{\exp\left(i\frac{\omega}{c}z\right) + \frac{1 - i\chi(\omega)}{1 + i\chi(\omega)}\exp\left(-i\frac{\omega}{c}z\right)\right\} + \text{c.c.}, \tag{2.6}$$

$$\mathbf{E}_L^P(\mathbf{r}, t) = \frac{1}{2}\mathbf{e}_x\tau\sqrt{2\pi}E_{0L}\exp\left(-\frac{r_\perp^2}{2R_L^2}\right)\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi} \times\exp\left(-i\omega t - \frac{(\omega-\omega_0)^2\tau^2}{2} - \frac{\omega}{c}\chi(\omega)z\right)\frac{2}{1 + i\chi(\omega)} + \text{c.c.} \tag{2.7}$$

We assume that there are many wave periods along the length of the laser pulse, $\omega_0\tau \gg 1$. In this case, the main contribution to the integrals in expressions (2.6) and (2.7) comes from a small vicinity of the laser carrier frequency, $|\omega - \omega_0| \le 1/\tau$. Taking the integral in formula (2.6), we arrive at the following expression for the electric field of the laser pulse in vacuum ($z < 0$):

$$\mathbf{E}_L^V(\mathbf{r}, t) = \frac{1}{2}\mathbf{e}_x E_{0L}\exp\left(-i\omega_0 t - \frac{r_\perp^2}{2R_L^2}\right)$$
$$\times\left\{\exp\left(i\frac{\omega_0}{c}z - \frac{\xi^2}{2L^2}\right)\right. \quad (2.8)$$
$$\left.+\frac{1-i\chi(\omega_0)}{1+i\chi(\omega_0)}\exp\left(-i\frac{\omega_0}{c}z - \frac{\eta^2}{2L^2}\right)\right\} + \text{c.c.},$$

where $\eta = z + ct$. This expression describes the waves that are incident on the plasma and those that are reflected from its boundary. In the case at hand, the incident wave is completely reflected from the boundary of an overcritical plasma ($\omega_p > \omega_0$). According to expression (2.7), the electric field of a laser pulse in a plasma has the form

$$\mathbf{E}_L^P(\mathbf{r}, t) = \frac{1}{2}\mathbf{e}_x\frac{2E_{0L}}{1+i\chi(\omega_0)}$$
$$\times\exp\left(-i\omega_0 t - \frac{t^2}{2\tau^2} - \frac{\omega_0}{c}\chi(\omega_0)z - \frac{r_\perp^2}{2R_L^2}\right) + \text{c.c.} \quad (2.9)$$

We can see from this formula that a laser pulse penetrates into a plasma to a small depth $l \approx c/(\omega_0\chi(\omega_0))$ on a time scale comparable to the pulse duration.

3. GENERATION OF LF RADIATION AT THE PLASMA BOUNDARY

In order to describe the generation of LF (terahertz) radiation in the reflection of laser pulse (2.1) from the boundary of a dense plasma, we use the hydrodynamic equation for the electron velocity $\mathbf{V}(\mathbf{r}, t)$, averaged over the high-frequency (HF) electromagnetic oscillations (see, e.g., [5]),

$$\frac{\partial}{\partial t}\mathbf{V} = \frac{e}{m_e}\mathbf{E} - \frac{1}{m_e}\nabla\phi, \quad (3.1)$$

and Maxwell's equations for the LF electric and magnetic fields, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$,

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}, \quad (3.2)$$

$$\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi e}{c}N_e(z)\mathbf{V}. \quad (3.3)$$

Equation (3.1) is written in the linear approximation in slowly varying quantities. In this equation, collisional and relativistic effects are ignored and the influence of the laser field is accounted for only through the averaged ponderomotive potential

$$\phi(\mathbf{r}, t) = \frac{1}{2}m_e\langle\mathbf{V}_L^2\rangle, \quad (3.4)$$

where $\mathbf{V}_L(\mathbf{r}, t) = (e/m_e)\int_{-\infty}^t dt'\mathbf{E}_L^P(\mathbf{r}, t')$ is the electron velocity in laser field (2.9) and the angle brackets denote averaging over HF oscillations. The linear approximation is valid under the inequalities $|\mathbf{V}|$, $V_{Te} \ll |\mathbf{V}_L| \ll c$, which imply that the electron velocity in the laser field is much lower than the speed of light but is much higher than both the electron thermal velocity V_{Te} and the slow electron drift velocity. Note that, according to [24], electron–ion collisions in the field of an intense laser pulse are governed by the motion of electrons in the laser field rather than by their thermal motion, $\nu_{e,i} \approx 4\pi Z e^4 N_e \ln\Lambda/(m_e^2\langle V_L^2\rangle^{3/2})$, where Z is the degree of plasma ionization and $\ln\Lambda$ is the Coulomb logarithm. Under the condition $\nu_{e,i}\tau < 1$, electrons and ions do not collide during the time of reflection of a laser pulse from the plasma boundary, so the plasma can be treated as being collisionless. Under the above restrictions, Eq. (3.1) can be written in the linear approximation in a form in which the term with the thermal pressure can be discarded and collisions can be ignored.

From Eqs. (3.1)–(3.3) we can obtain the following equations for the LF electric and magnetic fields:

$$\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_p^2(z)\mathbf{E} + c^2\nabla\times(\nabla\times\mathbf{E}) = \frac{\omega_p^2(z)}{e}\nabla\phi, \quad (3.5)$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2} + \omega_p^2(z)\mathbf{B} + c^2\nabla\times(\nabla\times\mathbf{B}) = 4\pi ec[\nabla N_e(z)\times\mathbf{V}]. \quad (3.6)$$

Equation (3.1) implies that, within the plasma (where the electron density is constant, $N_e(z) = N_{0e}$), the electric current $\mathbf{j} = eN_{0e}\mathbf{V}$ is purely potential and does not excite electromagnetic waves. However, at the plasma boundary (at $z = 0$), where the electron density has a jump, the electric current has the vortex component $\nabla\times\mathbf{j} = e[\nabla N_e(z)\times\mathbf{V}]$, which enters into the right-hand side of Eq. (3.6) for the magnetic field and serves as a source for generating terahertz electromagnetic radiation.

Equation (3.6) can be solved by applying the Fourier transformation in time and in the transverse spatial coordinates $\mathbf{r}_\perp = (x, y)$:

$$\mathbf{B}(\mathbf{r}, t) = \int\frac{d\omega d\mathbf{k}_\perp}{(2\pi)^3}\exp(-i\omega t + i\mathbf{k}_\perp\mathbf{r})\mathbf{B}(\omega, \mathbf{k}_\perp, z),$$
$$\mathbf{B}(\omega, \mathbf{k}_\perp, z) = \int dt dx dy\exp(i\omega t - i\mathbf{k}_\perp\mathbf{r})\mathbf{B}(\mathbf{r}, t), \quad (3.7)$$

where the transverse wave vector $\mathbf{k}_\perp$ has the components k_x and k_y. From Eqs. (3.5) and (3.6) we can see that, in its reflection from the boundary of a dense plasma, a laser pulse excites the azimuthal components B_ϕ of the LF magnetic field and also the radial and longitudinal (along the plasma density gradient) components, E_r and E_z, of the LF electric field.

Performing simple manipulations, we arrive at the following equation for the Fourier transformed LF azimuthal magnetic field component $B_\phi(\omega, \mathbf{k}_\perp, z)$, whose right-hand side is written in terms of the Fourier component $\phi(\omega, \mathbf{k}_\perp, z)$ of the ponderomotive potential:

$$\varepsilon(z)\frac{d}{dz}\left(\frac{1}{\varepsilon(z)}\frac{dB_\phi}{dz}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - k_\perp^2\right)B_\phi = k_\perp\frac{\omega}{c}\left\{\frac{\omega_p^2(z)}{\omega^2}\frac{d}{dz}\left(\frac{\phi}{e}\right) - \varepsilon(z)\frac{d}{dz}\left(\frac{1}{\varepsilon(z)}\frac{\omega_p^2(z)}{\omega^2}\frac{\phi}{e}\right)\right\}. \quad (3.8)$$

Here, we have introduced the notation $\varepsilon(z) = 1 - \omega_p^2(z)/\omega^2$ and $k_\perp^2 = k_x^2 + k_y^2$. The longitudinal, $E_z(\omega, \mathbf{k}_\perp, z)$, and radial, $E_r(\omega, \mathbf{k}_\perp, z)$, components of the electric field are related to the azimuthal magnetic field $B_\phi(\omega, \mathbf{k}_\perp, z)$ by the relationships

$$E_r = -\frac{ic}{\omega\varepsilon(z)}\left\{\frac{dB_\phi}{dz} + \frac{\omega}{c}k_\perp\frac{\omega_p^2(z)}{\omega^2}\frac{\phi}{e}\right\}, \quad (3.9)$$

$$E_z = -\frac{c}{\omega\varepsilon(z)}\left\{k_\perp B_\phi + i\frac{\omega^2}{c^2}\frac{\omega_p^2(z)}{\omega^2}\frac{\phi}{e}\right\}. \quad (3.10)$$

Solving Eq. (3.8) in vacuum ($z < 0$) and in plasma ($z > 0$) and using the continuity conditions for the tangential components E_r and B_ϕ of the electric and magnetic fields at the plasma boundary ($z = 0$), we can determine the LF electromagnetic fields over the entire space. In vacuum ($z < 0$), the solution to Eq. (3.8) for the Fourier component of the magnetic field has the form of a wave running away from the plasma boundary,

$$B_\phi(\omega, \mathbf{k}_\perp, z) = ik_\perp\frac{\omega}{c}\frac{\omega_p^2}{\omega^2}\frac{\exp(-ik_0z)}{k_0\varepsilon + ik}\frac{\phi(\omega, \mathbf{k}_\perp, z = 0)}{e}, \quad (3.11)$$

where $k = \sqrt{k_\perp^2 + (\omega^2/c^2)\chi^2(\omega)}$, $\varepsilon = 1 - \omega_p^2/\omega^2$, and $k_0 = \sqrt{\omega^2/c^2 - k_\perp^2}$. The magnetic field, which penetrates into the plasma ($z > 0$) to a skin depth on the order of $1/k$, is described by the expression

$$B_\phi(\omega, \mathbf{k}_\perp, z) = ik_\perp\frac{\omega}{c}\frac{\omega_p^2}{\omega^2}\frac{\exp(-kz)}{k_0\varepsilon + ik}\frac{\phi(\omega, \mathbf{k}_\perp, z = 0)}{e}. \quad (3.12)$$

The results obtained, specifically, formulas (3.11) and (3.12), show that the LF electromagnetic radiation in vacuum, in which we are interested here, is determined by the Fourier component of the ponderomotive potential at the plasma boundary. Taking into account formulas (3.4), (2.8), and (2.9), we can obtain the ponderomotive potential at $z = 0$ and also its Fourier transform, which has the form

$$\phi(\omega, \mathbf{k}_\perp, z = 0) = \frac{\omega_0^2\pi^{3/2}m_eV_E^2R_L^2\tau}{\omega_p^2}\exp\left(-\frac{\omega^2\tau^2}{4} - \frac{k_\perp^2R_L^2}{4}\right), \quad (3.13)$$

where $V_E = eE_{0L}/m_e\omega_0$ is the electron oscillatory velocity in the laser field.

We apply the inverse Fourier transformation in the spatial variables (see the first of formulas (3.7)) and take into account relationships (3.11) and (3.13) to represent the spectral density of the azimuthal magnetic field in vacuum in the form

$$B_\phi(\omega, \mathbf{r}) = i\frac{\omega_0}{\omega}\pi^{3/2}R_L^2\tau\frac{V_E}{c}E_{0L}\exp\left(-\frac{\omega^2\tau^2}{4}\right) \times \int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\frac{k_\perp}{k_0\varepsilon + ik}\exp\left(i\mathbf{k}_\perp\mathbf{r} - ik_0z - \frac{k_\perp^2R_L^2}{4}\right). \quad (3.14)$$

The integral in expression (3.14) can be calculated for large distances from the region of interaction of a laser pulse with the plasma boundary, $r = \sqrt{r_\perp^2 + z^2} \gg R_L, L$. Using the stationary-phase method and accounting for the contribution of the saddle point, we arrive at the following expression for the azimuthal magnetic field component in the wave zone:

$$B_\phi(\omega, \mathbf{r}) = \frac{\sqrt{\pi}\omega_0R_L^2\tau}{c}\frac{V_E}{2c}E_{0L}\frac{\sin\theta\cos\theta}{r} \times \frac{\exp\left(i\frac{\omega}{c}r - \frac{\omega^2\tau^2}{4} - \frac{\omega^2R_L^2}{4c^2}\sin^2\theta\right)}{\varepsilon\cos\theta + i\sqrt{\omega_p^2/\omega^2 - \cos^2\theta}}. \quad (3.15)$$

The electric field components in the wave zone are related to azimuthal magnetic field (3.15) by the relationships

$$E_r = -B_\phi\cos\theta, \quad E_z = -B_\phi\sin\theta, \quad (3.16)$$

where θ is the angle between the observation direction and the negative direction of the z axis, which is related to the conventional azimuthal angle ϑ by the formula $\theta = \pi - \vartheta$. Expressions (3.15) and (3.16) describe an LF electromagnetic wave that propagates into vacuum from the region of interaction of a laser pulse with the plasma boundary.

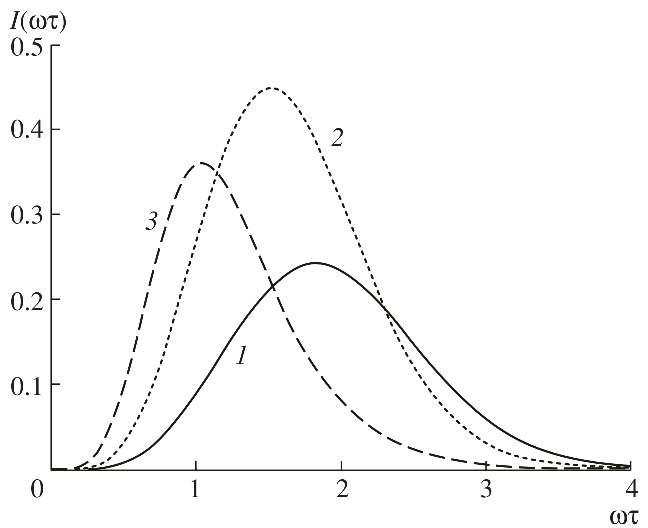

Fig. 1. Spectra of terahertz radiation from a laser pulse in its reflection from a dense plasma ($\omega_p\tau = 100$) for different values of R_L^2/L^2: (*1*) 0.25, (*2*) 1, and (*3*) 4.

4. SPECTRAL AND ANGULAR PARAMETERS OF THE LF RADIATION

Here, using expressions (3.15) and (3.16) for the Fourier transformed electric and magnetic fields, we consider the frequency and angular dependence of the LF electromagnetic radiation energy in vacuum. In order to investigate the energy parameters of the radiation generated by a laser pulse, we utilize the following expression for the time-integrated energy flux density at a certain point [25]:

$$\mathbf{P}(\mathbf{r}) = \int_{-\infty}^{+\infty} dt \frac{c}{4\pi}[\mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t)]. \quad (4.1)$$

The vector $\mathbf{P}(\mathbf{r})$ describes the electromagnetic energy that is emitted by the pulse during the entire time of its interaction with the plasma boundary and flows through a surface of unit area along the normal to it in the vicinity of a point with the position vector $\mathbf{r}$. The vector $\mathbf{P}(\mathbf{r})$ can also be represented in terms of the frequency integral,

$$\mathbf{P}(\mathbf{r}) = \int_0^{\infty} d\omega \mathbf{p}(\omega, \mathbf{r}), \quad (4.2)$$

of the spectral energy flux density $\mathbf{p}(\omega, \mathbf{r})$, which in turn is expressed through the Fourier components of the electric and magnetic fields in the form

$$\mathbf{p}(\omega, \mathbf{r}) = \frac{c}{8\pi^2}\{[\mathbf{E}(\omega, \mathbf{r}) \times \mathbf{B}^*(\omega, \mathbf{r})] + \text{c.c.}\}. \quad (4.3)$$

Taking into account relationships (3.15) and (3.16), we find from formula (4.3) that the vector $\mathbf{p}(\omega, \mathbf{r})$ is directed along the position vector $\mathbf{r}$ and that it can be represented in terms of the spectral density of the azimuthal magnetic field as

$$\mathbf{p}(\omega, \mathbf{r}) = \frac{c}{4\pi^2}|B_\phi(\omega, \mathbf{r})|^2 \mathbf{e}_r, \quad (4.4)$$

where $\mathbf{e}_r = \mathbf{r}/r$ is a unit vector in the direction of the position vector. Using expression (3.15), we can determine the energy $dW(\omega, \theta) = (\mathbf{p} \cdot \mathbf{e}_r)r^2 dO d\omega$ emitted by a laser pulse into the solid angle element $dO = 2\pi \sin\theta d\theta$ per unit frequency range $d\omega$ in vacuum:

$$\frac{dW(\omega, \theta)}{d\omega dO} = \frac{\omega_0^2 R_L^2 \tau}{\pi^{3/2} c^2} \frac{V_E^2}{2c^2} W_L \frac{\omega^4}{\omega_p^4} \times \frac{\sin^2\theta \cos^2\theta}{\cos^2\theta + (1 - 2\cos^2\theta)(\omega^2/\omega_p^2)} \times \exp\left[-\frac{\omega^2}{2c^2}(L^2 + R_L^2 \sin^2\theta)\right], \quad (4.5)$$

where $W_L = E_{0L}^2 \pi^{3/2} R_L^2 L/8\pi$ is the laser pulse energy.

In expression (4.5), we integrate over the solid angle elements dO (with the angle θ lying within the interval $0 \le \theta \le \pi/2$) to obtain the spectral energy of the excited LF radiation:

$$\frac{dW(\omega)}{d\omega} = \tau \frac{\omega_0^2 R_L^2 V_E^2}{\omega_p^2 L^2 c^2} \frac{W_L}{\sqrt{\pi}\omega_p^2 \tau^2}(\omega\tau)^4 \times \int_0^1 dy \frac{y^2(1 - y^2)}{y^2 + (1 - 2y^2)(\omega^2/\omega_p^2)} \times \exp\left\{-\frac{(\omega\tau)^2}{2}\left[1 + \frac{R_L^2}{L^2}(1 - y^2)\right]\right\}. \quad (4.6)$$

Let us consider the behavior of the normalized spectral energy density of the LF radiation, $I(x) = \frac{dW(\omega)}{d\omega\tau} / \left(\frac{\omega_0^2}{\omega_p^2} \frac{V_E^2}{c^2} \frac{W_L}{\sqrt{\pi}\omega_p^2 \tau^2}\right)$, as a function of the dimensionless frequency $x = \omega\tau$:

$$I(x) = \frac{R_L^2}{L^2} x^4 \int_0^1 dy \frac{y^2(1 - y^2)}{y^2 + (1 - 2y^2)(x^2/\omega_p^2 \tau^2)} \times \exp\left\{-\frac{x^2}{2}\left[1 + \frac{R_L^2}{L^2}(1 - y^2)\right]\right\}. \quad (4.7)$$

Dependence (4.7), which characterizes the spectrum of the LF radiation emitted by a laser pulse, is illustrated graphically in Fig. 1, which shows the results obtained for a fixed value of the parameter $\omega_p\tau$ and for different values of the ratio of the focal spot radius to the pulse length. We can see that the spectrum has a flat peak at the frequency $\omega_{max} \cong 1/\tau$, whose position shifts toward lower frequencies as the ratio R_L/L increases.

These results can be obtained analytically by integrating over angles in formula (4.7). In this way, we arrive at the following frequency dependence of the radiation energy:

$$I(x) = \frac{R_L^2}{L^2}x^4\exp\left(-\frac{x^2}{2}\right) \times\left\{\frac{\exp(-a)}{\sqrt{a}}\left(1+\frac{1}{2a}\right)\int_0^{\sqrt{a}}dt\exp(t^2)-\frac{1}{2a}\right\}, \quad (4.8)$$

where $a = x^2R_L^2/2L^2$. Equating the derivative of the function $I(x)$ to zero and using its power series expansion for small values $a < 1$, we obtain from formula (4.8) the position of the maximum in the radiation spectrum:

$$\omega_{max} = \frac{2}{\tau\sqrt{1+(4R_L^2/5L^2)}}. \quad (4.9)$$

For a tightly focused laser pulse ($R_L \ll L$), the radiation energy is maximum at the frequency $\omega_{max} = 2/\tau$. As the focal spot size increases, the maximum in the radiation spectrum shifts toward lower frequencies, in accordance with the numerical results presented in Fig. 1.

Taking the integral over frequency in expression (4.5), we find the angular distribution of the LF radiation energy emitted by a laser pulse:

$$\frac{dW(\theta)}{dO} = \frac{\omega_0^2}{\omega_p^2}\frac{R_L^2}{L^2}\frac{V_E^2}{2c^2}\frac{W_L}{\pi^{3/2}\omega_p^2\tau^2} \times\int_0^\infty dx\frac{x^4\sin^2\theta\cos^2\theta}{\cos^2\theta+(1-2\cos^2\theta)(x^2/\omega_p^2\tau^2)} \times\exp\left[-\frac{x^2}{2}\left(1+\frac{R_L^2}{L^2}\sin^2\theta\right)\right]. \quad (4.10)$$

The dependence of the dimensionless radiation energy $J(\theta) = \frac{dW(\theta)}{dO}\Big/\left(\frac{\omega_0^2}{\omega_p^2}\frac{V_E^2}{2c^2}\frac{W_L}{\pi^{3/2}\omega_p^2\tau^2}\right)$ on the angle θ, namely,

$$J(\theta) = \frac{R_L^2}{L^2}\int_0^\infty dx\frac{x^4\sin^2\theta\cos^2\theta}{\cos^2\theta+(1-2\cos^2\theta)(x^2/\omega_p^2\tau^2)} \times\exp\left[-\frac{x^2}{2}\left(1+\frac{R_L^2}{L^2}\sin^2\theta\right)\right], \quad (4.11)$$

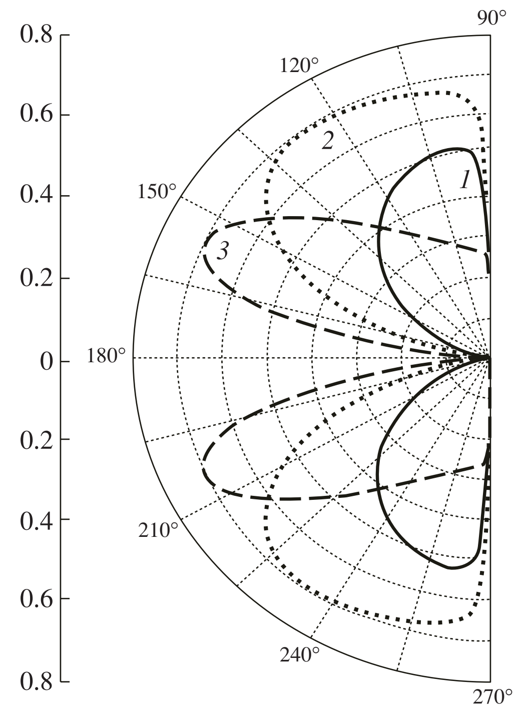

Fig. 2. Directional pattern of the terahertz radiation in the reflection of a laser pulse from a dense plasma ($\omega_p\tau = 100$) for different values of R_L^2/L^2: (*1*) 0.25, (*2*) 1, and (*3*) 4. For a given direction in which the radiation is emitted, the value of $J(\theta)$ (see formula (4.11)) is determined by the radius of the circle passing through the corresponding point of the directional pattern.

is depicted in Fig. 2 in the form of a directional pattern for different values of the ratio of the laser pulse radius to its length. A tightly focused laser pulse ($R_L \ll L$) radiates LF energy predominantly in the transverse direction ($\theta \cong \pi/2$). As the size of the focal spot of the laser pulse increases, the directional pattern is displaced toward smaller angles. A laser pulse with a large transverse size ($R_L \gg L$) emits radiation at small angles, almost along the normal to the plasma boundary.

Let us analyze angular dependence (4.11) analytically for different values of the ratio between the length of the laser pulse and the radius of its focal spot. For a tightly focused laser pulse ($R_L \ll L$), formula (4.11) becomes

$$J(\theta) = 3\sqrt{\frac{\pi}{2}}\frac{R_L^2}{L^2}\frac{\sin^2\theta\cos^2\theta}{\cos^2\theta+(1-2\cos^2\theta)(4/\omega_p^2\tau^2)}. \quad (4.12)$$

The angle at which the intensity of the emitted radiation is maximum is close to $\pi/2$ and is described by the relationship

$$\theta_{max} = \frac{\pi}{2}-\sqrt{\frac{2}{\omega_p\tau}}. \quad (4.13)$$

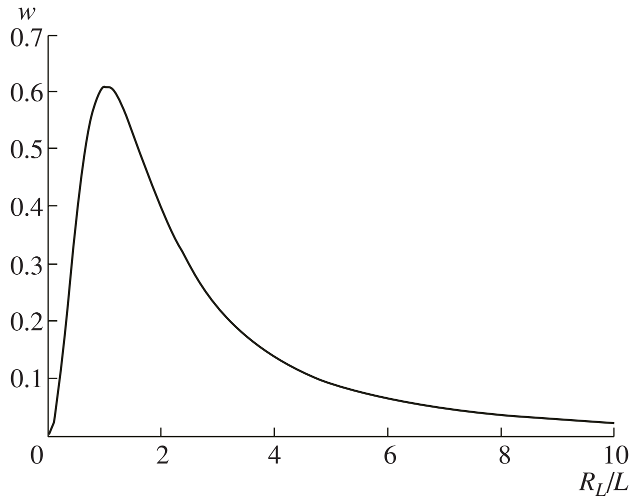

Fig. 3. Dimensionless total energy of the terahertz radiation from a laser pulse in its reflection from a dense plasma ($\omega_p\tau = 100$) as a function of the radius-to-length ratio R/L of the pulse.

For a laser pulse with a wide focal spot ($R_L \gg L$), formula (4.11) yields the following angular distribution of the radiated energy:

$$J(\theta) = 3\sqrt{\frac{\pi}{2}}\frac{R_L^2}{L^2}\frac{\theta^2}{[1+\theta^2(R_L^2/L^2)]^{5/2}}. \qquad (4.14)$$

In this case, radiation is emitted at small angles to the normal to the plasma boundary,

$$\theta_{\max} = \sqrt{2/3}\,L/R_L. \qquad (4.15)$$

By integrating in expression (4.6) over frequencies or in expression (4.10) over angles, we can calculate the total LF energy emitted by a laser pulse in its reflection from the boundary of a dense plasma:

$$W = \frac{\omega_0^2}{\omega_p^2}\frac{R_L^2}{L^2}\frac{V_E^2}{c^2}\frac{W_L}{\sqrt{\pi}\omega_p^2\tau^2}\int_0^\infty dx\int_0^1 dy\frac{x^4y^2(1-y^2)}{y^2+(1-2y^2)(x^2/\omega_p^2\tau^2)}$$
$$\times\exp\left\{-\frac{x^2}{2}\left[1+\frac{R_L^2}{L^2}(1-y^2)\right]\right\}. \qquad (4.16)$$

Simple expressions for the total radiation energy can be derived analytically in two limiting cases. For a tightly focused laser pulse ($R_L \ll L$), from formula (4.12) we obtain

$$W = \frac{\sqrt{2}}{\omega_p^2\tau^2}\frac{\omega_0^2}{\omega_p^2}\frac{R_L^2}{L^2}\frac{V_E^2}{c^2}W_L. \qquad (4.17)$$

In the opposite limit, i.e., for a laser pulse with a wide focal spot such that $R_L \gg L$, the expression for the total energy can be obtained with allowance for formula (4.14):

$$W = \frac{\sqrt{2}}{\omega_p^2\tau^2}\frac{\omega_0^2}{\omega_p^2}\frac{L^2}{R_L^2}\frac{V_E^2}{c^2}W_L. \qquad (4.18)$$

The energy is likely to be maximum for $R_L \approx L$, as is confirmed by a numerical analysis of expression (4.16). Figure 3 illustrates the results of investigating the dimensionless total energy $w = W/\left(\frac{\omega_0^2}{\omega_p^2}\frac{V_E^2}{c^2}\frac{W_L}{\sqrt{\pi}\omega_p^2\tau^2}\right)$ numerically as a function of the ratio between the radius and length of the laser pulse. Calculations from formula (4.16) show that, for $\omega_p\tau > 50$, the dimensionless energy has a maximum equal to $w_{\max} \approx 0.6$ for $R_L \approx L$. Based on this result, we can conclude that the LF radiation energy is maximum,

$$W_{\max} \approx 0.6\frac{\omega_0^2}{\omega_p^2}\frac{V_E^2}{c^2}\frac{W_L}{\sqrt{\pi}\omega_p^2\tau^2}, \qquad (4.19)$$

for a laser pulse whose length is close to the focal spot radius. Using formula (4.19), we can find the coefficient of energy conversion from a laser pulse to terahertz radiation:

$$T = W_{\max}/W_L \approx \frac{0.6}{\sqrt{\pi}\omega_p^2\tau^2}\frac{\omega_0^2}{\omega_p^2}\frac{V_E^2}{c^2}. \qquad (4.20)$$

Terahertz radiation energy (4.19) and conversion coefficient (4.20) are both inversely proportional to the electron plasma density squared. Consequently, it can be expected that the emitted radiation energy will be maximum at electron densities slightly above the critical value.

5. SPATIOTEMPORAL DISTRIBUTION OF THE LF ELECTROMAGNETIC FIELD

Let us consider the spatiotemporal distribution of the LF (terahertz) electromagnetic radiation in vacuum. We apply the inverse Fourier transformation in time to represent expression (3.15) for the magnetic field as

$$B_\phi(\mathbf{r}, t) = \frac{\sqrt{\pi}R_L^2L}{\omega_0}\frac{\omega_0^2}{c^2}\frac{V_E}{2c}E_{0L}\frac{\sin\theta\cos\theta}{r}$$
$$\times\int_0^{+\infty}\frac{d\omega}{2\pi}\frac{\exp\left\{i\frac{\omega}{c}(r-ct)-\frac{\omega^2}{4c^2}(L^2+R_L^2\sin^2\theta)\right\}}{\varepsilon(\omega)\cos\theta+i\sqrt{\omega_p^2/\omega^2-\cos^2\theta}} + \text{c.c.} \qquad (5.1)$$

The electric field components can be calculated from relationships (3.16). Using the saddle point method, we can analytically obtain asymptotic formulas for the integral in expression (5.1) in the wave zone under the conditions $|r-ct| \gg L, R_L$. Calculating the derivative of the power index of the exponential function in the integrand in expression (5.1) and equating it to zero, we find the position of the saddle point in the plane of the complex variable ω:

$$\omega_s = \frac{2ic(r-ct)}{L^2 + R_L^2\sin^2\theta}. \quad (5.2)$$

Deforming the integration contour so that it passes through the saddle point (5.2) along the path of steepest descent, we arrive at the following expression for the magnetic field of the LF radiation in vacuum under the condition $(r-ct)^2 \gg L^2 + R_L^2\sin^2\theta$:

$$B_\phi(\mathbf{r}, t) = \frac{\dfrac{\omega_0}{c}\dfrac{R_L^2 L}{\sqrt{L^2 + R_L^2\sin^2\theta}}\dfrac{V_E}{2c}E_{0L}\dfrac{\sin\theta\cos\theta}{r}\exp\left[-\dfrac{(r-ct)^2}{L^2 + R_L^2\sin^2\theta}\right]}{\cos\theta\left[1 + \dfrac{\omega_p^2(L^2 + R_L^2\sin^2\theta)^2}{4c^2(r-ct)^2}\right] + \sqrt{\cos^2\theta + \dfrac{\omega_p^2(L^2 + R_L^2\sin^2\theta)^2}{4c^2(r-ct)^2}}}. \quad (5.3)$$

The result obtained, namely, formula (5.3), indicates that, in vacuum, LF radiation propagates as an electromagnetic pulse whose duration is determined by the sizes of the laser pulse and depends on the observation angle θ. A laser pulse does not emit terahertz radiation exactly along the normal to the plasma boundary (θ = 0) and in a direction strictly transverse to the normal (θ = π/2). At small observation angles, the duration of the LF electromagnetic radiation pulse is comparable to that of the laser pulse. At observation angles close to θ = π/2, the LF radiation pulse duration is determined by both the length of the laser pulse and its transverse dimensions.

The results of a numerical analysis of the integral in expression (5.1) for the magnetic field of the terahertz radiation are illustrated in Fig. 4 in the form of a distribution of the dimensionless function $H(r, \theta, t) = B_\phi(r, \theta, t)/\left(\dfrac{R_L^2\omega_0}{\sqrt{\pi}c\omega_p^2\tau^2}\dfrac{V_E}{2c}E_{0L}\dfrac{\sin\theta\cos\theta}{r}\right)$, specifically,

$$H(r, \theta, t) = \mathrm{Re}\int_0^{+\infty} dx x^2 \frac{\exp\left\{ix\dfrac{r-ct}{L} - \dfrac{x^2}{4}\left(1 + \dfrac{R_L^2}{L^2}\sin^2\theta\right)\right\}}{\cos\theta\left(\dfrac{x^2}{\omega_p^2\tau^2} - 1\right) + i\dfrac{x^2}{\omega_p^2\tau^2}\sqrt{\dfrac{\omega_p^2\tau^2}{x^2} - \cos^2\theta}}. \quad (5.4)$$

Note that formula (5.3) and the distribution shown in Fig. 4 are similar to the results obtained in [22, 23] for the LF transition radiation emitted by a long ($\omega_p\tau \gg 1$) laser pulse at the boundary of a rarefied plasma.

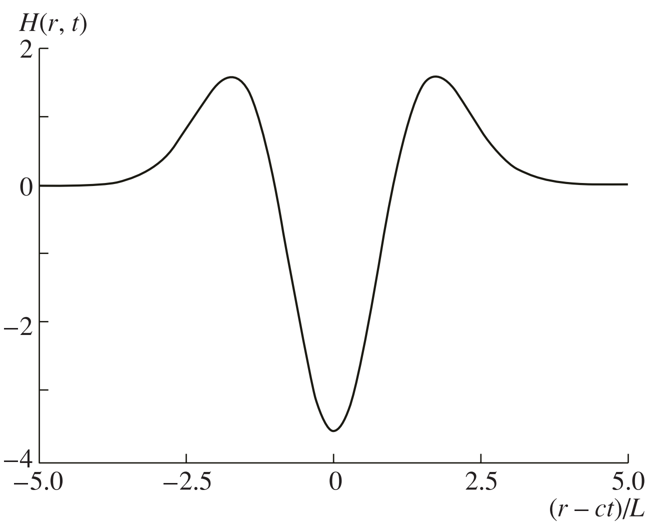

Fig. 4. Spatiotemporal distribution of the dimensionless magnetic field in a terahertz radiation pulse (5.4) generated in the reflection of a laser pulse (with $R_L^2 = 2L^2$) propagating at the angle θ = π/4 from the boundary of a dense plasma ($\omega_p\tau = 100$).

6. CONCLUSIONS

In the present paper, a study has been made of the excitation of LF (terahertz) electromagnetic wave fields in vacuum in the reflection of an incident short laser pulse from the boundary of an overcritical plasma. As a laser pulse is reflected from the plasma boundary, it induces a surface vortex electric current, which generates a short terahertz radiation pulse. The duration of the terahertz radiation pulse is determined by the time of interaction of the laser pulse with the plasma boundary and is comparable to the duration of the laser pulse. The spectrum of the LF radiation and its directional pattern depend strongly on the ratio of the length of the laser pulse to the diameter of its focal spot. In its reflection from the boundary of a dense plasma, a tightly focused laser pulse radiates LF electromagnetic energy predominantly in the transverse direction, the radiation spectrum being peaked at the frequency $\omega_{max} = 2/\tau$. As the radius of the focal spot of the laser pulse increases, this spectral line shifts toward lower frequencies and the directional pattern is displaced toward smaller angles.

The model considered here, namely, that of a plasma with a sharp boundary, makes it possible to substantially simplify the problem by calculating the electromagnetic fields with the help of the boundary conditions. This approximate model is valid when the width of the plasma boundary is less than the laser pulse length. In actuality, such conditions can be achieved, e.g., in experiments with femtosecond laser pulses focused on the surfaces of quartz (SiO_2) aerogels having a very low density, 0.01–0.04 g/cm^3 [26]. In this case, the electron density in the plasma produced through ionization of the target material may be only a few times higher than the critical density. With increasing density of the target material, the terahertz radiation energy decreases markedly, because it is inversely proportional to the electron density squared (see formula (4.19)). As for the interaction of laser pulses with solid-state targets, the efficiency with which terahertz radiation is generated in such conditions is likely to be very low. However, the results reported in the present paper were obtained in the collisionless plasma model and as such cannot be applied to plasmas with a solid-state density, in which the electron–ion collision frequency exceeds the reciprocal of the laser pulse duration and can be comparable to the laser frequency.

Let us estimate the energy of the terahertz electromagnetic radiation generated in the interaction of an incident, 0.8-μm 90-mJ 4.5-TW laser pulse with a peak intensity of 2×10^{17} W/cm^2, duration of 20 fs (corresponding a pulse length of 6 μm), and focal spot radius of 20 μm with the boundary of a SiO_2 aerogel with a density of 0.04 g/cm^3. In a laser-produced plasma with an average ion charge number of 10, the electron density is equal to 4×10^{21} cm^{-3}, which is almost two times the critical density for the given laser wavelength. In this case, the electron–ion collision frequency is $\nu_{e,i} \approx 3 \times 10^{11}$ s^{-1}, which is much less than the reciprocal of the laser pulse duration, $\nu_{e,i}\tau \approx 6 \times 10^{-3}$. From formulas (4.7) and (4.11), we can conclude that electromagnetic energy is radiated at a small angle to the plasma boundary, $\theta \approx 14°$, and the radiation spectrum is peaked at a frequency of about 5.6 THz. The terahertz radiation energy is equal to 0.3 μJ (see formula (4.19)), conversion coefficient (4.20) is equal to 3.2×10^{-6}, and the radiated power is estimated to be 15 MW. These estimates show that the generation mechanism under discussion here should be taken into account in developing broadband high-power frequency-tunable terahertz radiation sources.

ACKNOWLEDGMENTS

I am grateful to L.M. Gorbunov for valuable remarks. This work was supported in part by the Russian Foundation for Basic Research (project no. 07-02-92160-NTsNI_a) and by the Presidium of the Russian Academy of Sciences (program nos. 2, 16, 18).

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