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1997 J. Phys.: Condens. Matter 9 L591

(http://iopscience.iop.org/0953-8984/9/45/001)

J. Phys.: Condens. Matter **9** (1997) L591–L598. Printed in the UK PII: S0953-8984(97)86939-1

LETTER TO THE EDITOR

The electron density of states in terahertz-driven three-dimensional electron gases in quantizing magnetic fields

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Received 11 August 1997

Abstract. A detailed theoretical study of the density of states (DOS) is presented for free electrons in terahertz- (THz-) driven three-dimensional electron gases (3DEGs) in the presence of static magnetic fields. Applying the Green's function approach to a specific sample configuration where the electron–photon interactions can be included exactly, we have derived the steady-state DOS for electrons in a THz-driven 3DEG. The results obtained show that the presence of intense THz electromagnetic radiations will result in a reduction of the DOS in the low-energy regime in a 3DEG in quantizing magnetic fields. A photon-modified electron DOS will lead to the lifting of the Fermi level in a 3DEG system and, consequently, to the occupation of higher Landau levels at a fixed magnetic field. A more pronounced effect can be observed for relatively low-frequency and/or high-intensity radiations.

A very recently achieved experimental set-up [1] has made it possible to perform measurements under intense far-infrared (FIR) or terahertz (THz) laser radiations in the presence of strong magnetic fields. At present, free-electron lasers (FELs) can provide a tunable source of linearly polarized THz electromagnetic (EM) radiations, and static magnetic fields can be generated up to 60 T. It can be foreseen that the combination of intense THz radiations with high magnetic fields will make a major impact on the investigation and characterization of condensed matter materials, such as low-dimensional semiconductor systems and nanostructures.

When an electronic system (e.g. a semiconductor structure) is subjected to THz EM radiations and to quantizing magnetic fields, we enter a regime with different competing energies, such as the Fermi energy (E_F), cyclotron energy ($\hbar\omega_c$), photon energy ($\hbar\omega$), and phonon energy ($\hbar\omega_Q$). These energies (frequencies) can be on the scale of meV (THz). This offers us the possibility of observing photon-induced quantum resonance effects such as photon-modified Shubnikov–de Haas oscillations [2], FIR cyclotron absorptions [3], and magneto-photon–phonon resonances [4].

Like in other studies, in the study of THz-driven three-dimensional electron gases (3DEGs) in strong magnetic fields, the electronic density of states (DOS) is one of the central quantities required to determine and to understand almost all physically measurable properties. Hence, it is of value to examine how EM radiation affects such a fundamental quantity as the DOS for electrons in strong static magnetic fields, and this is the motivation

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0953-8984/97/450591+08$19.50

of the present study. In the absence of a magnetic field, the *dynamical* DOS in 3DEG and ideal 2DEG structures has been investigated very recently [5] by using the approach of the gauge-invariant spectral function. In the presence of a quantizing magnetic field, there is no gauge in which the electron Hamiltonian is translationally invariant [6]. Therefore, it may be difficult to derive the corresponding DOS using the gauge-invariant spectral function. In this study, we present a simple theoretical treatment for calculating the steady-state DOS for free electrons in a THz-driven 3DEG in the presence of high magnetic fields.

We consider the following configuration: the magnetic field $\boldsymbol{B}$ is applied along the z-direction of a 3DEG and the EM radiation field is polarized parallel to the magnetic field. In this case, the most convenient gauge for describing the two uniform fields is

$$\phi(\boldsymbol{R}, t) = 0 \qquad \text{and} \qquad \boldsymbol{A}(\boldsymbol{R}, t) = (0, Bx, A_z(t)) \tag{1}$$

where $\boldsymbol{R} = (x, y, z)$. Here, we have used the Landau gauge (Coulomb gauge) for the vector and scalar potentials induced by the static magnetic field (radiation field). The usage of the Coulomb gauge [7] allows us to choose the vector potential $\boldsymbol{A}_1 = (0, 0, A_z(t))$ and the scalar potential ϕ_1 for the radiation field such that $\boldsymbol{\nabla} \cdot \boldsymbol{A}_1 = 0$ and $\phi_1 = 0$. The gauge chosen here corresponds to a situation in which the charge density $\rho = 0$ and the current density $\boldsymbol{j} = 0$, which is true for the case of free electrons (i.e. in the absence of scattering, inhomogeneities, external driving fields, etc). Furthermore, after using the dipole approximation for the EM field, we can write $A_z(t) = A_0 \sin(\omega t)$ with ω being the frequency of the radiation. Thus, the single-electron Hamiltonian in this gauge can be written as

$$H_0(t) = \frac{1}{2m^*}[p_x^2 + (p_y - eBx)^2 + (p_z - eA_z(t))^2] \tag{2}$$

where m^* is the effective electron mass and $p_{x_j} = -\mathrm{i}\hbar\, \partial/\partial x_j$ is the momentum operator. The time-dependent Schrödinger equation: $\mathrm{i}\hbar\, \partial\Psi/\partial t = H_0(t)\Psi$ can be solved analytically and the electron wavefunction is obtained as

$$|N, k_y, k_z; t\rangle = |N, k_y, k_z; 0\rangle \mathrm{e}^{-\mathrm{i}[E_N(k_z)+2\gamma\hbar\omega]t/\hbar} \mathrm{e}^{\mathrm{i}r_0 k_z[1-\cos(\omega t)]} \mathrm{e}^{\mathrm{i}\gamma \sin(2\omega t)} \tag{3a}$$

where

$$|N, k_y, k_z; 0\rangle = (2^N N! \pi^{1/2} l)^{-1/2} \mathrm{e}^{\mathrm{i}(k_y y + k_z z)} \mathrm{e}^{-\xi^2/2} H_N(\xi) \tag{3b}$$

which has been normalized. Here, k_{x_j} is the electron wavevector along the x_j-direction, $l = (\hbar/eB)^{1/2}$ is the radius of the ground cyclotron orbit, $\xi = (x + l^2 k_y)/l$, the $H_N(x)$ are the Hermite polynomials, $E_N(k_z) = \hbar^2 k_z^2/2m^* + E_N$ is the electronic energy spectrum, $E_N = (N + 1/2)\hbar\omega_c$ is the energy of the Nth Landau level (LL) where $\omega_c = eB/m^*$ is the cyclotron frequency, $r_0 = eF_0/m^*\omega^2$ where F_0 is the strength of the radiation electric field, $\gamma = (eF_0)^2/(8m^*\hbar\omega^3)$, and $2\gamma\hbar\omega$ is the energy of the radiation field. We have used the relation $\boldsymbol{F} = \partial \boldsymbol{A}_1/\partial t = F_0 \cos(\omega t)$ with $F_0 = \omega A_0$.

With the time-dependent electron wavefunction obtained from the solution of the time-dependent Schrödinger equation, one can derive the Green's function in $(N, k_z; t)$-space for the system. In the present study, we generalize the general approaches documented in reference [8] to derive the Green's function for the current situation. From equation (3), we can calculate the probability amplitude, which describes a process in which if one adds an electron in a state $|k'_y, k'_z, N'\rangle$ at time t' to the system then the system will be in a state $|k_y, k_z, N\rangle$ at time t, through

$$\langle t'; k'_z, k'_y; N' | N, k_y, k_z; t\rangle = \delta_{N',N} \delta_{k'_y,k_y} \delta_{k'_z,k_z} R(N, k_z; t, t') \tag{4a}$$

where

$$R(N, k_z; t, t') = \mathrm{e}^{-\mathrm{i}[E_N(k_z)+2\gamma\hbar\omega](t-t')/\hbar} \mathrm{e}^{-\mathrm{i}r_0 k_z[\cos(\omega t)-\cos(\omega t')]} \mathrm{e}^{\mathrm{i}\gamma[\sin(2\omega t)-\sin(2\omega t')]}. \tag{4b}$$

Hence, by definition, the corresponding retarded propagator or Green's function for electrons should have the features

$$G^+(N', k'_y, k'_z; N, k_y, k_z; t > t') = \delta_{N',N}\delta_{k'_y,k_y}\delta_{k'_z,k_z}G^+(N, k_z; t > t') \tag{5}$$

and

$$G^+(N, k_z; t > t') = -\frac{\mathrm{i}}{\hbar}\Theta(t - t')R(N, k_z; t, t') \tag{6}$$

where $\Theta(x)$ is the unit-step function. Equation (6) is a two-time Green's function and satisfies

$$\left[\mathrm{i}\hbar\frac{\partial}{\partial t} - H_0(t)\right]G^+(N, k_z; t > t')|N, k_y, k_z; 0\rangle = \delta(t - t')|N, k_y, k_z; 0\rangle$$

because $\{[p_x^2 + (p_y - eBx)^2]/2m^* - E_N\}\psi_{N,k_y}(x, y) = 0$ has been solved in real space.

The Fourier transform (or average over time $t - t'$) of the retarded Green function is given by

$$\begin{aligned} G_{N,k_z}(E, t') &= \int_{-\infty}^{\infty} \mathrm{d}(t - t')\ \mathrm{e}^{\mathrm{i}(E+\mathrm{i}\delta)(t-t')/\hbar}G^+(N, k_z; t > t') \\ &= \sum_{m=-\infty}^{\infty} \frac{\mathcal{F}_m(k_z, t')}{E - E_N(k_z) - 2\gamma\hbar\omega - m\hbar\omega + \mathrm{i}\delta} \end{aligned} \tag{7a}$$

where an infinitesimal quantity $\mathrm{i}\delta$ has been introduced to make the integral converge. Here

$$\mathcal{F}_m(k_z, t') = (-1)^m F_m(k_z) \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_{m+n}(r_0k_z)\mathrm{e}^{\mathrm{i}[n\omega t' - \gamma\sin(2\omega t')]} \tag{7b}$$

where $J_m(x)$ is a Bessel function and

$$F_m(k_z) = \sum_{n=0}^{\infty} \frac{J_n(\gamma)}{1 + \delta_{n,0}}[J_{2n-m}(r_0k_z) + (-1)^{m+n}J_{2n+m}(r_0k_z)]. \tag{7c}$$

In this study, we are interested in a steady state where we can average over the initial time t'. After averaging t' over a period of the radiation field [5], the averaged Green function becomes

$$G^*_{N,k_z}(E) = \sum_{m=-\infty}^{\infty} \frac{F_m^2(k_z)}{E - E_N(k_z) - 2\gamma\hbar\omega - m\hbar\omega + \mathrm{i}\delta}. \tag{8}$$

The DOS for electrons in the Nth LL is determined by the imaginary part of the Fourier transform of the Green's function, namely

$$\begin{aligned} D_N(E) &= -\frac{g_s}{\pi}\frac{1}{2\pi l^2}\sum_{k_z} \mathrm{Im}\, G^*_{N,k_z}(E) \\ &= \frac{g_s}{4\pi^2 l^2}\sqrt{\frac{2m^*}{\hbar^2}}\sum_m \frac{\Theta(E - E_N - 2\gamma\hbar\omega - m\hbar\omega)}{\sqrt{E - E_N - 2\gamma\hbar\omega - m\hbar\omega}} \\ &\quad\times F_m^2\left(\sqrt{\frac{2m^*(E - E_N - 2\gamma\hbar\omega - m\hbar\omega)}{\hbar^2}}\right) \end{aligned} \tag{9}$$

where $g_s = 2$ accounts for the spin degeneracy, and we have taken into account the fact that the degeneracy of each LL is given by $1/2\pi l^2$ in unit area. When $F_0 = 0$ (i.e. $r_0 = \gamma = 0$), equation (9) becomes the well-known result obtained in the absence of the radiation, due

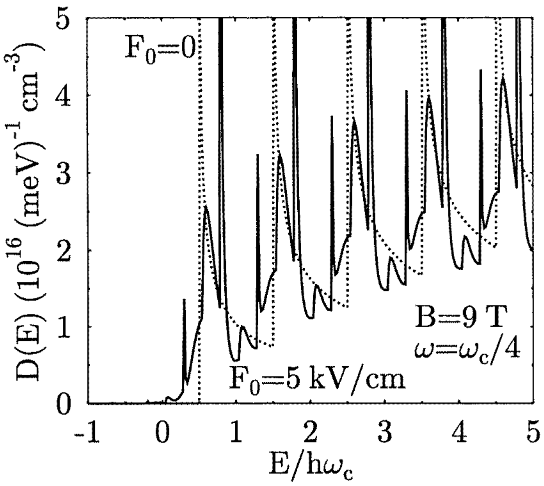

Figure 1. The total density of states ($D(E) = \sum_N D_N(E)$) as a function of the electron energy E at a fixed radiation frequency ω and a fixed magnetic field B for different radiation intensities: $F_0 = 0$ (dotted curve) and $F_0 = 5$ kV cm^{-1} (solid curve). ω_c is the cyclotron frequency. When $\omega = \omega_c/4$ at $B = 9$ T, $\omega/2\pi \simeq 0.95$ THz, $\hbar\omega \simeq 3.92$ meV and $\gamma \simeq 0.60$ at $F_0 = 5$ kV cm^{-1}. The effective electron mass is taken as $m^*/m_e = 0.0665$ (for the material GaAs) with m_e the electron rest mass.

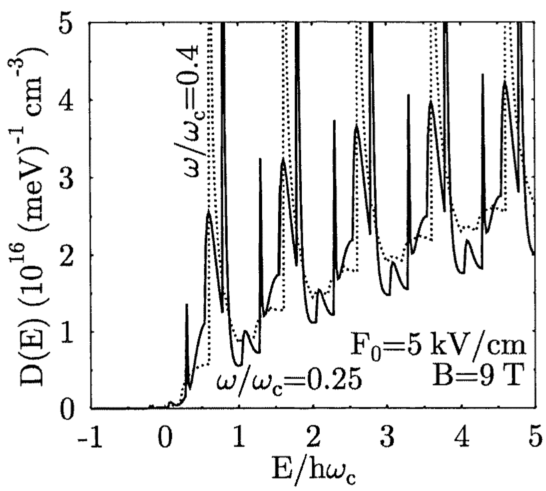

Figure 2. The total density of states as a function of the electron energy at a fixed radiation intensity and a fixed magnetic field for different radiation frequencies: $\omega/\omega_c = 0.4$ (dotted curve; here $\hbar\omega \simeq 6.27$ meV and $\gamma \simeq 0.15$) and 0.25 (solid curve; see figure 1).

to the feature that $J_m(0) = \delta_{m,0}$. In the above equations, $m = 1, 2, 3, \ldots$ $(-1, -2, -3, \ldots)$ corresponds to the absorption (emission) of $1, 2, 3, \ldots$ photons with the frequency ω.

The influence of the strength (F_0) and frequency (ω) of the THz radiation on the total DOS (i.e. $D(E) = \sum_N D_N(E)$) for electrons in a 3DEG at a fixed magnetic field (B) is shown in figures 1 and 2. From equation (9), we see that: (i) in the presence of the EM radiations, the energy of the electronic system is shifted by the energy of the radiation field

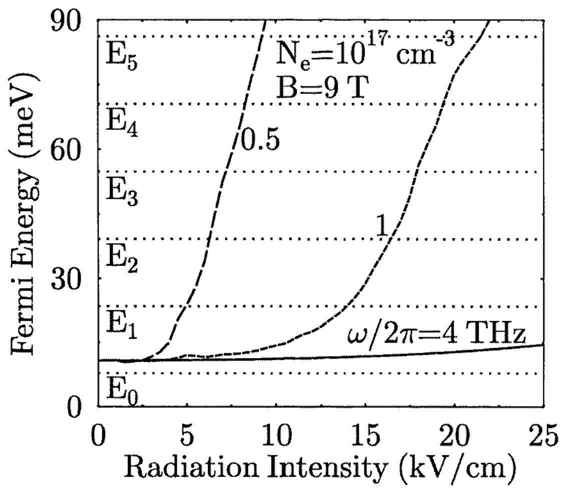

Figure 3. The Fermi energy E_F as a function of the radiation intensity F_0 at a fixed magnetic field for different radiation frequencies. Here, N_e is the electron density and $E_N = (N + 1/2)\hbar\omega_c$ is the energy of the Nth Landau level.

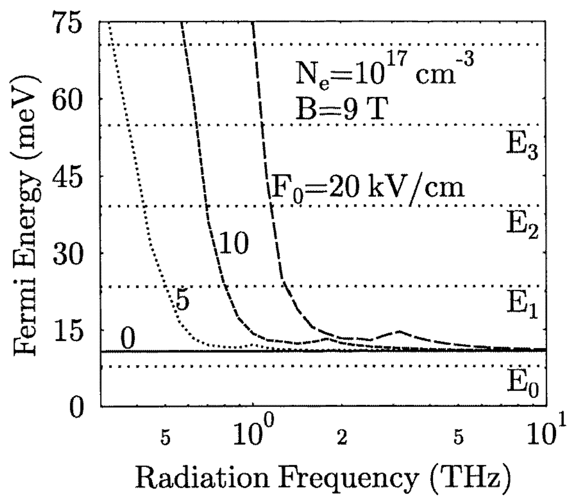

Figure 4. The Fermi energy as a function of the radiation frequency $\omega/2\pi$ at a fixed magnetic field for different strengths of the radiation.

$2\gamma\hbar\omega \sim (F_0/\omega)^2$. This is analogous to the blue shift of the absorption edge observed in, e.g., the dynamic Franz–Keldysh effect in the absence of the magnetic field [5]; (ii) the electrons in the system can interact with the radiation field via the processes with photon absorption and emission; and (iii) with increasing electron energy E, a contribution from the process of m-photon absorption ($-$) or emission ($+$) to the DOS becomes possible when the condition $E - E_N - 2\gamma\hbar\omega \mp m\hbar\omega \geqslant 0$ is satisfied. These can be observed in figure 1 (solid curve) when $E/\hbar\omega_c$ is around $\gamma/2 + N$, $\gamma/2 + N + 0.25$, $\gamma/2 + N + 0.5$, $\gamma/2 + N + 0.75$ and $\gamma/2 + N + 1$ (which correspond, respectively, to the processes with $m = -2, -1, 0, 1$ and 2). A similar feature can be seen in figure 2. We note that in sharp contrast to the case of $F_0 = 0$ (see figure 1), in the presence of the EM radiation the electron DOS can be present in the energy regime where $E - E_N - 2\gamma\hbar\omega < 0$ due to the contributions from the processes of the photon emission. The processes of optical absorption and emission may result in an increase in the DOS. However, in the presence of intense THz EM radiations and of strong magnetic fields, the maximum DOS appears when E approaches $E_N + 2\gamma\hbar\omega$ for the zero-photon process (because $\lim_{x\to 0} J_m(x) = \delta_{m,0}$). As a consequence, the overall DOS in the *low-energy regime* will be reduced, due to a large blue-shift induced by the radiation field ($2\gamma\hbar\omega \sim (F_0/\omega)^2$) and to the fact that $|J_m(x)| \leqslant 1$, in comparison with the fact that at $F_0 = 0$ (see figure 1). The DOS measures the maximum number of electrons which can occupy an energy range. The EM field applied will drive electrons out of the low-energy regime, so a reduced electron DOS in the low-energy regime can be achieved. Due to the limiting feature $\lim_{x\to 0} J_m(x) = \delta_{m,0}$, for radiation with relatively high frequency and/or low intensity, which leads to $r_0 \ll 1$ and $\gamma \ll 1$, the effects of the radiation on the DOS can be suppressed. Moreover, because $r_0 \sim F_0/\omega^2$ and $\gamma \sim F_0^2/\omega^3$, the radiation frequency has a stronger effect on the DOS.

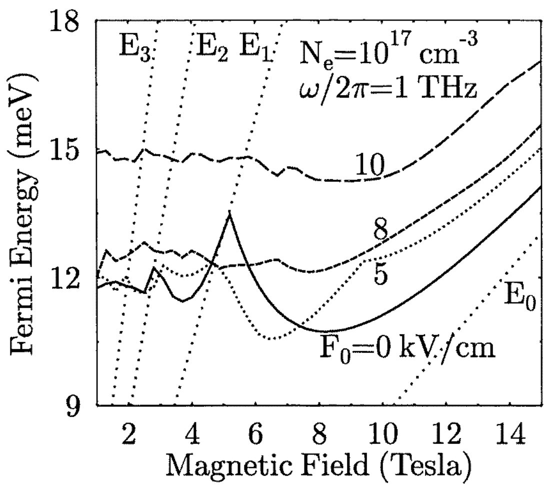

Figure 5. The Fermi energy in a 3DEG as a function of the magnetic field at a fixed radiation frequency for different radiation intensities.

A direct and important application of the DOS is that in determining the Fermi energy in an electronic system. Using the condition of electron number conservation, the Fermi energy E_F for a 3DEG subjected to EM radiation and to a magnetic field can be determined, for the case of the low-temperature limit ($T \to 0$) and of where the total electron density N_e in the system is not varied by the presence of the radiation field and of the magnetic field, by

$$N_e = \frac{1}{\pi^2 l^2} \sum_{N,m} \Theta(E_F - E_N - 2\gamma\hbar\omega - m\hbar\omega) \int_0^{\sqrt{2m^*(E_F - E_N - 2\gamma\hbar\omega - m\hbar\omega)/\hbar^2}} \mathrm{d}x\ F_m^2(x). \tag{10}$$

The dependence of the Fermi energy in a 3DEG on the strength and frequency of the THz driving fields at a fixed magnetic field is shown in figures 3 and 4. Because of the reduction of the DOS in the low-energy regime by the radiation field, especially in the low-frequency and high-intensity regimes, the electron occupation of the higher LLs can be observed for radiation with low frequency (see figure 4) and/or high intensity (see figure 3). For high-frequency (e.g. $\omega/2\pi = 4$ THz in figure 3 and $\omega/2\pi > 6$ THz in figure 4) and/or low-intensity (e.g. $F_0 \to 0$ in figure 3) EM fields, the Fermi energy depends very weakly on the radiation. The theoretical results indicate that in the low-frequency ($\omega \ll 1$) and/or high-intensity ($F_0 \gg 1$) limit where $2\gamma\hbar\omega$ is much larger than E_N and $\hbar\omega$, $E_F \sim 2\gamma\hbar\omega \sim (F_0/\omega)^2$. A significant conclusion that we draw from these results is that by varying the strength and/or frequency of the THz EM radiation, one can tune the electron population in different LLs very efficiently and, consequently, photon-modified Shubnikov–de Haas oscillations may be observed. In figure 5 we show the dependence of the Fermi energy in a 3DEG on the magnetic field at a fixed radiation frequency for different radiation intensities. It can be seen again that E_F increases with increasing F_0 for magnetic fields up to 14 T. The physical reason behind the strong effect of the EM radiation on the electron DOS and the Fermi energy can be understood on the basis of the fact that for a GaAs-based electron gas driven by an EM field with $F_0 \sim 1$ kV cm^{-1} and $\omega \sim 1$ THz, conditions such as $r_0[2m^*(E_F - E_N - 2\gamma\hbar\omega \mp m\hbar\omega)/\hbar^2]^{1/2} \sim 1$ and $\gamma \sim 1$ can be satisfied. As a consequence, (i) the energy of the electronic system is shifted by the energy of the EM field; (ii) the electrons in the system can interact with the radiation field via the processes of photon absorption and emission; and (iii) the features that are specific to electron–photon interactions can be exposed.

In the present study we have derived the DOS for noninteracting electrons in a THz-driven 3DEG in quantizing magnetic fields, using a Green's function approach and including the electron–photon interaction exactly. We have studied the influence of intense THz radiation and a strong magnetic field on the electron DOS and Fermi energy in a 3DEG structure. We found that: (1) the DOS and the Fermi energy for a THz-driven 3DEG will be strongly modulated by the frequency and strength of the radiation field; (2) applying an EM driving field to a 3DEG will result in a decrease in the DOS in the low-energy regime and, consequently, in an increase in the Fermi energy, due to the nature of the interactions between the electrons and EM radiation fields; (3) a stronger effect of the radiation on the DOS and the Fermi energy can be observed at relatively low frequencies and/or high intensities; (4) the processes of optical absorption and emission, including multiphoton absorption and emission, have a relatively weak effect on the Fermi energy in comparison with the effects caused by the energy shift; (5) by varying the frequency and/or strength of the THz radiation, the electron population in different Landau levels can be varied; and (6) the effects of the EM radiation on the electron DOS in strong magnetic fields are very similar to those observed in the dynamic Franz–Keldysh effect at $B = 0$ [5]. The Green's function and the DOS for noninteracting electrons in a THz-driven 3DEG in strong magnetic fields, obtained from this study, can be used for further investigations of the magneto-transport and magneto-optical properties in, e.g., semiconductor materials. It should be noted that, in the present study, we did not include the effects of electron correlations on the filling of the LLs. It is well known [9, 10] that, in the absence of an EM radiation field, these effects can change the population of the LLs significantly when E_F approaches the edge of a Landau band. The single-electron Green's function obtained here can also be used to study this problem when an EM field is present.

The phenomena predicted and discussed in this letter may be observed within the radiation intensity and frequency regimes of recently developed free-electron lasers such as the UCSB FELs [11] and the FELIX [1, 12]. We hope that the phenomena presented and discussed in this letter will be verified experimentally.

References

[1] Vaughan T A, Nicholas R J, Langerak C J G M, Murdin B N, Pidgeon C R, Mason N J and Walker P J 1996 *Phys. Rev.* B **53** 16 481

[2] Mena R A, Schacham S E, Haugland E J, Alterovitz S A, Young P G, Bibyk S B and Ringel S A 1995 *J. Appl. Phys.* **78** 6626

[3] Zhao Y, Tsui D C, Santos M B, Shayegan M, Ghanbari R A, Antoniadis D A and Smith H I 1995 *Phys. Rev.* B **51** 13 174 and references therein

[4] Xu W and Zhang C 1996 *Phys. Rev.* B **54** 4907

[5] Jauho A P and Johnsen K 1996 *Phys. Rev. Lett.* **76** 4576

[6] Pottier N and Calecki D 1982 *Physica* A **110** 471

[7] See, e.g.,
Shankar R 1980 *Principles of Quantum Mechanics* (New York: Plenum)

[8] See, e.g.,
Mattuck R D 1976 *A Guide to Feynman Diagrams in the Many-Body Problem* (New York: McGraw-Hill)

[9] Skudlarski P and Vignale G 1992 *Phys. Rev. Lett.* **69** 949
Skudlarski P and Vignale G 1993 *Phys. Rev.* B **48** 8547

[10] Xu W, Vasilopoulos P, Das M P and Peeters F M 1995 *J. Phys.: Condens. Matter* **7** 4419
Xu W and Vasilopoulos P 1995 *Phys. Rev.* B **51** 1694

[11] Asmar N G, Markelz A G, Gwinn E G, C̃erne J, Sherwin M S, Campman K L, Hopkins P E and Gossard A C 1995 *Phys. Rev.* B **51** 18 041
Asmar N G, C̃erne J, Markelz A G, Gwinn E G, Sherwin M S, Campman K L and Gossard A C 1996 *Appl. Phys. Lett.* **68** 829

[12] Murdin B N, Heiss W, Langerak C J G M, Lee S-C, Galbraith I, Strasser G, Gornik E, Helm M and Pidgeon C R 1997 *Phys. Rev.* B **55** 5171