Available online at www.sciencedirect.com
SCIENCE DIRECT®
Optics Communications 246 (2005) 429–435
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

Negative refraction at the interface of uniaxial anisotropic media ☆

Lu Yonghua *, Wang Pei, Yao Peijun, Xie Jianping, Ming Hai

Department of Physics, University of Science and Technology of China, JinZhai Road 96, Hefei 230026, China

Received 3 July 2004; received in revised form 3 October 2004; accepted 5 November 2004

Abstract

In this paper, negative refraction at the interface of uniaxial crystal is discussed in detail. It is pointed out that dispersion relation is more useful to narrate the extraordinary light refraction in uniaxial crystal, especially in the case of negative refraction. Generally, the negative refraction only exists within a small incident angle around 10°. The negative refraction here is very different from that in left-handed material where wave vector and energy flux are anti-parallel.

PACS: 78.20.Ci; 42.25.Gy
Keywords: Negative refraction; Uniaxial crystal; Equifrequency surface contour

1. Introduction

Negative refractive material (NRM) is a hot research field recently for its unique properties, such as "superlens", reversal of Doppler shift and Cerenkov radiation etc. This kind of material is firstly proposed and theoretically investigated by Veselago in 1968 [1], which is characterized by both negative permittivity ($\varepsilon < 0$) and negative permeability ($\mu < 0$). The basic phenomenon in such material is negative refraction across the interface of this NRM and ordinary material (such as air). Not only experiments [2–5] but also numerical simulations [6,7] have proved that metamaterial combining split resonant ring (SRR) [8] and metal rod array [9] really refract negatively within some proper frequency band.

However, it should be pointed out that negative refractive index is not necessary to produce negative refraction phenomenon. It has been

☆ Sponsored by the National Natural Science Foundation of China (90206002); the National High Technology Research and Development Program of China (2002AA313030); Natural science Foundation of Anhui Province (2004kj364zd); Project of Education Stimulation face to 21st century.

* Corresponding author. Tel.: +86 551 3603504; fax: +86 551 3603504/1073.

E-mail address: yhlu@ustc.edu.cn (L. Yonghua).

0030-4018/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2004.11.025

reported that negative refraction was also observed and interpreted in optical crystal [10] or uniaxial anisotropic material [11] where only partial components in its permittivity and permeability tensor are negative, even all the components are totally positive.

In our research, negative refraction will be verified theoretically at the interface of uniaxial crystal and normal material. The detailed refraction behavior will be investigated at such crystal interface and twin crystal interface. The negative refraction condition and the underlying physics will also be discussed.

2. Dispersion relation in uniaxial anisotropic media

It is well known that Maxwell equations and material equations are two foundations to describe the optical properties of material. In dealing with uniaxial anisotropic media (UAM) such as uniaxial crystal, electrical anisotropy induce a tensorial permittivity $\boldsymbol{\varepsilon}$ into the material equation $\boldsymbol{D} = \boldsymbol{\varepsilon} \cdot \boldsymbol{E}$ while it is a scalar in isotropic dielectric. Consequently, electric displacement vector $\boldsymbol{D}$ is no longer in the direction of electric vector $\boldsymbol{E}$. In principal axis coordinate system, permittivity tensor is as following:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}. \quad (1)$$

On the assumption that optic axis is in z-direction, $\varepsilon_x = \varepsilon_y = \varepsilon_o$ is named as ordinary permittivity and $\varepsilon_z = \varepsilon_e$ extraordinary permittivity. Light propagating in uniaxial crystal can be classified into ordinary or extraordinary wave, where ordinary wave with polarization perpendicular to the principal plane, propagates just the same as that in isotropic medium; extraordinary wave with polarization in the principal plane, has different propagating velocity depending on the angle between wave normal and the optic axis.

In order to derive the dispersion relation, plane wave $e^{i(\omega t - \vec{k}\cdot\vec{r})}$ is assumed to propagate in uniaxial crystal, and then electromagnetic wave obeys following Maxwell equations:

$$\begin{cases} \boldsymbol{k} \times \boldsymbol{E} = \frac{\omega}{c}\boldsymbol{B}, \\ \boldsymbol{k} \times \boldsymbol{H} = -\frac{\omega}{c}\boldsymbol{D}. \end{cases} \quad (2)$$

Combining with material equation, dispersion relation of uniaxial crystal can be deduced as

$$\left(\frac{\omega}{c}\right)^2 = \frac{k_x^2}{\varepsilon_o} + \frac{k_y^2}{\varepsilon_e}. \quad (3)$$

Therefore, equi-frequency contour in $\boldsymbol{k}$ space is an ellipse (Fig. 1).

Maxwell equations also indicate that vectors $\boldsymbol{D}$, $\boldsymbol{H}$, $\boldsymbol{k}$ constitute a right-handed set. However, energy flux vector $\boldsymbol{s}$ represents the real light propagating direction, which is defined as $\boldsymbol{s} = \boldsymbol{E} \times \boldsymbol{H}$. Because $\boldsymbol{D}$ and $\boldsymbol{E}$ in uniaxial crystal for extraordinary wave are not parallel to each other, energy flow propagating direction is no longer corresponded to wave vector $\boldsymbol{k}$. Fortunately, light-propagating direction can also be represented by group velocity $v_g = \nabla_k \omega(\boldsymbol{k})$, such a definition indicates that the direction of group velocity can be determined by the normal to equi-frequency contour and directed in increased frequency. Therefore, after drawing the dispersion relation curve, i.e., the equi-frequency contour in $\boldsymbol{k}$ space, extraor-

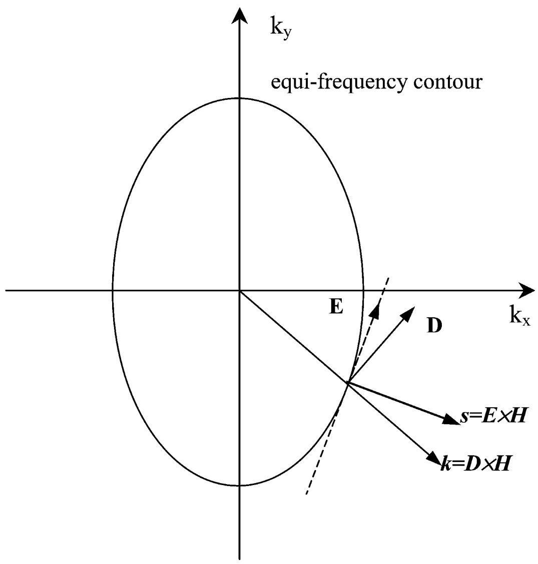

Fig. 1. Dispersion relation of uniaxial crystal. The energy flow direction $\boldsymbol{s}$ and wave vector direction $\boldsymbol{k}$ can both be determined from the dispersion relation.

dinary light propagating direction in uniaxial crystal can be graphically determined as shown in Fig. 1. In the following, it will be also shown that such graphical method can also be used to determine the refraction direction at the interface of uniaxial crystal instead of Snell's law in isotropic material.

3. Negative refraction at interface of UAM

It is well known that, refraction of extraordinary wave does not obey Snell's law; we will show that equi-frequency in $\boldsymbol{k}$ space is a useful tool to describe this kind of refraction. Some distinct phenomena such as negative refraction can be graphically depicted. Essentially, refraction is a kind of electromagnetic boundary problem, where boundary conditions are the main physics. Continuous condition of tangential component of electrical vector requires that tangential component of wave vector $\boldsymbol{k}$ is continuous too across the interface, that is, $k_{1y} = k_{2y}$ in Fig. 2. Of cause, angular frequency ω should keep unchanged across the interface because of energy conservation. Therefore, under the help of equi-frequency contour, refraction direction can be determined in following steps: First, the equi-frequency contour at a given frequency for the materials on both sides of the interface is drawn and the incident light is marked on it; Second, the wave vector of refraction on equi-frequency contour is determined by keeping the tangential wave vectors on both sides equal; Third, the refraction direction is determined according to $v_g = \nabla_k \omega(\boldsymbol{k})$, that is, the normal to the equi-frequency contour and directed in increased frequency.

As shown in Fig. 2, refraction at the interface of air-crystal (Fig. 2(a)), where optic axis is inclined with angle α to the interface, is schemed in Fig. 2(b). Optical dispersion relation in the air is $(\omega/c)^2 = k^2$, which is corresponded to a sphere in $\boldsymbol{k}$-space, thus wave vector and propagating direction is paralleled to each other. Because optic axis of the crystal is inclined with an angle α to the interface (k_y coordinate axis in $\boldsymbol{k}$-space), its dispersion relation defined in formula (3) should be rewritten in new coordinate system as

$$\left(\frac{\omega}{c}\right)^2 = \frac{(k_y \cos\alpha - k_x \sin\alpha)^2}{\varepsilon_o} + \frac{(k_y \sin\alpha + k_x \cos\alpha)^2}{\varepsilon_e}, \tag{4}$$

which is an inclined ellipse with its short axis along the optic axis.

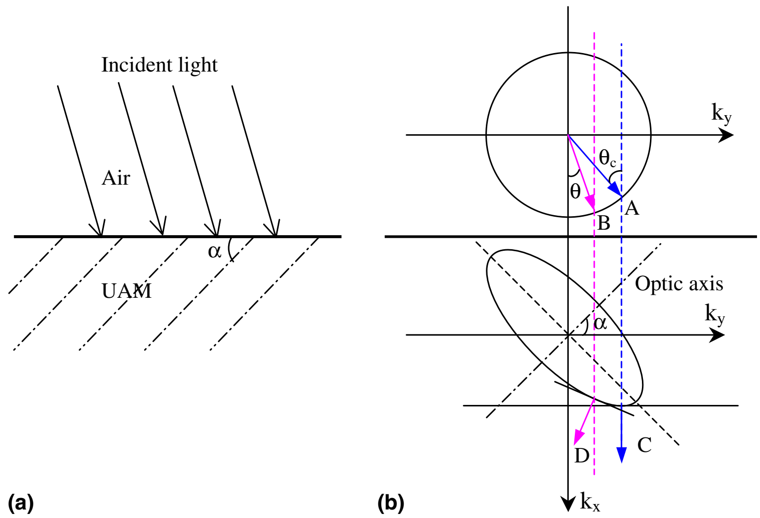

Fig. 2. (a) Interface of air and uniaxial crystal, the optic axis of crystal is inclined to the interface; (b) Diagram of equi-frequency contour for refraction at air-UAM interface, negative refraction range is depicted from the equi-frequency contour.

In Fig. 2(b), supposing that the incident angle from the air is θ, represented by arrow B, the corresponding refraction direction is arrow according to the laws described above. It is clear, when the incident light is in a proper range, for example between 0° and θ_c in Fig. 2(b), the refraction light and the incident light lay on the same side of the normal to the interface, that is, negative refraction. Here, negative refraction is produced without antiparallelity of group velocity and wave vector as that in left-handed material. In fact, such negative refraction can also be predicted graphically with Huygens' construction. As shown in Fig. 3, when the incident angle is so small that the tangential point T is on the left of the normal, the refraction angle of extraordinary light becomes negative, which is corresponded with the description in equi-frequency contour frame.

Using the same method, negative refraction in a twin uniaxial anisotropic material, proposed in [10], can also be graphically described. The twin structure means that optic axis of the crystal is reflective symmetrical to the interface, which is shown in Fig. 4(a) and its equi-frequency contour diagram is shown in Fig. 4(b). When the incident light lies between arrow A and C, for example arrow B, the corresponding refraction light is on the same side of normal of the interface, that is, negative refraction, which is observed in [10]. Comparing to the case of single crystal surface, negative refraction range of $\boldsymbol{k}$ vector is doubled in such twin structure. And the most important is that both incident and refraction light have almost the same refractive index in twin crystal, which means the energy can transfer across the interface nearly without reflection, which is identical with the result in [10] too.

4. Discussion

From above analysis based on equi-frequency contour, some qualitative conclusions about the refraction at the interface of single or twin-structured uniaxial crystal can be drawn as following: Light can refract both negatively and positively; In Figs. 2 and 4, while incidence is toward right, there is a maximum critical angle θ_c, light refracts negatively when indent angle is less than θ_c, and positively while incident angle is greater than θ_c;

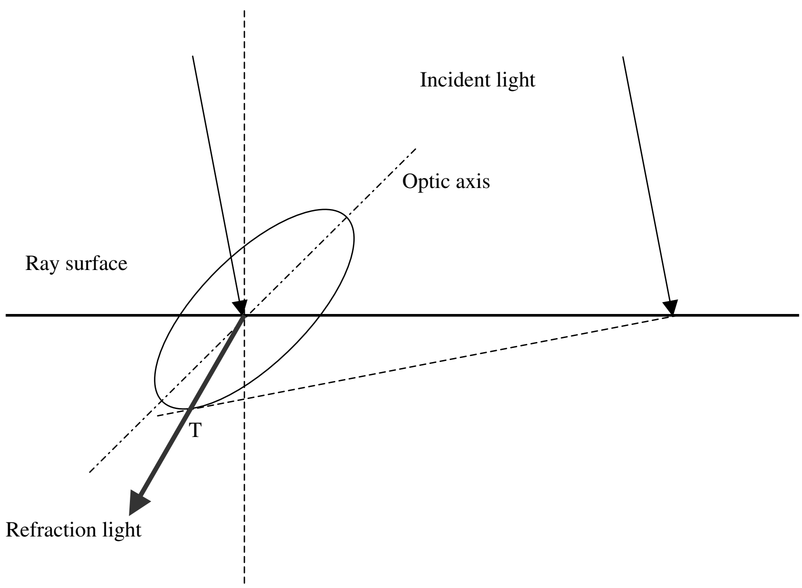

Fig. 3. Huygens' construction is used to indicate negative refraction at the interface of uniaxial crystal (positive crystal). Ray surface in uniaxial crystal is ellipsoid.

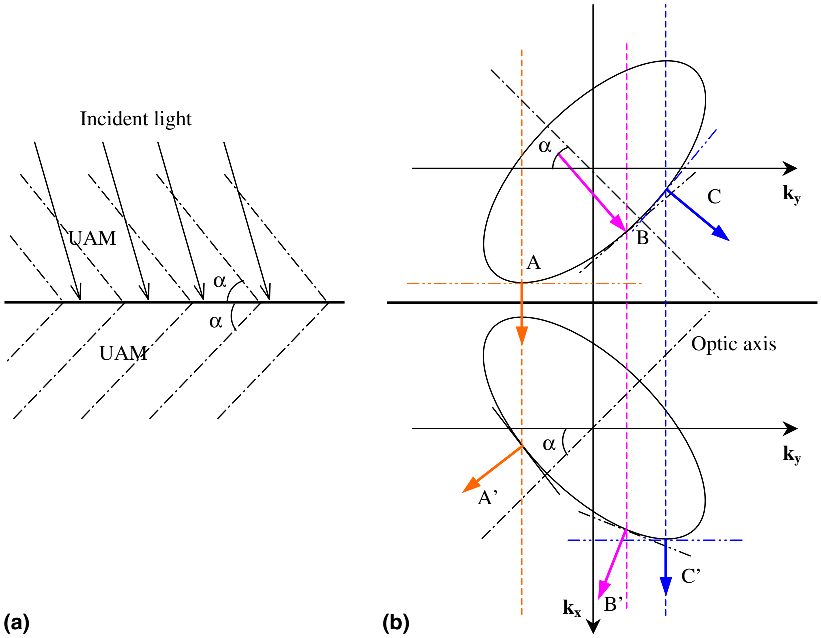

Fig. 4. (a) Interface of twin uniaxial crystal, their optic axes are coplanar with the incident plane and oriented symmetrically with respect to the interface; (b) Diagram of equi-frequency contour is used to depict negative refraction at twin UAM interface.

While incidence is toward left, light refracts always positively. Obviously, the critical angle θ_c is an important parameter and its dependence on the optic axis orientation of the crystal is the most important relationship. Both of them can be determined from the dispersion relation of the crystal.

In Fig. 2, the critical angle θ_c is corresponded to the refraction of arrow C, where $dk_x/dk_y = 0$. Applying this condition to dispersion relation of formula (4),

$$\frac{dk_x}{dk_y} = \frac{k_y\left(\frac{\cos^2\alpha}{\varepsilon_o} + \frac{\sin^2\alpha}{\varepsilon_e}\right) - k_x\left(\frac{1}{\varepsilon_o} - \frac{1}{\varepsilon_e}\right)\sin\alpha\cos\alpha}{k_y\left(\frac{1}{\varepsilon_o} - \frac{1}{\varepsilon_e}\right)\sin\alpha\cos\alpha - k_x\left(\frac{\sin^2\alpha}{\varepsilon_o} + \frac{\cos^2\alpha}{\varepsilon_e}\right)} = 0. \tag{5}$$

Thus,

$$k_y\left(\frac{\cos^2\alpha}{\varepsilon_o} + \frac{\sin^2\alpha}{\varepsilon_e}\right) - k_x\left(\frac{1}{\varepsilon_o} - \frac{1}{\varepsilon_e}\right)\sin\alpha\cos\alpha = 0. \tag{6}$$

Substitute (6) back into (4), the critical position k_{yc} is given in the following:

$$\left(\frac{\omega}{c}\right)^2 = k_{yc}^2 \times \left\{\frac{1}{\varepsilon_o}\left[\frac{1}{\left(\frac{\varepsilon_e}{\varepsilon_o} - 1\right)\cos\alpha}\right]^2 + \frac{1}{\varepsilon_e}\left[\frac{1}{\left(1 - \frac{\varepsilon_o}{\varepsilon_e}\right)\sin\alpha}\right]^2\right\}. \tag{7}$$

Combing with the dispersion relation in air: $\left(\frac{\omega}{c}\right)^2 = k_x^2 + k_y^2$, the critical incident angle θ_c can be achieved as

$$1 + \mathrm{ctg}^2\theta_c = \frac{1}{\varepsilon_o}\left[\frac{1}{\left(\frac{\varepsilon_e}{\varepsilon_o} - 1\right)\cos\alpha}\right]^2 + \frac{1}{\varepsilon_e}\left[\frac{1}{\left(1 - \frac{\varepsilon_o}{\varepsilon_e}\right)\sin\alpha}\right]^2. \tag{8}$$

Without loss of generality, YVO_4 crystal, which is a positive uniaxial crystal with $n_o = 1.9929$, $n_e = 2.2154$ at $\lambda = 0.63$ μm [12], is taken as an example. The dependence of critical incident angle θ_c on the optic axis orientation of crystal α is

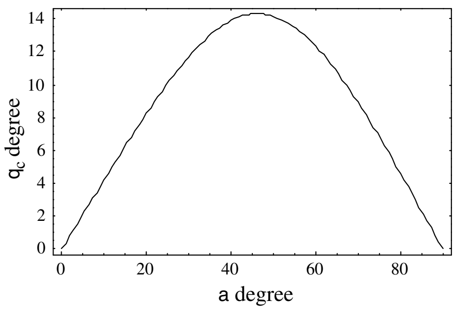

Fig. 5. The relationship between the critical incident angle θ_c and the oriented angle of optic axis α is displayed for YVO_4 crystal.

illustrated in Fig. 5 according to the above formula (8). When optic axis is parallel or perpendicular to the surface of crystal, light always refracts positively at arbitrary incidence. Otherwise, light refracts negatively within a narrow incidence range in the case of inclined optic axis orientation. The maximum critical incident angle of negative refraction exists around $\alpha = 45°$, for the example here, the maximum position is $\alpha = 46°$ and the maximum critical incident angle $\theta_c = 12.8°$.

For the twin structured uniaxial crystal interface, similar deduction can be carried out to give the dependence of critical incident angle and optic axis orientation as shown in Fig. 6. The result is very similar to the case of single crystal interface, only the maximum position is shifted to the left of $\alpha = 45°$, at $\alpha = 42°$ where the maximum

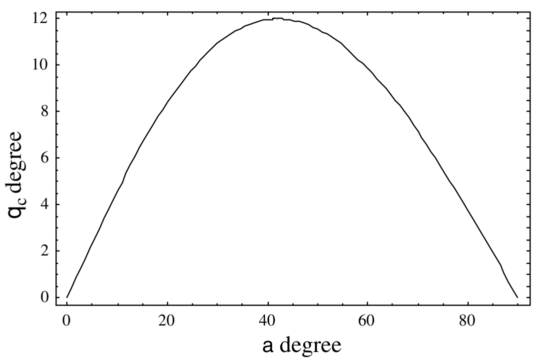

Fig. 6. The relationship between the critical incident angle θ_c and the oriented angle of optic axis α is displayed for YVO_4 crystal at twin structured interface.

critical incident angle is $\theta_c = 12.0$. The asymmetry in the dependence of the critical incident angle θ_c on the oriented angle of optic axis α is resulted from the anisotropy of the crystal. In reality, the asymmetry is generally not obvious because the difference of ordinary permittivity ε_o and extraordinary permittivity ε_e is very small. Also because the divergence between ε_o and ε_e is generally very small, reflection from the interface in the twin-structured uniaxial crystal is neglectable.

From all above discussions, it can be pointed out that negative refraction here is a natural result from the anisotropy of uniaxial crystal. It is different from the case of left-handed material although negative refraction in left-handed material can also be illustrated by using equi-frequency contour. Negative refraction in uniaxial crystal is limited within a narrow incidence angle range of around 10° and disobey Snell's law, on the other hand, in left-handed material, light refracts negatively in arbitrary incidence and obeys Snell's law if negative refractive index is introduced.

5. Conclusion

In conclusion, negative refraction takes place with not only left-handed material but also uniaxial anisotropic materials. Considering the energy flow defined as $v_g = \nabla_k \omega(\boldsymbol{k})$, negative refraction can be naturally illustrated by using equi-frequency contour in $\boldsymbol{k}$-space. Different from left-handed material, negative refraction related to uniaxial anisotropic material is limited within a narrow incidence range of about 10°. In the same theoretical frame, twin structured uniaxial crystal is convinced capable to produce negative refraction, and reflection at the interface is almost neglectable because of the nearly equal refractive index on the both sides.

References

[1] V.G. Veselago, Sov. Phys. USPEKI 10 (1968) 509.
[2] D.R. smith, Willie J. Padilla, D.C. Vier et al., Phys. Rev. Lett. 84 (2000) 4184.

[3] R.A. Shelby, D.R. Smith, S. Schultz, Science 292 (2001) 77.
[4] Andrew A. Houck, Jeffery B. Brock, Isaac L. Chuang, Phy. Rev. Lett. 90 (2003) 137401.
[5] C.G. Parazzoli, R.B. Greegor, K. Li et al., Phy. Rev. Lett. 90 (2003) 107401.
[6] P. Markos, C.M. Soukoulis, Phys. Rev. E 65 (2002) 036622.
[7] S. Foteinopoulou, E.N. Economou, C.M. Soukoulis, Phy. Rev. Lett. 90 (2003) 107402.
[8] J.B. Pendry, A.J. Holden, D.J. Robbins et al., IEEE Trans. Micro. Theory Techniques 47 (1999) 2075.
[9] J.B. Pendry, A.J. Holden, W.J. Stewart et al., Phys. Rev. Lett. 76 (1996) 4773.
[10] Yong Zhang, B. Fluegel, A. Mascarenhas, Phys. Rev. Lett. 91 (2003) 157404.
[11] I.V. Lindell, S.A. Tretyakov, K.I. Nikoskinen, S. Ilvonen, Microw. Opt. Technol. Lett. 31 (2001) 129.
[12] http://www.casix.com.