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NONLINEAR PHENOMENA

Nonlinear Mechanism for the Generation of Electromagnetic Fields in a Magnetized Plasma by the Beatings of Waves

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Abstract—The modulational instability in a plasma in a strong constant external magnetic field is considered. The plasmon condensate is modulated not by conventional low-frequency ion sound but by the beatings of two high-frequency transverse electromagnetic waves propagating along the magnetic field. The instability reduces the spatial scales of Langmuir turbulence along the external magnetic field and generates electromagnetic fields. It is shown that, for a pump wave with a sufficiently large amplitude, the effect described in the present paper can be a dominant nonlinear process.

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1. INTRODUCTION

The formation of a "plasmon condensate" in a weakly turbulent plasma is a well-known paradoxical phenomenon [1] in which the main nonlinear process—the decay of a Langmuir wave l into a Langmuir wave l' and an ion acoustic wave s ($l \longrightarrow l' + s$) [2]—gives rise to a wave energy flux toward the long-wavelength region, where there is no efficient mechanism for wave absorption. As the energy is pumped into Langmuir turbulence, it is stored in the long-wavelength region. Vedenov and Rudakov [3] showed that, because of the development of modulational instability, a uniformly distributed plasmon background can break into plasmon bunches. Zakharov [4] studied the modulational instability in its nonlinear stage, which is characterized by Langmuir wave collapse. The collapse leads the formation of cavities—regions with depressed ion density and elevated field (i.e., with elevated plasmon density). Galeev et al. [5] supposed that the modulational instability can eliminate the problem of plasmon condensate because of the reduction in the spatial scales of the waves.

In [6–8], a theory of Langmuir turbulence was developed that takes into account the wave energy flux to smaller scales due to Langmuir wave collapse and the subsequent onset of a linear mechanism for plasmon absorption.

All the papers cited above were aimed at studying turbulence in an unmagnetized plasma. In this case, the weak turbulence parameter W/nT should satisfy the condition $W/nT \gg \omega_{Be}^2/\omega_{pe}^2$ (provided that $\omega_{Be}^2/\omega_{pe}^2 \ll k_0^2\lambda_D^2$), where $W = \varepsilon_0 E^2/2$ is the wave energy density, E is the electric field amplitude, ε_0 is the dielectric constant, $\omega_{Be} = eB_0/m$ is the electron gyrofrequency, $\omega_{pe} = [e^2n/(\varepsilon_0 m)]^{1/2}$ is the electron Langmuir frequency, n is the electron density, m is the mass of an electron, T is the electron temperature, k_0 is the characteristic wavenumber in the turbulence spectrum, and $\lambda_D = [\varepsilon_0 T/(e^2n)]^{1/2}$ is the electron Debye radius. This condition, however, substantially limits the applicability of the results obtained in solving relevant problems. Zakharov [9] proposed a theory of plasma collapse that takes into account the effect of an external magnetic field such that $W/nT \ll \omega_{Be}^2/\omega_{pe}^2 \ll 1$. He showed that the wave energy is accumulated in the lowest frequency slow extraordinary wave. Krasnosel'skikh and Sotnikov [10] developed a theory of Langmuir wave collapse in a weak magnetic field such that $1 \gg \omega_{Be}^2/\omega_{Pe}^2 \geq k_0^2\lambda_D^2$. Even such a weak magnetic field changes the dispersion properties of Langmuir waves and, consequently, the nature of the modulational instability. In particular, the cavities in this situation occur in the form of pancakes flattened in the direction of the external magnetic field, along which the pump wave propagates: $l_\parallel \ll l_\perp$, where $l_\parallel$ and $l_\perp$ are the longitudinal and transverse sizes of the cavity, respectively. The collapse of wave structures continues and their spatial scales (wavelengths) become progressively smaller until Landau damping by resonant particles comes into play.

In [10], as well as in the numerical experiments of [11], a study was made of the case in which the wave electric field $\mathbf{E}$ is directed along the external magnetic field $\mathbf{B}_0$ ($\mathbf{E} \parallel \mathbf{B}_0$). Asaulov and Zakharov [12] investigated the case in which the wave electric field is transverse to the external magnetic field, $\mathbf{E} \perp \mathbf{B}_0$; this case can occur when a magnetized plasma is irradiated by an electromagnetic wave. It was shown that, for $\mathbf{E} \parallel \mathbf{B}_0$

(a longitudinal collapse), only a small fraction of the plasmon condensate energy is dissipated in the cavity, whereas for $\mathbf{E} \perp \mathbf{B}_0$ (a transverse collapse), the cavity tends to absorb all the energy that has been carried into it (the case of a strong collapse).

Of particular interest is the problem of plasma turbulence in a fairly strong external magnetic field, $\omega_{Be}^2/\omega_{pe}^2 \geq 1$. In such a plasma, experimental observations revealed filaments stretched out along the magnetic field $\mathbf{B}_0$ [13, 14]. It was supposed that these filaments are cavities stretched out in the direction of the external magnetic field. The case $\omega_{Be} \sim \omega_{pe}$ is not only of general physical interest but is also important for laboratory applications, as well as for studying ionospheric, magnetospheric, and interplanetary plasmas.

The case of strong magnetic fields ($\omega_{Be}^2 \gg \omega_{pe}^2$) is of special interest for studying astrophysical plasmas, e.g., pulsar magnetospheres. In this case, the problem of Langmuir plasmon condensate also presents considerable interest, especially in view of the specific properties of an electron–positron plasma. The reason is that, in such plasmas, the energy is accumulated in long-wavelength Langmuir waves with phase velocities V_{ph} exceeding the speed of light, $V_{\text{ph}} > c$. Another reason is that an electron–positron plasma does not contain heavy particles and, consequently, there is no low-frequency (LF) ion sound—a necessary component for the development of conventional Langmuir turbulence. In [15], it was assumed that the role of the LF wave component can be played by the beatings of two high-frequency (HF) electromagnetic waves. In that paper, consideration was given to the modulational instability in the electron–positron plasma of a pulsar magnetosphere, in which HF Langmuir waves with superluminal phase velocities, $V_{\text{ph}} > c$ (plasmon condensate), are modulated by the beatings of two HF electromagnetic waves, t and t'. The interaction of waves t and t' with a strongly magnetized plasma can trigger a modulational instability—a process that is accompanied by aperiodic generation of both longitudinal and transverse small-scale perturbations.

We think that, in this context, it is important to investigate the modulational instability of Langmuir waves that is caused by the beatings of two HF electromagnetic waves, t and t', in a conventional magnetized ($\omega_{Be}^2 \geq \omega_{pe}^2$) electron–ion plasma. In what follows, we will show that, when the amplitudes of the waves t and t' participating in the modulational instability are sufficiently large, the beating process can turn out to be more efficient than the main nonlinear interaction process ($l \longrightarrow l' + s$).

As was mentioned above, Asaulov and Zakharov [12] showed that, for $\mathbf{E} \perp \mathbf{B}_0$, the collapse is more efficient. In this case, the main nonlinear process is redistribution of the wave energy in k space and its accumulation in the narrow region around the direction of the magnetic field, followed by a two-dimensional longitudinal collapse.

The process of accumulation of the wave energy around the direction of the external magnetic field is more pronounced when the external magnetic field is sufficiently strong, $\omega_{Be}^2 \geq \omega_{pe}^2$, and when the role of the initial electric field $\mathbf{E}_\perp$ is played by the field induced by the beatings of two electromagnetic waves t and t' propagating along the external magnetic field. The electric and magnetic fields of the waves t and t' are equal to $\mathbf{E}^t$ and $\mathbf{B}^t$ and to $\mathbf{E}^{t'}$ and $\mathbf{B}^{t'}$, respectively, and are directed perpendicular to the external magnetic field $\mathbf{B}_0$.

For a weakly turbulent plasma in which all possible electromagnetic and electrostatic modes are excited, it is easy to choose two waves with mutually orthogonal fields $\mathbf{E}^t \perp \mathbf{E}^{t'}$ and $\mathbf{B}^t \perp \mathbf{B}^{t'}$ such that $\mathbf{E}^t \parallel \mathbf{B}^{t'}$ and $\mathbf{E}^{t'} \parallel \mathbf{B}^t$. In this case, the plasma will drift in crossed fields, namely, a periodically varying electric field $\mathbf{E}^t$ and an external magnetic field $\mathbf{B}_0$, with the velocity

$$\mathbf{U}_{\text{dr}} = \frac{\mathbf{E}^t \times \mathbf{B}_0}{B_0^2}. \tag{1}$$

The drift velocity $\mathbf{U}_{\text{dr}}$ is perpendicular to both $\mathbf{B}_0$ and $\mathbf{E}^t$ and, accordingly, to $\mathbf{B}^{t'}$.

As charged plasma particles drift with velocity (1) in the magnetic field $\mathbf{B}^{t'}$ of the second wave, they generate a nonlinear electric field directed along the external magnetic field $\mathbf{B}_0$,

$$E_z = (\mathbf{U}_{\text{dr}} \times \mathbf{B}^{t'})_z = \frac{1}{B_0^2}((\mathbf{E}^t \times \mathbf{B}_0) \times \mathbf{B}^{t'})_z, \tag{2}$$

in the interaction of two transverse electromagnetic waves such that

$$\frac{\omega^t - \omega^{t'}}{k_z^t - k_z^{t'}} \approx v_{z,T}, \tag{3}$$

where $v_{z,T}$ is the longitudinal component of the electron (or ion) thermal velocity, ω is the frequency, and k is the wave vector of the perturbations. In this case, all plasma particles will participate in the nonlinear collective process, which can thus be described in the hydrodynamic approach.

Our paper is organized as follows. In Section 2, we study the motion of charged plasma particles in their interaction with the fields of two electromagnetic waves t and t' and with the external magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$. In Section 3, we analyze the nonlinear dynamics of the modulational instability. To do this, we do not restrict ourselves to considering a certain branch of the dispersion relation but use a general wave equation. In Section 4, we discuss the results of our work, compare them with the results obtained for the classical modulational instability ($l \longrightarrow l' + s$), and also examine the limitations on the wave and plasma parameters.

2. MOTION OF CHARGED PARTICLES IN A MAGNETIZED PLASMA WITH TWO ELECTROMAGNETIC WAVES

Under the condition $\partial/\partial t \gg (\mathbf{v}\cdot\nabla)$ (or $\omega \gg \mathbf{k}\cdot\mathbf{v}$, where $\mathbf{v}$ is the hydrodynamic particle velocity), we can retain only the first term in the substantial derivative. In this case, the equation for the flux of charged plasma particles (electrons and ions) in an external equilibrium magnetic field $\mathbf{B}_0$ and in the electromagnetic fields of an incident wave t and the wave t' reflected from the particles has the form

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} = \frac{e_\alpha}{m_\alpha}[\mathbf{E} + \mathbf{E}' + \mathbf{v}\times(\mathbf{B}_0 + \mathbf{B} + \mathbf{B}')]. \quad (4)$$

Here, the subscripts $\alpha = e, i$ refer to the electrons and ions, respectively, and e_α and m_α are the charge and mass of the corresponding particle species. In what follows, we will primarily consider the electrons, also keeping in mind the ions.

The problem has two small parameters:

(i) the amplitude of the wave perturbations, which is much smaller than the amplitude of the external magnetic field, ($|\mathbf{E}|, |\mathbf{E}'|, |\mathbf{B}|, |\mathbf{B}'| \ll |\mathbf{B}_0|$), and

(ii) the energy of the wave perturbations, which is much smaller than the energy of the plasma particles, $|E|^2, |E'|^2 \ll mnv_T^2$, where n is the electron (ion) density and $v_T = (T/m)^{1/2}$ is the electron thermal velocity.

Therefore, the particle velocity $\mathbf{v}$ can be represented as the sum of the relevant small velocity components:

$$\mathbf{v} = \mathbf{v}_{0\perp} + \mathbf{v}_1 + \mathbf{v}_2, \quad (5)$$

where $\mathbf{v}_{0\perp}$ is the unperturbed particle velocity in the external magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$, $|\mathbf{v}_1| \ll |\mathbf{v}_{0\perp}|$ is the linear perturbation of the velocity $\mathbf{v}_{0\perp}$ under the action of the waves t and t', and $| |\mathbf{v}_2| \ll |\mathbf{v}_1|$ is the nonlinear velocity perturbation under the action of the waves t and t'.

Substituting expression (5) into Eq. (4), we obtain three coupled equations for $\mathbf{v}_{0\perp}$, $\mathbf{v}_1$, and $\mathbf{v}_2$.

In the zeroth approximation, we have

$$\frac{\partial \mathbf{v}_{0\perp}}{\partial t} = \frac{e}{m}(\mathbf{v}_{0\perp}\times\mathbf{B}_0). \quad (6)$$

For electrons, the solution to Eq. (6) has the form

$$v_{0x} = v_{0\perp}\cos\omega_{Be}t, \quad v_{0y} = -v_{0\perp}\sin\omega_{Be}t, \quad (7)$$

where $\omega_{Be} = eB_0/m$ is the electron gyrofrequency and $v_{0\perp}$ is an arbitrary constant velocity. For ions, solution (7) has the same form but with the replacement $\omega_{Be} \longrightarrow -\omega_{Bi} = -eB_0/m_i$, where m_i is the mass of an ion.

In the first approximation, we have the equation

$$\frac{\partial \mathbf{v}_1}{\partial t} = \frac{e}{m}[\mathbf{E} + \mathbf{E}' + \mathbf{v}_1\times\mathbf{B}_0 + \mathbf{v}_{0\perp}\times(\mathbf{B} + \mathbf{B}')]. \quad (8)$$

We represent the wave electric fields as

$$E_x = |E_\perp(\mathbf{r}, t)|\cos\omega^t t, \quad E_y = |E_\perp(\mathbf{r}, t)|\sin\omega^t t,$$
$$E'_x = |E'_\perp(\mathbf{r}, t)|\sin\omega^{t'} t, \quad E'_y = -|E_\perp(\mathbf{r}, t)|\cos\omega^{t'} t, \quad (9)$$

where ω^t and $\omega^{t'}$ are the frequencies of the electromagnetic waves t and t', respectively. The field components $\mathbf{E}_\perp(\mathbf{r}, t)$ and $\mathbf{E}'(\mathbf{r}, t)$ are slowly varying functions of the coordinate and time: $|\partial\mathbf{E}_\perp/\partial r| \ll |k_\perp^t|\,|\mathbf{E}_\perp|$, $|\partial\mathbf{E}_\perp/\partial t| \ll |\omega^t||\mathbf{E}_\perp|$, $|\partial\mathbf{E}'_\perp/\partial r| \ll |k_\perp^{t'}|\,|\mathbf{E}_\perp|$, and $|\partial\mathbf{E}'/\partial t| \ll |\omega^{t'}||\mathbf{E}'_\perp|$, where $k_\perp^t$ and $k_\perp^{t'}$ are the transverse wave vectors of the waves t and t', respectively. In this case, the solution to Eq. (8) can be represented as

$$v_{1x} = \frac{e}{m}\left(\frac{|\mathbf{E}_\perp|\sin\omega^t t}{\omega^t + \omega_{Be}} - \frac{|\mathbf{E}'_\perp|\sin\omega^{t'} t}{\omega^{t'} + \omega_{Be}}\right), \quad (10)$$

$$v_{1y} = -\frac{e}{m}\left(\frac{|\mathbf{E}_\perp|\cos\omega^t t}{\omega^t + \omega_{Be}} + \frac{|\mathbf{E}'_\perp|\cos\omega^{t'} t}{\omega^{t'} + \omega_{Be}}\right), \quad v_{1z} = 0. \quad (11)$$

As was mentioned above, for ions, it is necessary to make the replacement $\omega_{Be} \longrightarrow -\omega_{Bi}$ and $m \longrightarrow m_i$.

In the second approximation, we take into account the relationship $(\mathbf{v}_2\times\mathbf{B}_0)_z \equiv 0$ to obtain from Eq. (4) the equation

$$\frac{\partial \mathbf{v}_{2z}}{\partial t} = \frac{e}{m}(\mathbf{v}_1\times(\mathbf{B} + \mathbf{B}'))_z. \quad (12)$$

From the corresponding Maxwell's equation, we get

$$B_x = -\frac{E_y}{c}\cos\theta, \quad B_y = \frac{E_x}{c}\cos\theta, \quad (13)$$

where we have used the dispersion relation $\omega^t \cong |\mathbf{k}^t|c$ for electromagnetic waves and have introduced the angle θ between the magnetic field $\mathbf{B}_0$ and the wave vector $\mathbf{k}^t$ through the relationship $\cos\theta = k_z^t/|\mathbf{k}^t|$. Substituting expressions (10) and (11), as well as expressions (13) and (9), into Eq. (12) yields the following equation for the perturbed nonlinear longitudinal electron velocity:

$$\frac{\partial v_{2z}}{\partial t} = \frac{e^2|\mathbf{E}_\perp||E'_\perp|}{m^2c}\cos(\Delta\omega t)\left[\frac{\cos\theta'}{\omega^t + \omega_{Be}} - \frac{\cos\theta}{\omega^{t'} + \omega_{Be}}\right], \quad (14)$$

where $\Delta\omega = \omega^t - \omega^{t'}$. Note that the velocity component v_{2z} and, accordingly, the electric field component E_{2z} arise in the nonlinear interaction of the electromagnetic waves t and t' with charged plasma particles. We assume that, in the frame of reference in which the bulk plasma is at rest, charged particles do not move in the longitudinal direction, $v_{1z} = 0$, so we have $v_{2\perp} \equiv 0$. Hence, in the second approximation, the electric current density $\mathbf{j}_2$ has only the longitudinal component, $\mathbf{j}_{2z} \neq 0$. For $\cos\theta \approx \cos\theta'$ and $|\mathbf{E}_\perp| \approx |\mathbf{E}'_\perp|$, Eq. (14) gives

$$\frac{\partial v_{2z}}{\partial t} = -a\cos(\Delta\omega t), \qquad (15)$$

where

$$a \approx \frac{e^2|\mathbf{E}^t_\perp|^2\Delta\omega}{m^2 c\omega^{t2}}. \qquad (16)$$

With the corresponding initial condition ($v_{2z} \longrightarrow 0$ at $t \longrightarrow 0$), the solution to Eq. (15) can be rewritten as

$$v_{2z} = -\frac{a}{\Delta\omega}\sin(\Delta\omega t). \qquad (17)$$

For the ion plasma component, we should make the replacement $e \longrightarrow -e$ and $\omega_{Be} \longrightarrow -\omega_{Bi}$. Using expression (17) for the electrons and the corresponding expression for the ions, we can determine the density $j_z = en(\mathbf{v}^i_{2z} - \mathbf{v}^e_{2z})$ of the longitudinal current generated in the nonlinear interaction of the t and t' waves with the particles of a magnetized plasma.

Such a motion of the charged particles in a magnetized plasma can induce a relatively LF quasineutral perturbation of the plasma density, $\delta n/n_0$, due to the beatings of two HF waves t and t' (here, n_0 is the equilibrium plasma density). The expression for $\delta n/n_0$ can be derived from the continuity equation for the charged plasma particles by averaging it over the high frequency. As a result, we have

$$\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}\right) = -\frac{\partial}{\partial z}v_{2z}. \qquad (18)$$

Note that the physical quantities in Eq. (18), namely, the longitudinal velocity v_{2z} and LF density perturbation δn, are averaged over the high frequency; they result from the influence of the external sources (waves t and t') on the plasma. Consequently, by analogy with formulas (9) (see also expression (22) below), their coordinate dependence can be represented as $(v_{2z}, \delta n) \sim (v_{2z}(r, t), \delta n(r, t))\exp(-i\mathbf{k}^t \cdot \mathbf{r})$, where the amplitudes $v_{2z}(r, t)$ and $\delta n(r, t)$ are slowly varying functions of the coordinate and time. The first of these amplitudes is given by formula (17). The final formula for the second amplitude, i.e., the amplitude of the LF plasma density perturbations, can be obtained by inserting expression (17) for $v_{2z}(r, t)$ into Eq. (18):

$$\frac{\delta n}{n_0} = -i\frac{k^t_z a}{(\Delta\omega)^2}[1 - \cos(\Delta\omega t)]. \qquad (19)$$

Here, we assume that the energy of the waves t and t' is maintained by external sources. Such a state is typical of ionospheric, magnetospheric, and astrophysical plasmas, as well as of plasmas in laboratory devices in regimes with auxiliary heating.

3. NONLINEAR DYNAMICS OF LANGMUIR WAVES IN A MAGNETIZED PLASMA

We investigate the dynamics of the wave fields by using the set of Maxwell's equations, which can be reduced to the wave equation

$$\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu_0^{-1}\varepsilon_0^{-1}\nabla\times(\nabla\times\mathbf{E}) + \frac{1}{\varepsilon_0}\frac{\partial\mathbf{j}}{\partial t} = 0, \qquad (20)$$

where μ_0 and ε_0 are the permeability and permittivity of free space ($\varepsilon_0\mu_0 = 1/c^2$), respectively, and $\mathbf{j} = en(\mathbf{v}_i - \mathbf{v}_e) = \mathbf{j}_2$ is the second-order electric current density.

We consider a wave packet propagating at a small angle to the equilibrium magnetic field $\mathbf{B}_0$. We represent the LF (relative to the frequency ω^t) component $\mathbf{E}^l$ and the HF transverse component $\mathbf{E}^t$ of the generated electric field as

$$\mathbf{E} = \mathbf{E}^l + \mathbf{E}^t, \qquad (21)$$

where $E^t_z = 0$ and it is assumed that $\omega^t \approx k^t c \gg \omega^l$. We also represent the fields in the form

$$E^{l,t}_i = \frac{1}{2}(E^{l,t}_i(\mathbf{r}, t)\exp(i\omega^{l,t}t - i\mathbf{k}^{l,t}\cdot\mathbf{r}) + \text{c.c.}), \qquad (22)$$

where the subscript i stands for a Cartesian component ($i = x, y, z$) and $E^{l,t}_i(\mathbf{r}, t)$ is the slowly varying (in both space and time) electric field amplitude.

In the linear approximation, the current density $\mathbf{j}$ can be determined from the equation $\varepsilon_0^{-1}\partial\mathbf{j}/\partial t \cong \omega^2_{pe}\mathbf{E}$. With allowance for the fact that the beatings of two electromagnetic waves t and t' give rise to an LF quasineutral perturbation δn of the plasma density n (see Section 2), we arrive at the following expression for the current density in a weakly nonlinear plasma state:

$$\frac{1}{\varepsilon_0}\frac{\partial\mathbf{j}}{\partial t} \cong \omega^2_{p0}\left(1 + \frac{\delta n}{n_0}\right)\mathbf{E}, \qquad (23)$$

where the relative perturbation amplitude $\delta n/n_0$ is determined by expression (19) and by the relationship

$\omega_{p0} = [e^2 n_0/(\varepsilon_0 m)]^{1/2}$ and n_0 is the equilibrium electron (ion) density.

It should be noted that the form of wave equation (20) to be investigated depends substantially on the choice of the time and spatial scales of the problem. This is why, for definiteness, we assume that $\omega^l > \omega_{p0}$ and $\omega^l \gg k^l c$ for the longitudinal mode; $\omega^t \cong k^t c \gg \omega^l$ for the transverse mode; and $\partial/\partial t \sim \omega \gg \Delta\omega$, $k^t c \gg \Delta\omega$, and $\omega^t\omega \gg \omega_{p0}^2$ for the perturbations generated. Accordingly, the spatial scales of the perturbations are assumed to satisfy the inequalities $k^{l_i t} \gg \partial/\partial(x, y, z)$ and $k_z^{l_i t} \gg k_x^{l_i t}, k_y^{l_i t}$.

With allowance for these inequalities and also the inequality $\omega^t \gg \omega^l$, we insert expressions (21)–(23) into Eq. (20) to obtain the following set of coupled equations:

$$\frac{\partial E^t_{x,y}}{\partial t} = 0, \quad (24)$$

$$2i\omega^l \frac{\partial E^l_{x,y}}{\partial t} - \frac{ik^l_z}{\varepsilon_0\mu_0}\frac{\partial E^l_z}{\partial(x, y)} = -\omega_{p0}^2 \frac{\delta n}{n_0} E^l_{x,y}, \quad (25)$$

$$2i\omega^l \frac{\partial E^l_z}{\partial t} - \frac{ik^l_z}{\varepsilon_0\mu_0}\left(\frac{\partial E^l_x}{\partial x} + \frac{\partial E^l_y}{\partial y}\right) = -\omega_{p0}^2 \frac{\delta n}{n_0} E^l_z. \quad (26)$$

The set of Eqs. (25) and (26) has been derived in the zero-order approximation in the small parameter $\omega^l/\omega^t \ll 1$. This reflects the fact that, the process of modulation of the longitudinal waves does not influence the evolution of the amplitudes of the transverse waves,

$$E^t_\perp \cong \text{const}. \quad (27)$$

At the same time, the nonlinear terms on the right-hand sides of Eqs. (25) and (26), which are proportional to $\delta n/n_0$ and describe the nonlinear dynamics of the modulational instability, are determined by the amplitude $\mathbf{E}^t_\perp$ of the HF transverse waves (see formula (19)).

Without its nonlinear right-hand side, Eq. (25) coincides with the corresponding equation obtained in [16]. But the main difference between Eqs. (25) and (26) and those in [16] is in the nonlinear term on the right-hand side of Eq. (26): in [16], it was assumed that $\delta n/n_0 \sim |E^l_z|^2$, whereas in our study, the LF quasineutral plasma density perturbation is attributed to the beatings of two HF transverse waves and it is assumed that $\delta n/n_0 \sim |E^t_\perp|^2$.

In order to construct an equation for the longitudinal field component E^l_z, we find the mixed derivative $\partial^2 E^l_{x,y}/\partial t\partial(x, y)$ from Eq. (25). We then integrate Eq. (26) over time and substitute the mixed derivative so determined into the resulting equation to obtain the following equation:

$$\frac{\partial^2 E^l_z}{\partial t^2} - \frac{c^2}{4}\left(\frac{k^l_z c}{\omega^l}\right)^2 \Delta_\perp E^l_z - \frac{i}{2}\frac{\omega_{p0}^2}{\omega^l}\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}E^l_z\right) - \frac{i}{4}\frac{k^l_z c^2 \omega_{p0}^2}{\omega^{l^2}}\frac{\delta n}{n_0}\nabla_\perp \cdot \mathbf{E}^l_\perp = 0, \quad (28)$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian operator. We further assume that the LF electric field $\mathbf{E}^l$ satisfies the condition $\nabla_\perp \cdot \mathbf{E}^l_\perp \equiv 0$, under which Eq. (28) can be substantially simplified to the form

$$\frac{\partial^2 E^l_z}{\partial t^2} - \left(\frac{k^l_z}{2\varepsilon_0\mu_0\omega^l}\right)^2 \Delta_\perp \varepsilon^l_z - \frac{i}{2}\frac{\omega_{p0}^2}{\omega^l}\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}E^l_z\right) = 0. \quad (29)$$

We substitute expression (19) for $\delta n/n_0$ into Eq. (29) and represent the LF electric field amplitude as $E^l_z \sim \exp(-i\omega t + \mathbf{kr})$ to obtain the following dispersion relation for the frequency of the perturbation generated:

$$\omega^2 - k_0^2 k_\perp^2 c^2 + \frac{\omega_{p0}^2 k^t_z a}{2\omega^l \Delta\omega} = 0, \quad (30)$$

where $k_0^2 = (k^l_z c/(2\omega^l))^2$.

We can see that the instability develops under the condition

$$\left|\frac{E^t_\perp}{B_0}\right|^2 > \frac{k^{l^2}_z c^2}{\omega^{l^2}}\left|\frac{k^t_z c}{2\omega_{p0}}\right|\frac{k^2_\perp c^2}{\omega^2_{Be}}c^2, \quad (31)$$

and grows at the rate

$$\gamma_0 = \left|\frac{k^t_z a\omega_{p0}}{2\Delta\omega}\right|^{1/2} = \frac{e|E^t_\perp|}{mc}\left(\frac{\omega_{p0}}{2\omega^t}\right)^{1/2}. \quad (32)$$

The aperiodic growth of the longitudinal electric field E^l_z has a threshold in terms of the amplitude of the external HF transverse wave

$$|E^t_{\perp\text{thr}}| \sim (k^l_z c/\omega^l)^2 |k^t_z c/\omega_{p0}| (k_\perp c/\omega_{Be})^2 B_0 c^2. \quad (33)$$

Hence, the modulational instability that is caused by the beatings of two electromagnetic waves t and t' reduces the spatial scales of turbulence in a plasmon condensate (i.e., Langmuir turbulence with the frequency ω_{p0} and with the characteristic wavenumber $k_0 \longrightarrow 0$), thereby giving rise to perturbations with

$k_0 \neq 0$. This indicates that the energy is transferred from plasmons to longitudinal wave fields with frequencies $\omega^l \approx \omega_{p0}(1 + k^2\lambda_D^2/2)$.

Let us now investigate the possibility of generating an LF transverse vortex electric field. Using Eq. (25), we find

$$\frac{\partial}{\partial t}(\nabla \times \mathbf{E}^l)_z = i\frac{\omega_{p0}^2}{2\omega^l}\frac{\delta n}{n_0}(\nabla \times \mathbf{E}^l)_z. \quad (34)$$

Integrating this equation over time yields the expression

$$(\nabla \times \mathbf{E}^l)_z = C\exp\left[i\frac{\omega_{p0}^2}{2\omega^l}\int^t\frac{\delta n(t', \mathbf{r})}{n_0}dt'\right], \quad (35)$$

where C is an integration constant. Expression (35) describes the time evolution of the LF vortex electric field. Inserting formula (19) for $\delta n/n_0$ into expression (35) reduces the latter to the following final form:

$$(\nabla \times \mathbf{E}^l)_z = C\exp\left\{\left|\frac{E_\perp^t}{B_0}\right|^2\frac{\omega_{p0}^2\omega_{Be}^2}{2\Delta\omega\omega^l\omega^t}\left[t - \frac{\sin(\Delta\omega t)}{\Delta\omega}\right]\right\}. \quad (36)$$

From expressions (35) and (36) we see that the LF modulation of the perturbed plasma density δn (caused by the beatings of two external HF electromagnetic waves) leads to the generation of an LF electromagnetic field.

Note that, in the drift approximation (see formulas (1), (2)), the plasma density is modulated along the external magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$. In this case, however, not only the spatial scales of Langmuir waves (plasmon condensate) change but also the plasmon energy is transferred to electromagnetic waves (see formula (36)).

4. DISCUSSION OF THE RESULTS AND CONCLUSIONS

We have shown that HF electromagnetic waves generated by external or internal sources in a magnetized plasma can give rise to an LF modulation of the plasma density along an external equilibrium magnetic field as a result of the beatings of two electromagnetic waves such that $\Delta\omega = \omega^t - \omega^{t'}$ (here, the inverse effect of the modulational instability on HF fields is ignored).

We have found that the E_z^l component of the potential electric field generated during the modulational instability grows aperiodically and that this growth process is threshold in character. The LF density modulation, in turn, generates a vortex electromagnetic field $(\nabla \times \mathbf{E}^l)_z$.

The results obtained may be of interest for the theory of wave–wave or wave–particle nonlinear interactions in a magnetized plasma. The processes under investigation can occur in actual laboratory plasmas as well as in ionospheric, magnetospheric, and astrophysical plasmas. According to the theory developed here, the modulational instability can lead to an increase in the longitudinal and transverse momenta of charged particles [17].

The charged particles so accelerated, e.g., conjugate photoelectrons in the upper ionosphere and magnetosphere of the Earth, can cause the observed increase in the intensity of red emission from the ionospheric F region [18]. In laboratory plasmas, this effect can lead to the generation of accelerated particles.

In order to determine the efficiency of the modulational instability mechanism considered above, we compare the growth rate (32) of the aperiodic instability with the growth rate $\gamma_m^{\max}$ of the classical modulational instability ($l \longrightarrow l' + s$) [10],

$$\gamma_m^{\max} = \frac{\omega_{p0}(m/M)^{1/2}|E_0|}{2(n_0T_e/\varepsilon_0)^{1/2}}, \quad (37)$$

where $\mathbf{E}_0$ is the amplitude of the pump Langmuir wave, M is the mass of an ion, and T_e is the electron temperature. If the amplitude of the external HF transverse electric field $|\mathbf{E}_\perp^t|$ and the amplitude of the pump Langmuir wave $|E_0|$ satisfy the inequality

$$|E_\perp^t| > \left(\frac{\omega^t}{2\omega_{p0}}\right)^{1/2}\frac{c}{v_{Te}}\left(\frac{m}{M}\right)^{1/2}|E_0|, \quad (38)$$

then the modulational instability considered in the present paper is more efficient than the conventional modulational instability of Langmuir waves.

Of course, this comparison of growth rates (32) and (37) is somewhat incorrect because, in [10], the magnetic field was assumed to be weak, $1 \gg \omega_B^2/\omega_p^2 \geq k_0^2\lambda_D^2$, whereas our study was done for strong magnetic fields, $\omega_B^2/\omega_p^2 \gtrsim 1$. In addition, in [12], it was mentioned that, if the electric field in the nonlinear stage of the modulational instability is purely longitudinal ($\mathbf{E} \parallel \mathbf{B}_0$), then the wave energy is only partially absorbed by plasma particles. However, in the case of a purely transverse electric field ($\mathbf{E} \perp \mathbf{B}_0$), the particles absorb the wave energy more efficiently.

In our case (i.e., when the external magnetic field is strong and the plasma density is modulated by the beatings of two HF electromagnetic waves propagating along the external magnetic field), the absorption of the wave energy by plasma particles in a plasmon condensate plays an important role.

It should be emphasized that the particle drift in crossed alternating electric and constant magnetic fields plays a governing role in the development of the modulational instability of a strongly magnetized plasma.

With the problem as formulated, we can state that the developing modulational instability not only changes the spatial scales of Langmuir turbulence in a plasmon condensate but also leads to the transfer of plasmon energy to electromagnetic perturbations.

In our study, the beatings of two waves t and t' is chosen in such a way that the modulational instability develops at the Langmuir branch of turbulence. However, the approach developed here also allows one to investigate the modulational instability of a LF ion acoustic mode; this problem is to be considered in subsequent papers.

REFERENCES

1. L. A. Artsimovich and R. Z. Sagdeev, *Plasma Physics for Physicists* (Atomizdat, Moscow, 1979) [in Russian].
2. V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tekh. Fiz. **32**, 1291 (1962) [Sov. Phys. Tech. Phys. 7, 955 (1962).
3. A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR **159**, 767 (1964) [Sov. Phys. Dokl. **9**, 1073 (1964)].
4. V. E. Zakharov, Zh. Éksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].
5. A. A. Galeev, V. N. Oraevskiĭ, and R. Z. Sagdeev, Pis'ma Zh. Éksp. Teor. Fiz. **16**, 194 (1972) [JETP Lett. **16**, 136 (1972)].
6. A. A. Galeev, R. Z. Sagdeev, Yu. S. Sigov, et al., Fiz. Plazmy **1**, 10 (1975) [Sov. J. Plasma Phys. **1**, 5 (1975)].
7. A. A. Galeev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Zh. Éksp. Teor. Fiz. **73**, 1352 (1977) [Sov. Phys. JETP **46**, 711 (1977)].
8. G. Pelletier, Phys. Rev. Lett. **49**, 782 (1982).
9. V. E. Zakharov, Pis'ma Zh. Éksp. Teor. Fiz. **21**, 479 (1975) [JETP Lett. **21**, 221 (1975)].
10. V. V. Krasnosel'skikh and V. I. Sotnikov, Fiz. Plazmy **3**, 872 (1977) [Sov. J. Plasma Phys. **3**, 491 (1977)].
11. A. S. Lipatov, Pis'ma Zh. Éksp. Teor. Fiz. **26**, 516 (1977) [JETP Lett. **26**, 377 (1977)].
12. Zh. A. Asaulov and V. E. Zakharov, Fiz. Plazmy **11**, 1331 (1985) [Sov. J. Plasma Phys. **11**, 762 (1985)].
13. P. J. Christiansen, V. K. Jain, and L. Stenflo, Phys. Rev. Lett. **46**, 1333 (1981).
14. M. J. Giles, Phys. Rev. Lett. **47**, 1606 (1981).
15. G. Z. Machabeli, S. V. Vladimirov, and D. B. Melrose, Phys. Rev. E **59**, 4552 (1999).
16. Sh. G. Amiranashvili and A. M. Ignatov, Fiz. Plazmy **21**, 390 (1995) [Plasma Phys. Rep. **21**, 368 (1995)].
17. G. Z. Machabeli, S. V. Vladimirov, D. B. Melrose, and Q. Luo, Phys. Rev. E **65**, 3608 (2002).
18. G. D. Aburjania, G. Z. Machabeli, and I. S. Nanobashvili, Geomagn. Aéron. **42**, 94 (2002).

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