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Traveling waves in twisted nematic liquid crystal cells

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Abstract

We have described a novel reorientation mechanism in the form of the traveling waves, under influence of an external electric field, directed parallel to both glass plates, which occur in the twisted nematic cell (TNC). It is found that the slowest velocity of the traveling front is proportional to the field strength, and, approximately, in three times higher than the front velocity corresponding to the non-traveling solution. The value of the critical electric field E_{cr} which may excite the traveling waves in the TNC in π times less than the value of the threshold electric field E_{th} corresponding to the untwisted geometry.

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One of the most useful and well understood phenomena in the physics of liquid crystals (LCs) is the field induced distortion of a LC cell [1]. This Letter describes a new mechanism of the director $\hat{\mathbf{n}}$ reorientation in the form of the traveling waves occurring in the nematic phase sandwiched between two parallel surfaces, when the director on the upper surface is at right angle to the director on the lower surface, both alignments being within the plane of the solid surfaces, under applying an electric field $E > E_{\text{cr}}$ parallel to an uniformly oriented twisted nematic (TN) cell. The value of the critical electric field E_{cr} which excite the traveling wave $\phi(z - vt)$ running in the twisted cell with the minimal velocity v_m, in π times less than the value of the threshold electric field [1] E_{th}, corresponding to the untwisted cell. With increase an external field $E > E_{\text{cr}}$, the reorientation processes in a TN cell exhibit a number of relaxation regimes which characterized by rotation of the director in the plane (x–y) parallel to both glass plates. The torques acting on the director $\hat{\mathbf{n}}$ may excite the traveling wave spreading along the normal to both boundaries (directed parallel to the z-axis), whose resemblance to a kink wave increase with increasing of an applied electric field (directed parallel to the y-axis). Taking into account that the TN cells are important elements of the flat screen displays, their switching dynamics needs to be examined in the details. The dynamic equation describing the reorientation of the director distortion in the gap between two glass plates is maintained by elastic, electric and viscous torques as [2] $\mathbf{T}_{\text{el}} + \mathbf{T}_{\text{elast}} + \mathbf{T}_{\text{vis}} = 0$. Here we localize of our attention on the azimuthal anchoring, when the polar angle θ is fixed, and the anchoring energy is a function of the surface azimuthal angle ϕ only. In the case of planar geometry $\hat{\mathbf{n}} = (\cos\phi(z), \sin\phi(z), 0)$ and absence of flow, the dimension form of the torque balance equation can be written as [2]

$$\gamma_1 \phi_t(t, z) = K_2 \phi_{zz}(t, z) + \Delta \sin 2\phi(t, z), \tag{1}$$

where $\phi_t(t, z) = \partial\phi(t, z)/\partial t$, $\phi_{zz}(t, z) = \partial^2\phi(t, z)/\partial z^2$, γ_1 is the rotational viscosity coefficient, K_2 is the twist elastic constant, $\Delta = \frac{\epsilon_0 \epsilon_a E^2}{2}$, ϵ_0 is the absolute dielectric permittivity of free space, $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$ is the dielectric anisotropy of the TN, $\epsilon_\parallel$ and $\epsilon_\perp$ are the dielectric constants parallel and perpendicular to the director $\hat{\mathbf{n}}$, respectively.

Note that any physical effect that reorients the director induces flow $\mathbf{v}$ in the LC phase, which, in turn, is coupled to the director. It is important to stress here that in the case of the planar geometry $\hat{\mathbf{n}} = (n_x(z), n_y(z), 0) = (\cos\phi(z), \sin\phi(z), 0)$, the viscous torque reduces to $\mathbf{T}_{\text{vis}} = -\gamma_1 \phi_t(t, z)\mathbf{k}$. Physically,

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this means that the induced flow $\mathbf{v}$ is not coupled to the director $\hat{\mathbf{n}}$ and the orientational relaxation process of the director to its equilibrium orientation in the twisted nematic cell can be derived only from the balance of the viscous, elastic, and electric torques without accounting for the Navier–Stokes equation for the velocity field [3]. In the case of the strong anchoring, the torque balance transmitted to the surface assumed that the azimuthal angle has to satisfy the boundary conditions

$$\phi(z)_{z=-d/2} = 0, \quad \phi(z)_{z=d/2} = \frac{\pi}{2}, \tag{2}$$

whereas the initial orientation of the director is disturbed parallel to the external field $\mathbf{E}$, with $\phi(t=0,z) = \frac{1}{\sigma}\varphi((z-z_1)/\sigma)$, and then allowed to relax to its equilibrium value $\phi_{\text{eq}}(z)$. Here $\varphi(q/\sigma)$ is the Gaussian (normal) distribution function, σ is the dispersion, and $-d/2 \leqslant z_1 \leqslant d/2$. Because, in our case, the field $\mathbf{E}$ is aligned parallel to the y-direction, the state $\phi_{z=d/2}(z) = \frac{\pi}{2}$ is now stable, and front $\phi(z - vt)$ starts to move away from the one edge ($z = d/2$) of the cell to their second edge ($z = -d/2$). However, this raises a number of questions. How fast will such a front move and how much influence have both the external electric field and the boundary conditions on the resemblance of a traveling wave to a kink wave? The answer to these questions will be given based on the analysis of the torque balance equation together with the appropriate initial and the boundary conditions. The front minimal velocity v_m can be determined by substituting $\phi(t,z) \sim \exp[-E\sqrt{\frac{\epsilon_0\epsilon_a}{K_2}}(z-vt)]$, into linearized form of Eq. (1), and one sees that the slowest velocity has a value [4] $v_m = 2\sqrt{\frac{\epsilon_0\epsilon_a K_2}{\gamma_1^2}}E$, and a wave narrowest thickness κ is inversely proportional to the electric field strength $\kappa = \sqrt{\frac{K_2}{\epsilon_0\epsilon_a}}\frac{1}{E}$. Hence, if we have $E \geqslant E_{\text{cr}} = \frac{1}{d}\sqrt{\frac{K_2}{\epsilon_0\epsilon_a}}$, only then the traveling wave, with the minimal velocity v_m, is short enough to fit in the cell length. To be able to observe the evolution of the traveling disturbance in time with velocity v, we consider the dimensionless analog of Eq. (1)

$$\phi_\tau(\tau,z) = \phi_{zz}(\tau,z) + \frac{1}{2}\sin 2\phi(\tau,z), \tag{3}$$

where $\tau = \frac{\epsilon_0\epsilon_a E^2}{\gamma_1}t$ is the dimensionless time, $\bar{z} = \sqrt{\frac{\epsilon_0\epsilon_a E^2}{K_2}}z = z/\kappa$ is the dimensionless direction through the cell thickness, and the overbar in the last equation has been eliminated.

It should be pointed out that the different problems may described in terms of the reaction–diffusion equation $\psi_\tau = \psi_{zz} + f(\psi)$, where $f(\psi)$ is a nonlinear term with at least two equilibrium points [5,6]. It has been established rigorously [4] that sufficiently localized initial conditions evolve asymptotically into traveling wave front $\psi(z - v\tau)$ joining two equilibrium states, and for a wide class reaction terms $f(\psi)$, with $f(0) = 0$ and $f'(0) = 1$, has a traveling wave solution propagating at speed $\bar{v}$, where $2[(\frac{\partial f(\psi)}{\partial\psi})_{\psi=0}]^{1/2} \leqslant \bar{v} \leqslant 2[\sup_{\psi\in[0,1]}(\frac{f(\psi)}{\psi})]^{1/2}$. Both limits are equal to 2 and the traveling wave solution $\psi(z - v\tau)$ is defined in an infinitely large interval $(-\infty, +\infty)$, whereas in our case, Eq. (1) with the twisted (I) boundary conditions $\phi(z)_{z=-d/2} = 0$, $\phi(z)_{z=d/2} = \frac{\pi}{2}$ is defined in the limited interval $[-d/2, d/2]$, and the traveling front $\phi(z - v\tau)$ starts to move away from the one edge ($z = d/2$) of the cell to their second edge ($z = -d/2$) and the wave speed $v \geqslant v_m$. So, the our aim is investigate numerically, by means of a standard relaxation method [6], the relaxation process which assume an exciting the traveling wave in the TN cell, and answer on the questions, how fast will such front move and how much influence both the external electric field E and the boundary conditions on the velocity u_R of the relaxation process. We assume here that the relaxation process is composed of the process before forming of the traveling wave and after forming, with the further propagation of the traveling front in the TN cell until getting of the equilibrium orientation. To be able to observe the evolution of the traveling wave in time we transform to a coordinate system (moving at the dimensionless wave speed v) $q = z/\kappa - v\tau$, and obtain

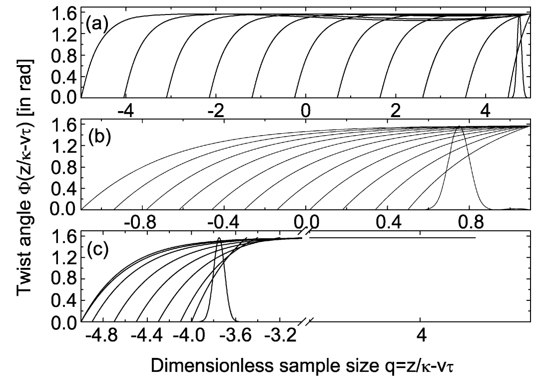

Fig. 1. Wave shape evolution of the azimuthal angle $\phi(q)$ ($q = z/\kappa - v\tau$ is a dimensionless direction through the cell thickness) from the one edge ($q = q_2$) of the cell to their second edge ($q = q_1$), calculated using Eq. (4), with the planar boundary conditions (5), at $E/E_{\text{cr}} = 10.0$, and with the initial condition $\phi(q,\tau=0) = \frac{1}{\sigma}\varphi((q-q_3)/\sigma)$, where $q_3 = 4.75$ and $\sigma = 0.25$, close to the upper restricted surface. Here $q_2 = 5$ and $q_1 = 5 - v\tau$ (a); (b) Same as (a), but $E/E_{\text{cr}} = 2.2$. Here $q_2 = 1.1$, $q_1 = 1.1 - v\tau$, $q_3 = 0.7$ and $\sigma = 0.02$. (c) Same as (a), but with the initial condition $\phi(q,\tau=0) = \frac{1}{\sigma}\varphi((q-q_4)/\sigma)$, where $q_4 = -3.7$ and $\sigma = 0.25$, close to the lower restricted surface.

$$v\phi_q(q) + \phi_{qq}(q) + \frac{1}{2}\sin 2\phi(q) = \phi_\tau(q). \tag{4}$$

The relaxation of the director $\hat{\mathbf{n}}$ to its equilibrium orientation, which is described by the angle $\phi(q)$, being initially disturbed parallel to the external field $\mathbf{E}$, with $\phi(q,\tau=0) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$ ($(q_1+q_2)/2 < q_3 \leqslant q_2$) close to the upper restricted surface, have been investigated by a standard numerical relaxation method [7], with the boundary conditions

$$\phi(q)_{q=q_1} = 0, \quad \phi(q)_{q=q_2} = \frac{\pi}{2}. \tag{5}$$

Here $q_1 = d/(2\kappa) - v\tau$ and $q_2 = d/(2\kappa)$ are the dimensionless positions for lower and upper boundaries. Result of calculations for number of dynamical regimes is shown in Figs. 1(a) and (b). In all these cases the angle $\phi(q)$ has been initially disturbed parallel to the external field $E = 10.0E_{\text{cr}}$ (Fig. 1(a)) and $E = 2.2E_{\text{cr}}$ (Fig. 1(b)), with $\phi(q,\tau=0) = \frac{1}{\sigma}\varphi((q-q_3)/\sigma)$ close to the upper restricted surface. It is found that the torque exerted on the director excite the traveling wave, directed to be normal to these boundaries, with the dimensionless front ve-

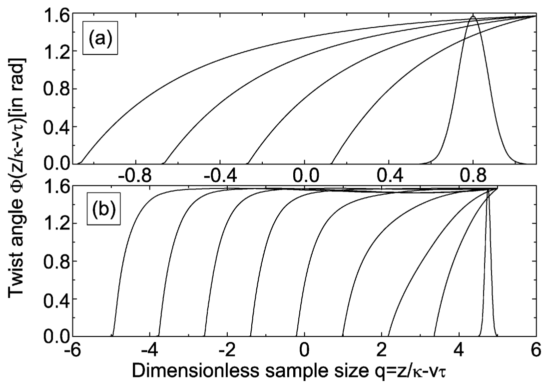

Fig. 2. Wave shape evolution of the azimuthal angle $\phi(q)$ from the one edge ($q = q_2$) of the cell to their second edge ($q = q_1$), calculated using Eq. (4), with the boundary conditions (6), at $\frac{A\kappa}{2K_2}\sin 2\Delta\phi = 1.0$ and under action of the electric field $E/E_{\rm cr} = 2.2$. Here $q_2 = 1.1$ and $q_1 = 1.1 - v\tau$; (b) Same as (a), but $E/E_{\rm cr} = 10.0$. Here $q_2 = 5$ and $q_1 = 5 - v\tau$. In both these cases the initial condition $\phi(\tau = 0, q) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$ $((q_1 + q_2)/2 < q_3 \leqslant q_2)$ close to the upper restricted surface.

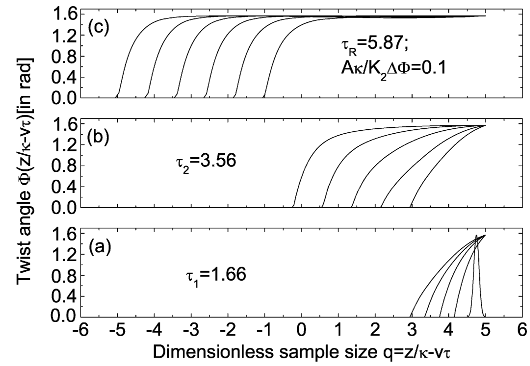

Fig. 3. Plot of three stages of the director evolution in the TN cell, calculated using Eq. (4), with the boundary conditions (6), at $\frac{A\kappa}{2K_2}\sin 2\Delta\phi = 0.1$ and $E/E_{\rm cr} = 10.0$. In the final stage (c) the dimensionless front velocity has a value 2.

locity value $v = 2.0$. Notes that the resemblance of a traveling wave to a kink wave increase with increasing of the electric field. The calculations show that the initial state ($\phi(q, \tau = 0) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$) in an applied field E can be reoriented by the propagation of a kink like wave with the constant shape and speed to the equilibrium state $\phi_{\rm eq}(q)$, at the value of the external electric field $E \geqslant 10.0E_{\rm cr}$. It is also found that the relaxation regime, under the influence of the electric, elastic, and viscous torques, may excite the traveling wave spreading along z-axis only under applying an electric field $E > E_{\rm cr}$ (see Fig. 1). Consider now the TN cell under applying an external electric field, when the director $\hat{\mathbf{n}}$ is strongly anchored to upper boundary plate and weakly anchored to lower boundary plate, and the anchoring energy takes the form [1] $W_{\rm az} = \frac{1}{2}A\sin^2(\phi_s - \phi_0)$, where A is the anchoring strength, ϕ_s and ϕ_0 are the azimuthal angles corresponding to the director orientation on the boundary plate and easy axis $\hat{\mathbf{e}}$, respectively. The torque balance transmitted to the surface assumed that the director angle has to satisfy the boundary conditions

$$\big(\partial\phi(q)/\partial q\big)_{q=q_1} = \frac{A\kappa}{2K_2}\sin 2\Delta\phi,$$
$$\phi(q)_{q=q_2} = \pi/2, \tag{6}$$

where $\Delta\phi = \phi_s - \phi_0$. For the case of 4-*n-octyl*-4′-*cyanobiphenyl* (8*CB*), at $T = 307$ K, $K_2 \sim 8$ pN [8], and the experimental data for A, obtained using different experimental techniques, are varied between 10^{-4} and 10^{-6} J/m^2. In the case of $\Delta\phi \leqslant 10°$ [2], for narrow TN cell up to $d \sim 10.0$ μm, the combination of $\frac{A\kappa}{2K_2}\sin 2\Delta\phi$ varied between 0.1 and 1.0. The relaxation of the director $\hat{\mathbf{n}}$ to its equilibrium orientation, which is described by the angle $\phi(q)$, being initially disturbed parallel to the external field $\mathbf{E}$, with $\phi(q, \tau = 0) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$ $((q_1 + q_2)/2 < q_3 \leqslant q_2)$ close to the upper restricted surface, for a number of $E/E_{\rm cr}$ values 2.2 (Fig. 2(a)), 10.0 (Fig. 2(b)), and at $\frac{A\kappa}{2K_2}\sin 2\Delta\phi = 1.0$, are shown in Fig. 2. It is found that the influence of the anchoring strength A on the dimensionless relaxation time τ_R has a weak effect. With increasing of $\frac{A\kappa}{2K_2}\sin 2\Delta\phi$ from 0.1 up to 1.0, the value of the relaxation time τ_R increase small from 5.87 to 5.88, with the following increasing, in the case of the strong anchoring, up to 5.94. The relaxation criterion $\epsilon = |(\phi(\tau_R) - \phi_{\rm eq})/\phi_{\rm eq}|$ for calculating procedure was chosen equal to be 10^{-4}, and the numerical procedure was then carried out until a prescribed accuracy was achieved. It should be pointed out that the full reorientation process can be subdivided, at least, in two stages. First one, correspond to evolution of the initial disturbance ($\phi(q, \tau = 0) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$) to a kink like wave during the larger time term ($\tau_1 + \tau_2 \sim 3.56$) (see Figs. 3(a) and (b)), whereas the second stage is characterized by the running kink like wave, with the constant shape and dimensionless speed $v = 2.0$, under action of the external electric field ($E/E_{\rm cr} = 10.0$), with the shorter time term ($\tau_k = \tau_R - \tau_2 - \tau_1 \sim 2.31$), until getting of the equilibrium orientation (see Fig. 3(c)). The dimension average velocity $u_R = d/t_R = (\epsilon_0\epsilon_a E^2 d)/(\gamma_1\tau_R)$ corresponding to evolution of the disturbance state to the equilibrium orientation, under acting of the external electric field $E/E_{\rm cr}$, are shown in Fig. 4. Here $t_R = t_{1,2} + t_k$ is the dimension time which the director $\hat{\mathbf{n}}$ spend for the full reorientation process from the initial state to the equilibrium orientation, $t_{1,2}$ is the part of the relaxation time which the director spend for reorientation process before forming of the traveling wave, and t_k is the time which the kink like wave spend for spreading along z-axis until getting of the equilibrium orientation. Calculations show a weak effect of the anchoring strength A (see curves (4) (weak anchoring) and (5) (strong anchoring) in Fig. 4) on the average velocity u_R of the director $\hat{\mathbf{n}}$ relaxation to its equilibrium orientation in the twisted TN cell. It should be mentioned here that the dimensionless analog of Eq. (1) with the boundary conditions (2) has also non-traveling solution $\phi(\tau, z)$. In that case, the relaxation of the $\hat{\mathbf{n}}$ to its equilibrium orientation $\hat{\mathbf{n}}_{\rm eq}$, has also been investigated by a standard relaxation method and result was shown in Fig. 3 of Ref. [2]. It was found that the effect of the external field E on the relaxation time τ_R decreases as the magnitude of E increases and saturates at $E/E_{\rm cr} \sim 20.0$ [2]. Having obtained the orientational relaxation time τ_R, which was calculated using Eq. (3), for the case of non-traveling solution $\phi(\tau, z)$ [2], both

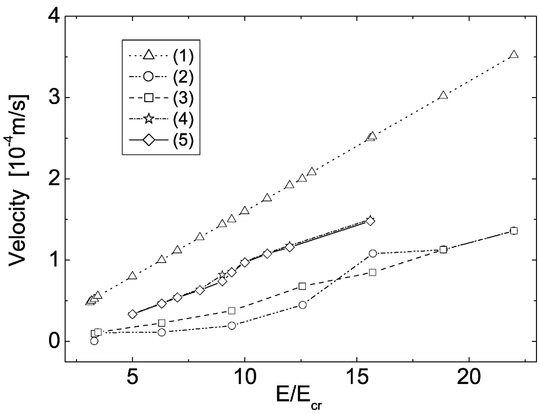

Fig. 4. The velocity of the director relaxation to its equilibrium orientation as a function of the electric field $E/E_{\rm cr}$. Different curves from the top to the bottom correspond to v_m (curve (1)), $v_{\rm I}$ curve (2) for the twisted boundary conditions, $v_{\rm II}$ curve (3) for the untwisted boundary conditions, u_R (curve (4)) for the weak anchoring conditions $(A\kappa/2K_2)\sin 2\Delta\phi = 1.0$, and (curve (5)) for the strong anchoring conditions, respectively. All calculations are done for the 10 μm 8*CB* TN cell.

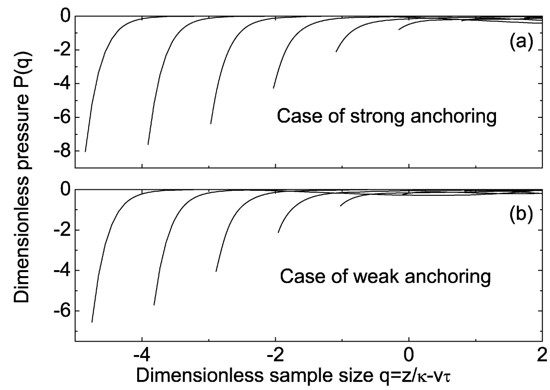

Fig. 5. Pressure $P(q)$ shape evolution from the one edge ($q = q_2$) of the cell to their second edge ($q = q_1$), calculated using Eq. (7), with the boundary conditions (5), and under action of the electric field $E/E_{\rm cr} = 10.0$. Here $q_2 = 5.0$ and $q_1 = 5.0 - v\tau$; (b) Same as (a), but with the boundary conditions (6), where $\frac{A\kappa}{2K_2}\sin 2\Delta\phi = 1.0$. In both these cases the initial condition $\phi(q, \tau = 0) = \frac{1}{\sigma}\varphi((q - q_3)/\sigma)$ $((q_1 + q_2)/2 < q_3 \leqslant q_2)$ close to the upper restricted surface.

for the boundary conditions (I) ($\phi(-\frac{d}{2}) = 0$, $\phi(\frac{d}{2}) = \frac{\pi}{2}$) and (II) ($\phi(\pm\frac{d}{2}) = 0$), one can calculate the average velocity $v_{\rm I,II} = d/t_R$ of the front $\phi(\tau, z)$ which start to move from the one edge of the cell to their second edge. Result of calculations $v_{\rm I,II}$ for the 10 μm TN 8*CB* cell is also shown in Fig. 4 (curves (2), for the twisted boundary conditions (I), and (3), for the untwisted boundary conditions (II), respectively). It is found that the slowest velocity v_m of the traveling front, approximately, in 3 times higher than the average velocity front $v_{\rm I}$ ($v_{\rm II}$) corresponding to the non-traveling solution, and in 1.5 times higher than the average velocity u_R of the director reorientation from the initial state to the equilibrium orientation with including of the kink like running contribution. It is important to stress here that the position of the initial disturbance $\phi(q, \tau = 0) = \frac{1}{\sigma}\varphi(q/\sigma)$ has influence on the mechanism of the traveling wave formation. Indeed, when the angle $\phi(q)$ has initially disturbed in the point $q_1 \leqslant q_4 < (q_1 + q_2)/2$ close to the lower restricted surface (unstable) (see Fig. 1(c)), for instance, by a focused Gaussian laser light [9], the traveling wave regime, after getting by the front $\phi(q)$ to the second edge of the cell, will changed into the non-traveling regime with the boundary conditions (2), whereas in the case when the point of the initial disturbance close to the upper (stable) surface, the traveling wave regime, after fast formation, can cover the full interval $[-d/2, d/2]$. In the first case, under action of the electric field $E/E_{\rm cr} = 10.0$ and the boundary condition $(A\kappa/2K_2)\sin 2\Delta\phi = 1.0$, the dimensionless relaxation time is equal to be 5.91, whereas in the second case, is equal to be 5.88. Notes that the relaxation behavior of $\phi(z - vt)$ in the form of the traveling wave probably can be observed in polarized white light. Taking into account that the director reorientation takes place in the narrow area of the LC sample (the width of the traveling wave) under influence of the electric field E, for instance, $E > 10.0E_{\rm cr}$ or 2×10^{-4} [C/m^2] for the 10 μm nematic 8*CB* cell, the traveling wave can be visualized in polarized white light as a dark strip running along the normal to both glass plates, with the velocity $v \sim 10^{-4}$ m/s.

Taking into account that there is no hydrodynamical flow because the director, representing the average molecular alignment, can rotate only within x–y plane without any translational motion, the dimensionless dissipation function $\mathcal{D}$ in the pure twist geometry, is given by [3] $\mathcal{D} = \phi_\tau^2(q)$, whereas the arbitrary dimensionless pressure P is given by [3]

$$P(q) = P_{\rm el}(q) - P_{\rm elast}(q) - \int (\partial\mathcal{D}/\partial\phi_\tau)\phi_q(q)\,dq = -\phi_q^2(q), \qquad (7)$$

where $P_{\rm el}(q) = \frac{1}{2}\sin^2\phi(q)$ and $P_{\rm elast}(q) = -\frac{1}{2}\phi_q^2(q)$ are both dimensionless contributions to the total arbitrary pressure P due to the electric and elastic forces, respectively. The dimensionless values of $P(q)$, in the case of strong (see Eq. (5)) and weak (see Eq. (6)) boundary conditions, under influence of the external electric field $E/E_{\rm cr} = 10.0$, as function of the dimensionless size $q = z/\kappa - v\tau$, on the final stage of evolution, is given in Fig. 5. Calculations show that the effect of the external electric field on the pressure $P(q)$, on the first stage of evolution, is characterized by the sharp decreasing of absolute magnitude of the pressure, up to, practically, zero value, with following monotonic increasing of the absolute value, on the final stage of evolution, up to ~ 8, in the case of the strong anchoring (Fig. 5(a)), and ~ 6.5, in the case of the weak anchoring (Fig. 5(b)), respectively. It is corresponds to dimension values of absolute pressure $\mathcal{P}(q) = (K_2/\kappa^2)P(q)$ in ~64 pN/μm^2, and ~52 pN/μm^2, respectively. Calculations also show that the pressure profile $P(q)$ will be determined by the balance of the rate of change in the elastic and electric energy with the viscous dissipation, and the front $P(q)$ start to move from the one edge of the cell to their second edge, and, finally, the pressure profile can get the lower plate after the time τ_R. According to our calculations, the relaxation time τ_R is the time which the director $\hat{\mathbf{n}}$ spend for the full reorientation process from the initial state ($\phi(q, \tau = 0)$) to their equilibrium orientation $\phi_{\rm eq}(q)$. Physically, this means that on the lower plate will be acted an extra pressure in $\sim 64\ \mathrm{pN}/\mu\mathrm{m}^2$, in the case of the strong anchoring, and in $\sim 52\ \mathrm{pN}/\mu\mathrm{m}^2$, in the case of the weak anchoring, respectively, during a very short period. Notes, that the anchoring effect decrease the magnitude of the absolute pressure up to 17%. So, by fixing of the extra pressure, which will be acted on the lower plate, one can measured, by using of an appropriate set up, the relaxation time τ_R, corresponding to evolution of the disturbance state to the equilibrium orientation in the form of the traveling wave.

It should be pointed out that in the present study we are primarily focused on the temperature range far from the second order phase transition temperature, because in the vicinity of a nematic–smectic A phase transition temperature both coefficients γ_1 and K_2 give rise to singularities. As results, the velocity $v_m = 0$ and the relaxation time t_R give rise to infinity [3], what in good agreement with the recent experimental data [10].

References

[1] P.G. de Gennes, J. Prost, The Physics of Liquid Crystals, second ed., Oxford Univ. Press, Oxford, 1995.
[2] A.V. Zakharov, A.A. Vakulenko, Phys. Rev. E 72 (2005) 021712.
[3] I.W. Stewart, The Static and Dynamic Continuum Theory of Liquid Crystals, Taylor and Francis, London, 2004.
[4] A. Kolmogorov, et al., Bull. Univ. Moscow, Ser. Int. A 1 (1937) 1.
[5] D.G. Aronson, H.F. Weinberger, Lect. Notes Math. 446 (1975) 3.
[6] W. van Saarloos, Phys. Rev. A 37 (1988) 211.
[7] I.S. Berezin, N.P. Zhidkov, Computing Methods, fourth ed., Pergamon Press, Oxford, 1965.
[8] P.P. Karat, N.N. Madhusudana, Mol. Cryst. Liq. Cryst. 40 (1977) 239.
[9] T. Matsui, M. Ozaki, K. Yoshino, Phys. Rev. E 69 (2004) 061715.
[10] D. Kamada, et al., Mol. Cryst. Liq. Cryst. 441 (2005) 129.