Quantum versus classical domains for teleportation with continuous variables

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By considering the utilization of a classical channel without quantum entanglement, fidelity $F_{classical}=\frac{1}{2}$ has been established as setting the boundary between classical and quantum domains in the teleportation of coherent states of the electromagnetic field [S. L. Braunstein, C. A. Fuchs, and H. J. Kimble, J. Mod. Opt. **47**, 267 (2000)]. We further examine the quantum-classical boundary by investigating questions of entanglement and Bell-inequality violations for the Einstein-Podolsky-Rosen states relevant to continuous variable teleportation. The threshold fidelity for employing entanglement as a quantum resource in teleportation of coherent states is again found to be $F_{classical}=\frac{1}{2}$. Likewise, violations of local realism onset at this same threshold, with the added requirement of overall efficiency $\eta>\frac{2}{3}$ in the unconditional case. By contrast, recently proposed criteria adapted from the literature on quantum-nondemolition detection are shown to be largely unrelated to the questions of entanglement and Bell-inequality violations.

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I. INTRODUCTION

As proposed by Bennett *et al.* [1], the protocol for achieving quantum teleportation is the following. Alice is to transfer an unknown quantum state $|\psi\rangle$ to Bob, using, as the sole resources, some previously shared *quantum entanglement* and a *classical channel* capable of communicating measurement results. Physical transport of $|\psi\rangle$ from Alice to Bob is excluded at the outset. Ideal teleportation occurs when the state $|\psi\rangle$ enters Alice's sending station and the *same* state $|\psi\rangle$ emerges from Bob's receiving station.

Of course, in actual experiments [2–5], the ideal case is *unattainable* as a matter of principle. The question of operational criteria for gauging success in an experimental setting, therefore, cannot be avoided. We previously proposed that a minimal set of conditions for claiming success in the laboratory are the following [6].

(1) An unknown quantum state (supplied by a third party Victor) is input physically into Alice's station from an outside source.

(2) The ''recreation'' of this quantum state emerges from Bob's receiving terminal available for Victor's independent examination.

(3) There should be a quantitative measure for the quality of the teleportation, and, based upon this measure, it should be clear that shared entanglement enables the output state to be ''closer'' to the input state than could have been achieved if Alice and Bob had utilized a classical communication channel alone.

In Ref. [6], it was shown that the fidelity F between input and output states is an appropriate measure of the degree of similarity in criterion (3). For an input state $|\psi_{in}\rangle$ and an output state described by the density operator $\hat{\rho}_{out}$, the fidelity is given by [7]

$$F=\langle \psi_{in}|\hat{\rho}_{out}|\psi_{in}\rangle. \tag{1}$$

To date only the experiment of Furusawa *et al.* [4] achieved unconditional experimental teleportation as defined by the three criteria above [6,8,9]. Based upon the original analysis of Vaidman for teleportation of continuous quantum variables [10], this experiment was carried out in the setting of continuous quantum variables with input states $|\psi_{in}\rangle$ consisting of coherent states of the electromagnetic field, with an observed fidelity $F_{\rm expt}=0.58\pm0.02$ having been attained. This benchmark is significant because it can be demonstrated [4,6] that quantum entanglement is the critical ingredient in achieving an average fidelity greater than $F_{classical}=\frac{1}{2}$ *when* the input is an absolutely random coherent state [11].

Against this backdrop, several recent authors suggested that the appropriate boundary between the classical and quantum domains in the teleportation of coherent states should be consistent with a fidelity $F=\frac{2}{3}$ [12–15]. Principal concerns expressed by these authors include the distinction between entanglement or nonseparability and possible violations of Bell's inequalities.[1] In Ref. [14] the violation of a certain Heisenberg-type inequality (HI) is introduced to characterize shared entanglement, leading to the condition $F>\frac{2}{3}$ being required for the declaration of successful teleportation. This criterion, based upon the Heisenberg-type inequality as well the bulk of the analyses in Refs. [12–14], is related to previous work on inference at a distance first introduced in the quantum nondemolition (QND) measurement literature. In a similar spirit, it was also suggested that the threshold $F>\frac{2}{3}$ is required by a criterion having to do with a certain notion of reliable ''information exchange'' [15].

The purpose of the present paper is to revisit the question of the appropriate point of demarcation between classical

[1]Since the terms ''entanglement'' and ''nonseparability'' are used interchangeably in the quantum information community, we will treat them as synonyms to eliminate further confusion. We will refer to violations of Bell's inequalities explicitly whenever a distinction must be made between entanglement and local realism *per se*.

and quantum domains in the teleportation of coherent states of the electromagnetic field. Our approach will be to investigate questions of nonseparability and violations of Bell inequalities for the particular entangled state employed in the teleportation protocol of Ref. [16]. Of significant interest will be the case with losses, so that the relevant quantum states will be mixed states. Our analysis supports the following principal conclusions.

(1) By application of the work of Duan *et al.* [17], Simon [18], and Tan [19], we investigate the question of entanglement. We show that the states employed in the experiment of Ref. [4] are nonseparable, as was operationally confirmed in the experiment. Moreover, we study the issue of nonseparability for mixed states over a broad range in the degree of squeezing for the initial Einstein-Podolsky-Rosen state, in the overall system loss, and in the presence of thermal noise. This analysis reveals that EPR mixed states that are nonseparable do indeed lead to a fidelity of $F>F_{classical}=\frac{1}{2}$ for the teleportation of coherent states. Hence, in keeping with criterion (3) above, the threshold fidelity for employing entanglement as a quantum resource is precisely the same as was deduced in the previous analysis of Ref. [6]. Within the setting of quantum optics, this threshold corresponds to the standard benchmark for manifestly quantum or nonclassical behavior, namely, that the Glauber-Sudarshan phase-space function takes on negative values, here for any bipartite nonseparable state [20]. By contrast, a fidelity consistent with the value $F=\frac{2}{3}$ championed in Refs. [12–15] is essentially unrelated to the threshold for entanglement (nonseparability) as well as to the boundary for the nonclassical character of the EPR state.

(2) By application of the work of Banaszek and Wodkiewicz [21,22], we explore the possibility of violations of Bell inequalities for the EPR (mixed) states employed in the teleportation of continuous quantum-variables states. We find direct violations of a Clauser-Horne-Shimong-Holt (CHSH) inequality [23] over large domains in fidelity F subject to the requirements that $F>F_{classical}=\frac{1}{2}$ and that the overall efficiency $\eta>\frac{2}{3}$. Significant is a regime both of entanglement (nonseparability) and violation of a CHSH inequality, for which the teleportation fidelity $F<\frac{2}{3}$ and for which the criteria of Refs. [14,15] fail. Hence, teleportation with $\frac{1}{2}<F<\frac{2}{3}$ is possible with EPR (mixed) states which do not admit a local hidden variable description. $F>\frac{2}{3}$ does not provide a relevant criterion for delineating the quantum and classical domains with respect to violations of Bell's inequalities for the EPR states.

(3) By adopting a protocol analogous to that employed in *all* previous experimental demonstrations of violations of Bell's inequalities [24–26], scaled correlation functions can be introduced for continuous quantum variables. In terms of these scaled correlations, the EPR mixed state used for teleportation violates a generalized version of the CHSH inequality, though nonideal detector efficiencies require a ‘‘fair sampling’’ assumption for this. These violations set in for $F>F_{classical}=\frac{1}{2}$, and were recently observed in a setting of low detection efficiency [27]. This experimental verification of a violation of a CHSH inequality (with a fair sampling assumption) again refutes the purported significance of the threshold $F=\frac{2}{3}$.

Note that these results are in complete accord with the prior treatment of Ref. [6], that demonstrated that, in the absence of shared entanglement between Alice and Bob, there is an upper limit for the fidelity for the teleportation of randomly chosen coherent states given by $F_{classical}=\frac{1}{2}$. Nothing in Refs. [12–15] called this analysis into question. By contrast, we find no support for a special significance to the threshold fidelity $F=\frac{2}{3}$ in connection to issues of separability and Bell-inequality violations. Instead, as we will show, it is actually the value $F_{classical}=\frac{1}{2}$ that heralds entrance into the quantum domain with respect to these very same issues.

All this is not to say that teleportation of coherent states with increasing degrees of fidelity beyond $F_{classical}=\frac{1}{2}$ to $F>\frac{2}{3}$ is not without significance. In fact, as tasks of ever-increasing complexity are to be accomplished, there will be corresponding requirements to improve the fidelity of teleportation yet further. Moreover, there are clearly diverse quantum states other than coherent states that one might desire to teleport, including squeezed states, quantum superpositions, entangled states [19,28], and so on. The connection between the ‘‘intricacy’’ of such states and the requisite resources for achieving high-fidelity teleportation was discussed in Ref. [16], including the example of the superposition of two coherent states,

$$|\alpha\rangle+|-\alpha\rangle, \tag{2}$$

which for $|\alpha|\gg 1$ requires an EPR state with an extreme degree of quantum correlation.

Similarly, the conditional variances contained in the Heisenberg-type inequalities are in fact quite important for the inference of the properties of a *system* given the outcomes of measurements made on a *meter* following a *system-meter* interaction. Such quantities are gainfully employed in quantum optics in many settings, including realization of the original EPR *gedanken* experiment [29–31] and of back-action evading measurement and quantum nondemolition (QND) detection [32]. However, even within the restricted context of QND detection, it is worth emphasizing that the usual inequalities imposed upon these inference variances, together with so-called information transfer coefficients, provide necessary and not sufficient conditions for successful back-action evading measurement [33].

Something that we would like to stress apart from the details of any particular teleportation criterion is the apparent growing confusion in the community that equates quantum teleportation experiments with fundamental tests of quantum mechanics. The purpose of such tests is generally to compare quantum mechanics to other potential theories, such as local realistic hidden-variable theories [14,34,35]. In our view, experiments in teleportation have nothing to do with this. They instead represent investigations *within* quantum mechanics, demonstrating only that a particular task can be accomplished with the resource of quantum entanglement and cannot be accomplished without it. This means that violations of

Bell's inequalities are largely irrelevant as far as the original proposal of Bennett *et al.* [1] is concerned, as well as for experimental implementations of that protocol. In a theory which allows states to be cloned, there would be no need to discuss teleportation at all—unknown states could be cloned and transmitted, with a fidelity arbitrarily close to 1.

These comments notwithstanding, there are nevertheless attempts to link the idea of Bell-inequality violations with the fidelity of teleportation. It is to the details of this and other linkages that we now turn. The remainder of the paper is organized as follows. In Sec. II, we extend the prior work of Ref. [6] to a direct treatment of the consequences of shared entanglement between Alice and Bob, beginning with an explicit model for the mixed EPR states used for teleportation of continuous quantum variables. In Sec. III we review the criteria based upon Heisenberg-type inequalities and "information content," in preparation for showing their inappropriateness as tools for the questions at hand. In Sec. IV, we explicitly demonstrate the relationship between entanglement and fidelity, and find the same threshold $F_{classical}=\frac{1}{2}$ as in our prior analysis [6]. The value $F=\frac{2}{3}$ is shown to have no particular distinction in this context. In Secs. V and VI, we further explore the role of entanglement with regard to violations of a CHSH inequality, and provide a quantitative boundary for such violations. Again, $F_{classical}=\frac{1}{2}$ provides the point of entry into the quantum domain, with $F=\frac{2}{3}$ having no notoriety. Our conclusions are collected in Sec. VII. Of particular significance, we point out that the teleportation experiment of Ref. [4] did indeed cross from the classical to the quantum domain, just as advertised previously.

II. EPR STATE

The teleportation protocol we consider is that of Braunstein and Kimble [16], for which the relevant entangled state is the so-called two-mode squeezed state. This state is given explicitly in terms of a Fock-state expansion for two modes (1,2) by [36,37]

$$|\mathrm{EPR}\rangle_{1,2}=\frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n|n\rangle_1|n\rangle_2, \tag{3}$$

where r measures the amount of squeezing required to produce the entangled state. Note that, for simplicity, we consider the case of two single modes for the electromagnetic field; the extension to the multimode case for fields of finite bandwidth can be found in Ref. [38].

The pure state of Eq. (3) can be equivalently described by the corresponding Wigner distribution W_{EPR} over the two modes (1,2),

$$\begin{aligned}W_{\mathrm{EPR}}&(x_1,p_1;x_2,p_2)\\&=\frac{4}{\pi^2}\frac{1}{\sigma_+^2\sigma_-^2}\exp(-[(x_1+x_2)^2+(p_1-p_2)^2]/\sigma_+^2\\&\quad-[(x_1-x_2)^2+(p_1+p_2)^2]/\sigma_-^2),\end{aligned} \tag{4}$$

where $\sigma_\pm$ are expressed in terms of the squeezing parameter by

$$\begin{aligned}\sigma_+^2&=e^{+2r},\\\sigma_-^2&=e^{-2r},\end{aligned} \tag{5}$$

with $\sigma_+^2\sigma_-^2=1$. Here the canonical variables (x_j,p_j) are related to the complex field amplitude α_j for mode $j=(1,2)$ by

$$\alpha_j=x_j+ip_j. \tag{6}$$

In the limit of $r\rightarrow\infty$, Eq. (4) becomes

$$C\,\delta(x_1-x_2)\,\delta(p_1+p_2), \tag{7}$$

which makes a connection to the original EPR state of Einstein, Podolsky, and Rosen [29–31].

Of course, W_{EPR}, as given above, is for the ideal, lossless case. Of particular interest with respect to experiments is the inclusion of losses, which arise, for example, form finite propagation and detection efficiencies. Rather than deal with any detailed setup (e.g., as treated in explicit detail in Ref. [30]), here we adopt a generic model of the following form. Consider two identical beam splitters each with a transmission coefficient η, one for each of the two EPR modes. We take $0\leq\eta\leq1$, with $\eta=1$ for the ideal, lossless case. The input modes to the beam splitter 1 are taken to be $(1',a')$, while for beam splitter 2 the modes are labeled by $(2',b')$. Here, the modes $(1',2')$ are assumed to be in the state specified by the ideal W_{EPR} as given in Eq. (4) above, while the modes (a',b') are taken to be independent thermal (mixed) states, each with a Wigner distribution

$$W(x,p)=\frac{1}{\pi\left(\bar{n}+\frac{1}{2}\right)}\exp\{-(x^2+p^2)/(\bar{n}+1/2)\}, \tag{8}$$

where $\bar{n}$ is the mean thermal photon number for each of the modes (a',b').

The overall Wigner distribution for the initial set of input modes $(1',2'),(a',b')$ is then just the product

$$W_{\mathrm{EPR}}(x_{1'},p_{1'};x_{2'},p_{2'})W(x_{a'},p_{a'})W(x_{b'},p_{b'}). \tag{9}$$

The standard beam-splitter transformations lead in a straightforward fashion to the Wigner distribution for the output set of modes $(1,2),(a,b)$, where, for example,

$$\begin{aligned}x_1&=\sqrt{\eta}x_{1'}-\sqrt{1-\eta}x_{a'},\\x_a&=\sqrt{\eta}x_{a'}+\sqrt{1-\eta}x_{1'}.\end{aligned} \tag{10}$$

We require W_{EPR}^{out} for the (1,2) modes alone, which is obtained by integrating over the (a,b) modes. A straightforward calculation results in the following distribution for the mixed output state,

$$W_{\rm EPR}^{out}(x_1,p_1;x_2,p_2) = \frac{4}{\pi^2}\frac{1}{\overline{\sigma}_+^2\overline{\sigma}_-^2}\exp(-[(x_1+x_2)^2+(p_1-p_2)^2]/\overline{\sigma}_+^2 -[(x_1-x_2)^2+(p_1+p_2)^2]/\overline{\sigma}_-^2), \quad (11)$$

where $\overline{\sigma}_\pm$ are given by

$$\overline{\sigma}_+^2=\eta e^{+2r}+(1-\eta)(1+2\overline{n}),$$
$$\overline{\sigma}_-^2=\eta e^{-2r}+(1-\eta)(1+2\overline{n}). \quad (12)$$

Note that $W_{\rm EPR}^{out}$, as above, follows directly from $W_{\rm EPR}$ in Eq. (4) via the simple replacements $\sigma_\pm \to \overline{\sigma}_\pm$. Relevant to the discussion of Bell inequalities in Secs. V and VI is the fact that $\overline{\sigma}_+^2\overline{\sigma}_-^2>1$ for any $r>0$ and $\eta<1$. Although the simple ''beam-splitter'' model is sufficient for our current discussion, a more detailed dynamical model was considered in Refs. [39,40] for continuous variable teleportation in the presence of dissipation.

III. ALTERNATE CRITERIA FOR TELEPORTATION OF COHERENT STATES

In Ref. [6] the boundary between classical and quantum domains in the teleportation of coherent states was determined to be $F_{classical}=\frac{1}{2}$, based upon an analysis of teleportation in the absence of shared entanglement (i.e., *Alice* and *Bob* employ a classical channel alone). Several recent authors instead argued in favor of alternate criteria for determining successful teleportation of coherent states [12–15]. In this section, we recapitulate the critical elements of these analyses, and state their criteria in the present notation, with particular attention to the work of Refs. [14,15]. Critical discussions of the criteria of Refs. [12,13] can be found in Refs. [6,38]. In subsequent sections we proceed further with our own analysis of entanglement and possible violations of Bell's inequalities for the EPR state of Eq. (11), and to an investigation of their relevance to the delineation of the appropriate quantum-classical boundary in teleportation.

Turning first to criteria arising from the QND literature [12–14], we recall the following statement with reference to Eq. (21) of Ref. [14]: ''As a criteria for non-separability [by which is meant violations of Bell's inequalities], we will use the EPR argument: two different measurements prepare two different states, in such a way that the product of conditional variances (with different conditions) violates the Heisenberg principle.''

This statement takes a quantitative form in terms of the following conditional variances expressed in the notation of the preceding section for EPR beams (1,2),

$$V_{x_i|x_j}=\langle\Delta x_i^2\rangle-\frac{\langle x_ix_j\rangle^2}{\langle\Delta x_j^2\rangle},$$
$$V_{p_i|p_j}=\langle\Delta p_i^2\rangle-\frac{\langle p_ip_j\rangle^2}{\langle\Delta p_j^2\rangle}. \quad (13)$$

with $(i,j)=(1,2)$ and $i\neq j$. Note that, for example, $V_{x_2|x_1}$ gives the error in the knowledge of the canonical variable x_2 based upon an estimate of x_2 from a measurement of x_1, and likewise for the other conditional variances. These variances were introduced in Refs. [30,31] in connection with an optical realization of the original *gedanken* experiment of Einstein, Podolsky, and Rosen [29]. An apparent violation of the uncertainty principle arises if the product of inference errors is below the uncertainty product for one beam alone. For example, $V_{x_2|x_1}V_{p_2|p_1}<\frac{1}{16}$ represents such an apparent violation, since $\Delta x_2^2\Delta p_2^2\geqslant\frac{1}{16}$ is demanded by the canonical commutation relation between x_2 and p_2, with $\Delta x_{1,2}^2=\frac{1}{4}=\Delta p_{1,2}^2$ for the vacuum state [30,31].

This concept of inference at a distance has been elevated to ''a criteria for nonseparability [i.e., violation of Bell's inequalities]'' [14], namely, that the domain of local realism should be determined by the conditions

$$V_{x_2|x_1}V_{p_2|p_1}\geqslant\frac{1}{16} \quad \text{and} \quad V_{x_1|x_2}V_{p_1|p_2}\geqslant\frac{1}{16}. \quad (14)$$

As shown in Refs. [30,31] for the states under consideration, the conditional variances of Eq. (13) are simply related to the following (unconditional) variances:

$$\Delta x_{\mu_{ij}}^2=\langle(x_i-\mu_{ij}x_j)^2\rangle,$$
$$\Delta p_{\nu_{ij}}^2=\langle(p_i-\nu_{ij}p_j)^2\rangle. \quad (15)$$

If we use a measurement of x_j to estimate x_i, then $\Delta x_{\mu_{ij}}^2$ is the variance of the error when the estimator is chosen to be $\mu_{ij}x_j$, and likewise for $\Delta p_{\nu_{ij}}^2$. For an optimal estimate, the parameters (μ_{ij},ν_{ij}) are given by [30,31]

$$\mu_{ij}^{\rm opt}=\frac{\langle x_ix_j\rangle}{\langle\Delta x_j^2\rangle}, \quad \nu_{ij}^{\rm opt}=\frac{\langle p_ip_j\rangle}{\langle\Delta p_j^2\rangle}, \quad (16)$$

and, in this case,

$$V_{x_i|x_j}=\Delta x_{\mu_{ij}^{\rm opt}}^2, \quad V_{p_i|p_j}=\Delta p_{\nu_{ij}^{\rm opt}}^2. \quad (17)$$

The condition in Eq. (14) that attempts to define the domain of local realism can then be reexpressed as

$$\Delta x_{\mu_{21}}^2\Delta p_{\nu_{21}}^2\geqslant\frac{1}{16}, \quad \Delta x_{\mu_{12}}^2\Delta p_{\nu_{12}}^2\geqslant\frac{1}{16}, \quad (18)$$

where we assume the optimized choice and drop the superscript ''opt.''

To make apparent the critical elements of the discussion, we next assume symmetric fluctuations as appropriate to the

EPR state of Eq. (11), $\mu_{ij}=\mu_{ji}\equiv\mu$ and $\nu_{ij}=\nu_{ji}\equiv\nu$, with $\mu=-\nu$. Note that within the context of our simple model of the losses, the optimal value of μ is given by

$$\mu^{\mathrm{opt}}=\frac{\eta\sinh 2r}{(1-\eta)+\eta\cosh 2r}, \tag{19}$$

where in the limit $r\gg 1$, $\mu\to 1$ for $\eta>0$. For this case of symmetric fluctuations, the HI of Eq. (18) becomes

$$\Delta x_\mu^2\Delta p_\mu^2\geq\frac{1}{16}, \tag{20}$$

where

$$\begin{aligned}\Delta x_\mu^2&=\langle(x_1-\mu x_2)^2\rangle=\langle(x_2-\mu x_1)^2\rangle,\\ \Delta p_\mu^2&=\langle(p_1+\mu p_2)^2\rangle=\langle(p_2+\mu p_1)^2\rangle.\end{aligned} \tag{21}$$

In the limit $r\gg 1$, $\mu\to 1$ for $\eta>0$, so that the Heisenberg-type inequality becomes

$$\Delta x^2\Delta p^2\geq\frac{1}{16}. \tag{22}$$

Here $(\Delta x^2,\Delta p^2)$ are as defined in Eq. (21); now, with $\mu=1$,

$$\begin{aligned}\Delta x^2&=\langle(x_1-x_2)^2\rangle,\\ \Delta p^2&=\langle(p_1+p_2)^2\rangle,\end{aligned} \tag{23}$$

where from Eq. (11), we have that $\Delta x^2+\Delta p^2=\overline{\sigma_-^2}$ for the EPR beams (1,2).

The claim of Grangier and Grosshans [14] is that the inequality of Eq. (20) serves as ''the condition for no useful entanglement between the two beams,'' where by ''useful'' they refer explicitly to ''the existence of quantum nonseparability (violation of Bell's inequalities).'' The variances of Eqs. (21) and (23) are also related to criteria developed within the setting of quantum nondemolition detection [32].

Relevant to the discussion in Sec. IV will be to note that in general the inequality

$$V_1V_2\geq\frac{a^2}{4} \tag{24}$$

implies that

$$V_1+V_2\geq V_1+\frac{a^2}{4V_1}\geq a, \tag{25}$$

so that the purported criterion Eq. (20) from Ref. [14] for *classical* teleportation leads to

$$\Delta x_\mu^2+\Delta p_\mu^2\geq\frac{1}{2}, \tag{26}$$

which for $r\gg 1$ becomes

$$\Delta x^2+\Delta p^2\geq\frac{1}{2}, \tag{27}$$

with $(\Delta x^2,\Delta p^2)$ as defined in Eq. (23).

Apart from the criteria of Eqs. (26) and (27), an alternative requirement for the successful teleportation of coherent states has been introduced in Ref. [15], namely, that ''the information content of the teleported quantum state is higher than the information content of any (classical or quantum) copy of the input state, that may be broadcasted classically.''

To quantify the concept of ''information content'' these authors introduced a ''generalized fidelity'' describing not the overlap of quantum states as is standard in the quantum information community, but rather the conditional probability $P(\alpha|I)$ that a particular coherent state $|\alpha\rangle$ was actually sent given ''the available information *I*.'' In effect, Ref. [15] considered the following protocol. Victor sends some unknown coherent state $|\alpha_0\rangle$ to Alice, with Alice making her best attempt to determine this state [41], and sending the resulting measurement outcome to Bob as in the standard protocol. Bob then does one of two things. In the first instance, he forwards only this classical message with Alice's measurement outcome to Victor without reconstructing a quantum state. In the second case, he actually generates a quantum state conditioned upon Alice's message and sends this state to Victor, who must then make his own measurement to deduce whether the teleported state corresponds to the one that he initially sent. The requirement for successful teleportation is that the information gained by Victor should be greater in the latter case where quantum states are actually generated by Bob than in the former case where only Alice's classical measurement outcome is distributed. It is straightforward to show that exceeding the bound set by Eq. (27) is sufficient to ensure that this second criteria is likewise satisfied for the teleportation of a coherent state $|\alpha\rangle$, albeit with the same caveat expressed in Ref. [11], namely that neither the set *S* of initial states $\{|\psi_{in}\rangle\}$ nor the distribution $P(|\psi_{in}\rangle)$ over these states is specified. We now turn to an evaluation of the foregoing criteria placing special emphasis on the issues of entanglement and violations of Bell's inequalities, specifically because these are the concepts that were emphasized in the work of Ref. [14].

IV. ENTANGLEMENT AND FIDELITY

A. Nonseparability of the EPR beams

To address the question of the nonseparability of the EPR beams, we refer to the papers of Duan *et al.* [17] and Simon [18], as well as related work by Tan [19]. For the definitions of (x_i,p_i) that we have chosen for the EPR beams (1,2), a sufficient condition for nonseparability (without an assumption of Gaussian statistics) is that

$$\Delta x^2+\Delta p^2<1, \tag{28}$$

where Δx^2 and Δp^2 are defined in Eq. (23). This result follows from Eq. (3) of Duan *et al.* with $a=1$ (and from a similar more general equation in Simon) [42]. Note that Duan *et al.* had $\Delta x_i^2=\frac{1}{2}=\Delta p_i^2$ for the vacuum state, while our definitions lead to $\Delta x_i^2 = \frac{1}{4} = \Delta p_i^2$ for the vacuum state, where for example, $\Delta x_1^2 = \langle x_1{}^2 \rangle$, and that the EPR fields considered have zero mean.

Given the Wigner distribution $W_{\rm EPR}^{out}$ as in Eq. (11), we find immediately that

$$\Delta x^2 + \Delta p^2 = 2\frac{\overline{\sigma_-^2}}{2} = \eta e^{-2r} + (1-\eta)(1+2\bar{n}). \quad (29)$$

For the case $\bar{n}=0$, the resulting state is *always entangled for any $r>0$ even for $\eta \ll 1$*, in agreement with the discussion in Duan *et al.* [17]. For nonzero $\bar{n}$, the state is entangled so long as

$$\bar{n} < \frac{\eta[1-\exp(-2r)]}{2(1-\eta)}. \quad (30)$$

We emphasize that in the experiment of Furusawa *et al.* [4], for which $\bar{n}=0$ is the relevant case, the above inequality guarantees that teleportation was carried out with entangled (i.e., nonseparable) states for the EPR beams, independent of any assumption about whether these beams were Gaussian or pure states. Explicitly, the measured variances for the work of Ref. [4] were $\Delta x^2 \approx (0.8 \times \frac{1}{2}) \approx \Delta p^2$, so that $\Delta x^2 + \Delta p^2 \approx 0.8 < 1$.

In contrast to the condition for entanglement given in Eq. (28), the discussion of Sec. III instead requires exceeding the more stringent condition of Eq. (27) for successful teleportation. Although the EPR beams are indeed entangled whenever Eq. (28) is satisfied, entanglement in the domain

$$\frac{1}{2} \leqslant \Delta x^2 + \Delta p^2 < 1 \quad (31)$$

is termed in Ref. [14] as not "useful" and in Ref. [13](b) as not "true EPR entanglement."

With regard to the QND-like conditions introduced in Refs. [12–14], we note that more general forms for the nonseparability condition of Eq. (28) are given in Refs. [17,18]. Of particular relevance is a condition for the variances of Eq. (15) for the case of symmetric fluctuations as for EPR state in Eq. (11), $\mu_{ij} = \mu_{ji} \equiv \mu$ and $\nu_{ij} = \nu_{ji} \equiv \nu$, with $\mu = -\nu$. Consider, for example, the first set of variances in Eq. (21), namely,

$$\Delta x_\mu^2 = \langle (x_2 - \mu x_1)^2 \rangle \quad \text{and} \quad \Delta p_\mu^2 = \langle (p_2 + \mu p_1) \rangle^2, \quad (32)$$

as would be appropriate for an inference of (x_2, p_2) from a measurement (at a distance) of (x_1, p_1). Although $\mu = 1$ is certainly the case relevant to the actual teleportation protocol of Ref. [16], Alice and Bob are surely free to explore the degree of correlation between their EPR beams and to test for entanglement by any means at their disposal, including simple measurements with $\mu \neq 1$.

In this case of general μ, a sufficient condition for entanglement of the EPR beams (1,2) may be obtained using Eq. (11) of Ref. [18] yielding

$$\Delta x_\mu^2 + \Delta p_\mu^2 < \frac{(1+\mu^2)}{2}, \quad (33)$$

which reproduces Eq. (28) for $\mu = 1$. Although the experiment of Ref. [4] explicitly recorded variances only for the case $\mu = 1$, the EPR experiment of Ref. [30] chose $\mu < 1$, in correspondence to the degree of correlation between the EPR beams. This original realization of the EPR experiment achieved $\Delta x_\mu^2 = [(0.835 \pm 0.008) \times \frac{1}{4}]$ and $\Delta p_\mu^2 = [(0.837 \pm 0.008) \times \frac{1}{4}]$ for $\mu^2 = 0.58$ [30], so that

$$\Delta x_\mu^2 + \Delta p_\mu^2 = (0.42 \pm 0.01) < 0.79 = \frac{(1+\mu^2)}{2}. \quad (34)$$

With the hindsight provided by the nonseparability criteria of Refs. [17,18], we see that the experiment of Ref. [30] represents the first demonstration of the unconditional generation and detection of bipartite entangled states (i.e., so-called *deterministic* production of entanglement), there within the setting of continuous quantum variables.

More generally, it is straightforward to show that the EPR mixed state of Eq. (11) satisfies the entanglement criteria Eq. (32) for any $r > 0$ with $\mu^{\min} < \mu \leqslant 1$. Here $\mu^{\min}$ sets the threshold for the onset of entanglement in Eq. (32), where

$$\mu^{\min} = \frac{2\sinh^2(r/2)}{\sinh(2r)} \quad (35)$$

independent of η. In contrast to the choice $\mu = \mu^{\rm opt}$ as in Eq. (19) which minimizes the conditional variances $(\Delta x_\mu^2, \Delta p_\mu^2)$, the value $\mu = 1$ maximizes the degree of entanglement in terms of the largest fractional deviation of $(\Delta x_\mu^2 + \Delta p_\mu^2)$ below $(1+\mu^2)/2$ [43]. This result is in satisfying correspondence with the actual teleportation protocol, namely, that $\mu = 1$ as appropriate there actually maximizes the degree of entanglement for given (r, η).

To connect these results with the inequalities introduced in Sec. III, we note that Eq. (33) for nonseparability implies that

$$\Delta x_\mu^2 \Delta p_\mu^2 < \frac{(1+\mu^2)^2}{16}, \quad (36)$$

which is in the form of a violation of a Heisenberg-type inequality. Note that for $\bar{n} = 0$, this inequality is satisfied for any $r > 0$ and $0 < \eta \leqslant 1$, now with $\mu^{\min} < \mu \leqslant 1$ as above. For $r \gg 1$ and $\eta > 0$, $\mu \to 1$, and Eq. (36) becomes

$$\Delta x^2 \Delta p^2 < \frac{1}{4}. \quad (37)$$

By contrast, application of the alternate conditions from Sec. III leads to the requirement

$$\Delta x_\mu^2 \Delta p_\mu^2 < \frac{1}{16}. \quad (38)$$

Within the setting of our current model, this condition can be satisfied for any $r>0$ only so long as the efficiency $\eta>\frac{1}{2}$ [44]. Again, for $r\gg1$ and $\eta>0$, $\mu\rightarrow1$, so that Eq. (38) becomes

$$\Delta x^2\Delta p^2<\frac{1}{16}. \tag{39}$$

Although these conditional variances and related criterion are quite useful in the analysis of back-action evading measurement for quantum nondemolition detection, they apparently have no direct relevance to the question of entanglement, for $\mu=1$ or otherwise.

These various inequalities can be viewed in somewhat more general terms by noting that Eq. (8) of Ref. [18] demands that the sum of variances for *any* bipartite state satisfy the condition

$$\frac{|1-\mu^2|}{2}\leqslant\Delta x_\mu^2+\Delta p_\mu^2, \tag{40}$$

so that entangled states that satisfy Eq. (33) are further constrained by

$$\frac{|1-\mu^2|}{2}\leqslant(\Delta x_\mu^2+\Delta p_\mu^2)<\frac{(1+\mu^2)}{2}. \tag{41}$$

The sum of variances $\Delta x_\mu^2+\Delta p_\mu^2$ for the EPR (mixed) state of Eq. (11) ranges continuously between these bounds. As discussed in connection with Eq. (33) above, for $\overline{n}=0$ the EPR (mixed) state drops below the upper bound to become entangled for any $r>0$ so long as $\eta\neq0$ and $\mu^{\min}<\mu\leqslant1$. It approaches the lower bound for $r\gg1$ with $\eta=1$. By contrast, the criteria of Refs. [12–14] [e.g., the inequality of Eq. (26) from Ref. [14]] effectively split the difference between these two limits by defining the quantum-classical boundary to be set by $\Delta x_\mu^2+\Delta p_\mu^2=\frac{1}{2}$.

In this regard, it is worth emphasizing that Ref. [29] nowhere contains the Heisenberg-type inequalities discussed above and in Sec. III, which were first introduced by Reid and Drummond [31]. The states originally considered by EPR [i.e., Eqs. (7), (8), (9), (11), and (15) of Ref. [29]] are instead δ-correlated pure states, and have inference variances equal to zero (e.g., $V_{x_2|x_1}=0=V_{p_2|p_1}$ and $\Delta x^2=0=\Delta p^2$). For finite degrees of correlation, the quantitative boundary at which the EPR argument fails is provided not by the Heisenberg-type inequalities of Eq. (38), but rather by the analysis of Refs. [17,18] for mixed as well as for pure states, which leads instead to Eq. (36).

Although the boundaries expressed by the nonseparability conditions of Eqs. (28) and (33) are perhaps not so familiar in quantum optics, we stress that these criteria are associated quite directly with the standard condition for nonclassical behavior adopted by this community. Whenever Eqs. (28) and (33) are satisfied, the Glauber-Sudarshan phase-space function takes on negative values [20], which for almost 40 years has heralded entrance into a manifestly quantum or nonclassical domain. It is difficult to understand how the authors in Refs. [12–15] proposed to move from $\Delta x^2+\Delta p^2=1$ to $\Delta x^2+\Delta p^2=\frac{1}{2}$ without employing quantum resources in the teleportation protocol (as is required when the Glauber-Sudarshan *P* function is not positive definite), with their own work offering no suggestion of how this is to be accomplished.

B. Fidelity

Turning next to the question of the relationship of entanglement of the EPR beams [as quantified in Eq. (28)] to the fidelity attainable for teleportation *with these beams*, we recall from Eq. (2) of Ref. [4] that

$$F=\frac{1}{1+\overline{\sigma_-^2}}, \tag{42}$$

where this result applies to teleportation of coherent states [45] . When combined with Eq. (29), we find that

$$F=\frac{1}{1+(\Delta x^2+\Delta p^2)}. \tag{43}$$

The criterion of Eq. (28) for nonseparability then guarantees that nonseparable EPR states as in Eqs. (4) and (11) (be they mixed or pure) are sufficient to achieve

$$F>F_{classical}=\frac{1}{2}, \tag{44}$$

whereas separable states must have $F\leqslant F_{classical}=\frac{1}{2}$, although we emphasize that this bound applies for the average fidelity for coherent states distributed over the entire complex plane [6]. More general cases for the distribution of coherent states are treated in the Appendix.

We thereby demonstrate that the condition $F>F_{classical}=\frac{1}{2}$ for quantum teleportation as established in Ref. [6] coincides with that for nonseparability (i.e., entanglement) of Refs. [17,18] for the EPR state of Eq. (11). Note that, for $\overline{n}=0$, we have

$$F=\frac{1}{2-\eta(1-e^{-2r})}, \tag{45}$$

so that the entangled EPR beams considered here (as well as in Refs. [12–15]) provide a sufficient resource for beating the limit set by a classical channel alone for any $r>0$, so long as $\eta>0$. In fact, the quantities $(\Delta x^2,\Delta p^2)$ are readily measured experimentally, so that the entanglement of the EPR beams can be operationally verified, as discussed in Sec. II A [4,30]. We stress that independently of any further assumption, the condition of Eq. (28) is sufficient to ensure entanglement for pure or mixed states [46,47].

The dependence of fidelity *F* on the degree of squeezing *r* and efficiency η, as expressed in Eq. (45), is illustrated in Fig. 1. Here, in correspondence to an experiment with fixed overall losses and variable parametric gain in the generation of the EPR entangled state, we show a family of curves, each of which is drawn for constant η as a function of *r*. Clearly, $F>F_{classical}=\frac{1}{2}$, and hence nonseparability results in each

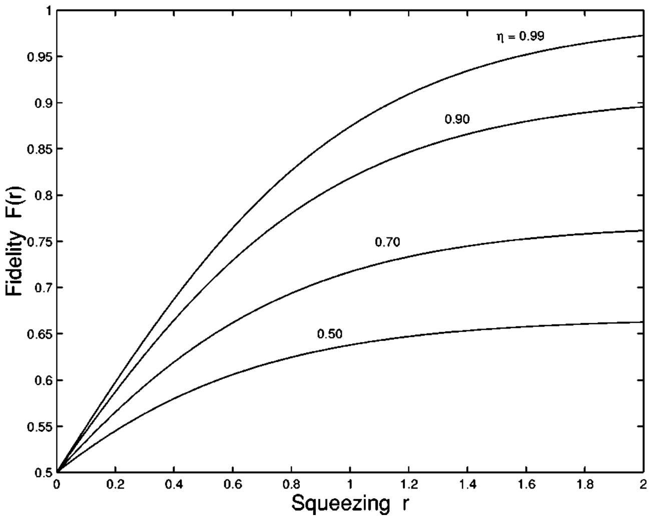

FIG. 1. Fidelity F as given by Eq. (45), vs the degree of squeezing r for a fixed efficiency η. From top to bottom, the curves are drawn with $\eta=\{0.99,0.90,0.70,0.50\}$, in correspondence to an increasing loss $(1-\eta)$. Note that $F_{classical}=\frac{1}{2}$ provides a demarcation between separable and nonseparable states (mixed or otherwise), while $F=\frac{2}{3}$ is apparently of no particular significance, the contrary claims of Refs. [12–15] notwithstanding. Note that for $\eta=1$, $r=\ln 2/2=0.3466$ gives $F=\frac{2}{3}$, corresponding to -3 dB of squeezing. In all cases, $\bar{n}=0$.

case. Although Refs. [12–15] would require fidelity $F>\frac{2}{3}$ (which results for $\Delta x^2+\Delta p^2<\frac{1}{2}$) for quantum teleportation of coherent states, this purported criterion has no apparent significance with respect to issues of entanglement, other than as a bound for $\eta=0.5$.

In this regard it is worth noting that violations of the Heisenberg-type inequality as in Eq. (38) can be attained for any $r>0$ so long as the efficiency $\eta>\frac{1}{2}$. Since it is the quantity $(\Delta x^2+\Delta p^2)$ and not $(\Delta x_\mu^2+\Delta p_\mu^2)$ that determines the fidelity [Eq. (43)], the threshold for violations as in Eq. (38) is thus fidelity $F_{classical}=\frac{1}{2}$ and not the value $F=\frac{2}{3}$ championed in Ref. [14]. In effect, these authors employed $F>\frac{2}{3}$, only to warranty that $\eta>\frac{1}{2}$, so that it is then possible to achieve a violation of the specific Heisenberg-type inequality with $\mu=1$ as expressed in Eq. (39). However, more generally, we have shown that the Heisenberg-type inequalities with optimized μ can be violated for any $F>F_{classical}=\frac{1}{2}$ if $\eta>\frac{1}{2}$.

As for the criterion of ''information content'' described in Sec. III [15], we note that it can be easily understood from the current analysis and the original discussion in Ref. [16]. Each of the interventions by Alice and Bob represent one unit of added vacuum noise that will be convolved with the initial input state in the teleportation protocol (the so-called *quduties*). The following two situations are compared in Ref. [15]: (i) Bob directly passes the classical information that he receives to Victor, and (ii) Bob generates a quantum state in the usual fashion that is then passed to Victor. The ''information content'' criterion demands that Victor should receive the same information in these two cases, which requires that $\overline{\sigma_-^2}=\Delta x^2+\Delta p^2<\frac{1}{2}$, and hence $F>\frac{2}{3}$. That is, as the degree of correlation between the EPR beams is increased, there comes a point for which $\Delta x^2+\Delta p^2=\frac{1}{2}$, and for which each of Alice and Bob's excess noise has been reduced from 1 quduty each to $\frac{1}{2}$ quduty each. At this point, the entire resulting noise of $\frac{1}{2}+\frac{1}{2}=1$ quduties is (arbitrarily) assigned to Alice, with then the perspective that Bob's state recreation adds no noise. Of course one could equally well make the complementary assignment, namely, 1 quduty to Bob and none to Alice (again in the case with $\overline{\sigma_-^2}=\frac{1}{2}$). The point that seems to have been missed in Ref. [15] is that key to quantum teleportation is the transport of quantum states. Clearly it is true that ''there is *no* extra noise associated to the reconstruction: given a measured β, one can exactly reconstruct the coherent state $|\beta\rangle$, by using a deterministic translation of the vacuum [15].'' However, while Bob can certainly make such a state deterministically, it is an altogether different matter for Victor to receive a classical number from Bob in case (i) as opposed to the actual quantum state in case (ii). In this latter case, apart from having a physical state instead of a number, Victor must actually make his own measurement with the attendant uncertainties inherent in $|\beta\rangle$ then entering. Analogously, transferring measurement results about a qubit, without recreating a state at the output (i.e., without sending an actual *quantum state* to Victor), is not what is normally considered to constitute quantum teleportation relative to the original protocol of Bennett *et al.* [1].

Turning next to the actual experiment of Ref. [4], we note that a somewhat subtle issue is that the detection efficiency for Alice of the unknown state was not 100%, but rather was $\eta_A^2=0.97$. Because of this, the fidelity for classical teleportation (i.e., with vacuum states in place of the EPR beams) did not actually reach $\frac{1}{2}$, but was instead $F_0=0.48$. This should not be a surprise, since there is nothing to ensure that a given classical scheme will be optimal and actually reach the bound $F_{classical}=\frac{1}{2}$. Hence the starting point in the experiment with $r=0$ had $F_0<F_{classical}$; the EPR beams with $r>0$ (which were in any event entangled by the above inequality) then led to increases in fidelity from F_0 upward, exceeding the classical bound $F_{classical}=\frac{1}{2}$ for a small (but not infinitesimal) degree of squeezing. Note that the whole effect of the offset $F_0=0.48<\frac{1}{2}$ can be attributed to the lack of perfect (homodyne) efficiency at Alice's detector for the unknown state. In the current discussion for determining the classical bound in the *optimal* case, we instead set Alice's detection efficiency $\eta_A^2=1$; then, as shown above, classical teleportation will achieve $F=\frac{1}{2}$.

Independent of such considerations, we reiterate that the nonseparability condition of Refs. [17,18] applied to the EPR state of Eqs. (4) and (11) leads to the same result $F_{classical}=\frac{1}{2}$ [Eqs. (43) and (44)] as did our previous analysis, based upon teleportation with only a classical communication channel linking Alice and Bob [6]. This convergence further supports $F_{classical}=\frac{1}{2}$ as the appropriate quantum-classical boundary for the teleportation of coherent states, the claims of Refs. [12–15] notwithstanding. Relative to the original work of Bennett *et al.* [1], exceeding the bound $F_{classical}=\frac{1}{2}$ for the teleportation of coherent can be accomplished with a classical channel and entangled (i.e., nonseparable)

EPR states, be they mixed or pure, as is made clear by the above analysis and as has been operationally confirmed [4].

We should however emphasize that the above conclusions concerning nonseparability and teleportation fidelity apply to the specific case of the EPR state as in Eq. (11), for which inequality Eq. (28) represents both a necessary and sufficient criterion for nonseparability according to Refs. [17,18]. More generally, for arbitrary entangled states, nonseparability does not necessarily lead to $F>\frac{1}{2}$ in coherent-state teleportation [46,47].

V. BELL'S INEQUALITIES

The papers by Banaszek and Wodkiewicz [21,22] provides our point of reference for a discussion of Bell's inequalities. In these papers, the authors introduced an appropriate set of measurements that lead to a Bell inequality of the CHSH type. More explicitly, Eq. (4) of Ref. [21] gives the operator $\hat{\Pi}(\alpha;\beta)$ whose expectation values are to be measured. Banaszek and Wodkiewicz pointed out that the expectation value of $\hat{\Pi}(\alpha;\beta)$ is closely related to the Wigner function of the field being investigated, namely,

$$W(\alpha;\beta)=\frac{4}{\pi^2}\Pi(\alpha;\beta), \tag{46}$$

where $\Pi(\alpha;\beta)=\langle\hat{\Pi}(\alpha;\beta)\rangle$.

For the entangled state shared by Alice and Bob in the teleportation protocol, we identify W_{EPR}^{out} as the relevant Wigner distribution for the modes (1,2) of interest, so that

$$\begin{aligned}\Pi_{\mathrm{EPR}}^{out}&(x_1,p_1;x_2,p_2)\\&=\frac{1}{\overline{\sigma_+^2}\,\overline{\sigma_-^2}}\exp\{-[(x_1+x_2)^2+(p_1-p_2)^2]/\overline{\sigma_+^2}\\&\quad-[(x_1-x_2)^2+(p_1+p_2)^2]/\overline{\sigma_-^2}\}.\end{aligned} \tag{47}$$

Banaszek and Wodkiewicz showed that $\Pi_{\mathrm{EPR}}^{out}(x_1,p_1;x_2,p_2)$ directly gives the correlation function that would otherwise be obtained from a particular set of observations over an ensemble representing the field with density operator $\hat{\rho}$, where the actual measurements to be made are as described in Refs. [21,22]. In simple terms, $\hat{\Pi}_{\mathrm{EPR}}^{out}(0{,}0;0{,}0)$ is the parity operator for separate measurements of photon number on modes (1,2), with then nonzero (x_i,p_i) corresponding to a "rotation" on the individual mode i that precedes its parity measurement.

The function constructed by Banaszek and Wodkiewicz to test for local hidden variable theories is denoted by $\mathcal{B}$, and is defined by

$$\begin{aligned}\mathcal{B}(\mathcal{J})=&\,\Pi_{\mathrm{EPR}}^{out}(0{,}0;0{,}0)+\Pi_{\mathrm{EPR}}^{out}(\sqrt{\mathcal{J}},0;0{,}0)\\&+\Pi_{\mathrm{EPR}}^{out}(0{,}0;-\sqrt{\mathcal{J}},0)-\Pi_{\mathrm{EPR}}^{out}(\sqrt{\mathcal{J}},0;-\sqrt{\mathcal{J}},0),\end{aligned} \tag{48}$$

where $\mathcal{J}$ is a positive (real) constant. As shown in Refs. [21,22], any local theory must satisfy

$$-2\leqslant\mathcal{B}\leqslant 2. \tag{49}$$

As emphasized by Banaszek and Wodkiewicz for the lossless case, $\Pi_{\mathrm{EPR}}^{out}(0{,}0;0{,}0)=1$ "describes perfect correlations . . . as a manifestation of . . . photons always generated in pairs."

There are several important points to be made about this result. In the first place, in the ideal case with no loss ($\eta=1$), there is a violation of the Bell inequality of Eq. (49) for any $r>0$. Further, this threshold for the onset of violations of the CHSH inequality coincides with the threshold for entanglement as given in Eq. (28), which likewise is the point for surpassing $F_{classical}=\frac{1}{2}$ as in Eqs. (43) and (44), and as shown in our prior analysis of Ref. [6] which is notably based upon a quite different approach.

Significantly, there is absolutely nothing special about the point $r=\ln 2/2\approx 0.3466$ (i.e., the point for which $\exp[-2r]=0.5$ and for which $F=\frac{2}{3}$ for the teleportation of coherent states). Instead, any $r>0$ leads to a nonseparable EPR state, to a violation of a Bell inequality, and to $F>F_{classical}=\frac{1}{2}$ for the teleportation of coherent states. There is certainly no surprise here since we are dealing with pure states for $\eta=1$ [48].

We next examine the case with $\eta<1$, which is clearly of interest for any experiment. Figure 2 illustrates the behavior of $\mathcal{B}$ as a function of $\mathcal{J}$ for various values of the squeezing parameter r and of the efficiency η. Note that throughout our analysis in this section, we make no attempt to search for optimal violations, but instead follow dutifully the protocol of Banaszek and Wodkiewicz as expressed in Eq. (48) for the case with losses as well.

From Fig. 2 we see that for any particular set of parameters (r,η), there is an optimum value $\mathcal{J}_{\max}$ that leads to a maximum value for $\mathcal{B}(\mathcal{J}_{\max})$, which is a situation analogous to that found in the discrete variable case. By determining the corresponding value $\mathcal{J}_{\max}$ at each (r,η), in Fig. 3 we construct a plot that displays the dependence of $\mathcal{B}$ on the squeezing parameter r for various values of efficiency η. Note that all cases shown in the figure lead to fidelity $F>F_{classical}$.

For $\frac{2}{3}<\eta\leqslant 1$ there are regions in r that produce direct violations of the Bell inequality considered here, namely, $\mathcal{B}>2$ [49]. In general, these domains with $\mathcal{B}>2$ contract toward smaller r with increasing loss $(1-\eta)$. In fact as r increases, η must become very close to unity in order to preserve the condition $\mathcal{B}>2$, where, for $r\gg 1$,

$$2(1-\eta)\cosh(2r)\ll 1. \tag{50}$$

This requirement is presumably associated with the EPR state becoming more "nonclassical" with increasing r, and hence more sensitive to dissipation [50]. Stated somewhat more quantitatively, recall that the original state $|\mathrm{EPR}\rangle_{1,2}$ of Eq. (3) is expressed as a sum over correlated photon numbers for each of the two EPR beams (1,2). The determination of $\mathcal{B}$ derives from (displaced) parity measurements on the beams

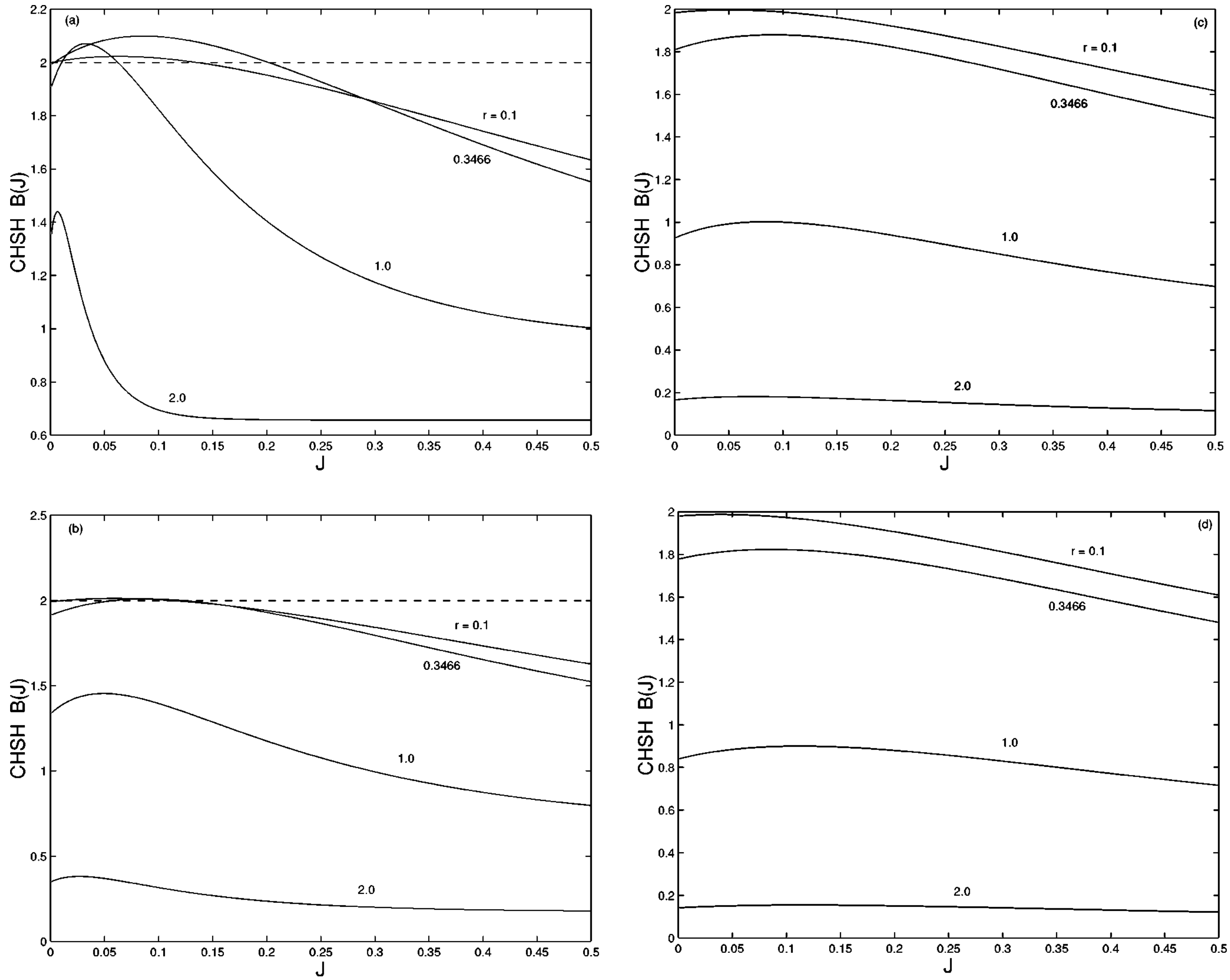

FIG. 2. The function $\mathcal{B}(\mathcal{J})$ from Eq. (48) as a function of $\mathcal{J}$ for various values of (r,η). Recall that $\mathcal{B}>2$ heralds a direct violation of the CHSH inequality, with the dashed line $\mathcal{B}=2$ shown. In each of plots (a)–(d), a family of curves is drawn for fixed efficiency η and four values of $r=\{0.1,\ln 2/2,1.0,2.0\}$. (a) $\eta=0.99$, (b) $\eta=0.90$, (c) $\eta=0.70$, and (d) $\eta=0.50$; and, in all cases, $\bar{n}=0$.

(1,2) (i.e., projections onto odd and even photon number), so that $\mathcal{B}$ should be sensitive to the loss of a single photon. The mean photon number $\bar{n}_i$ for either EPR beam goes as $\sinh^2 r$, with then the probability of losing no photons after encountering the beam splitter with transmission η scaling as roughly $p_0 \sim [\eta]^{\bar{n}_i}$. We require that the total probability for the loss of one or more photons to be small, so that

$$(1-p_0)\ll 1, \tag{51}$$

and hence, for $(1-\eta)\ll 1$ and $r\gg 1$, that

$$(1-\eta)\bar{n}_i \sim (1-\eta)\exp(2r)\ll 1, \tag{52}$$

in correspondence to Eq. (50) [51].

On the other hand, note that small values of r in Fig. 3 lead to direct violations of the CHSH inequality $\mathcal{B}>2$ with much more modest efficiencies [50]. In particular, note that for $r=\ln 2/2 \approx 0.3466$ and $\eta=0.90$, $F<\frac{2}{3}$ [from Eq. (45)]. This case and others like it provide examples for which mixed states are nonseparable and yet directly violate a Bell inequality, but for which $F\leqslant\frac{2}{3}$. Such mixed states do not satisfy the criteria of Refs. [12–15], yet these are states for which $\frac{1}{2}<F\leqslant\frac{2}{3}$ and $\mathcal{B}>2$. There remains the possibility that $F>\frac{2}{3}$ might be sufficient to warranty that mixed states in this domain would satisfy that $\mathcal{B}>2$, and hence to exclude a description of the EPR state in terms of a local hidden variables theory.

To demonstrate that this is emphatically not the case, we further examine the relationship between the quantity $\mathcal{B}$ relevant to the CHSH inequality and the fidelity F. Figure 4 shows a parametric plot of $\mathcal{B}$ versus F for various values of the efficiency η. The curves in this figure are obtained from plots as in Figs. 1 and 3, by eliminating the common dependence on r. From Fig. 4, we are hard pressed to find any indication that the value $F=\frac{2}{3}$ is in any fashion noteworthy with respect to violations of the CHSH inequality. In particular, for efficiency $\eta\simeq 0.90$ most relevant to current experimental capabilities, the domain $F>\frac{2}{3}$ is one largely devoid

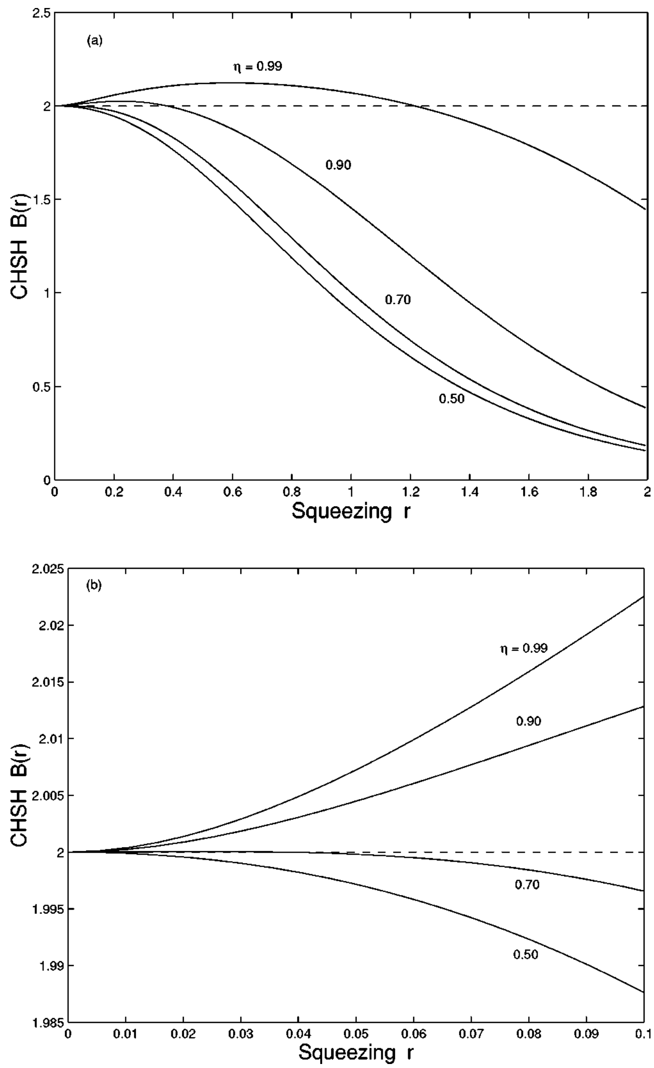

FIG. 3. (a) The quantity $\mathcal{B}$ from Eq. (48) as a function of r for various values of efficiency $\eta=\{0.99,0.90,0.70,0.50\}$, as indicated. At each point in (r,η), the value of $\mathcal{J}$ that maximizes $\mathcal{B}$ has been chosen. Recall that $\mathcal{B}>2$ heralds a direct violation of the CHSH inequality, with the dashed line $\mathcal{B}=2$ shown. Also note that $F>\frac{1}{2}$ for all $r>0$. (b) An expanded view of $\mathcal{B}$ in the small-r region $r\leqslant 0.1$. Note that in the case $\eta=0.70$, $\mathcal{B}>2$ for small r. In all cases, $\overline{n}=0$.

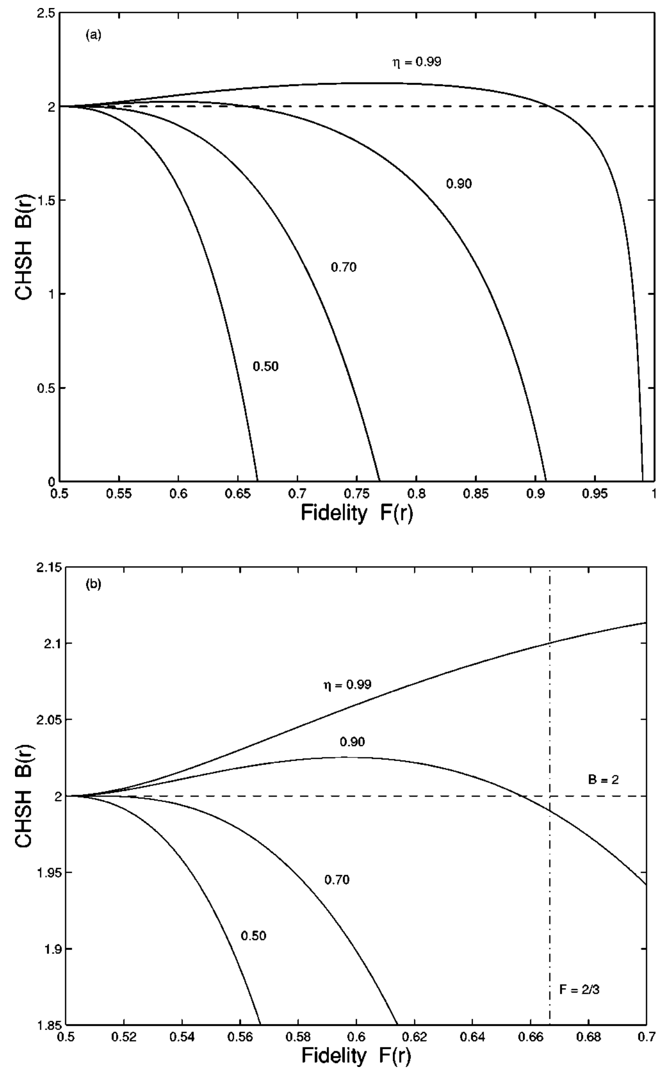

FIG. 4. (a) A parametric plot of the CHSH quantity $\mathcal{B}$ [Eq. (48)] vs fidelity F [Eq. (45)]. The curves are constructed from Figs. 1 and 3 by eliminating the r dependence, now over the range $0\leqslant r\leqslant 5$, with r increasing from left to right for each trace. The efficiency η takes on the values $\eta=\{0.99,0.90,0.70,0.50\}$ as indicated; in all cases, $\overline{n}=0$. Recall that $\mathcal{B}>2$ heralds a direct violation of the CHSH inequality, with the dashed line $\mathcal{B}=2$ shown. (b) An expanded view around $\mathcal{B}=2$. Note that $\mathcal{B}>2$ is impossible for $F\leqslant F_{classical}=\frac{1}{2}$, but that $\mathcal{B}>2$ for $F>F_{classical}$ in various domains (including for $\eta=0.70$ at small r). The purported boundary $F=\frac{2}{3}$ championed in Refs. [12–15] is seen to have no particular significance. Conversely, $F=\frac{2}{3}$ provides absolutely no warranty that $\mathcal{B}>2$ for $F>\frac{2}{3}$, nor does it preclude $\mathcal{B}>2$ for $F<\frac{2}{3}$.

of instances with $\mathcal{B}>2$, in contradistinction to the claim that this domain is somehow "safer" [14] with respect to violations of Bell's inequalities. Moreover, contrary to the dismissal of the domain $\frac{1}{2}<F\leqslant\frac{2}{3}$ as not being manifestly quantum, we see from Fig. 4 that there are in fact regions with $\mathcal{B}>2$. Overall, the conclusions in Ref. [14] related to the issues of violation of a Bell inequality and of teleportation fidelity are simply not supported by an actual quantitative analysis.

To conclude this section, we would like to inject a note of caution concerning any discussion involving issues of testing Bell's inequalities and performing quantum teleportation. We have placed them in juxtaposition here to refute the claims of Grangier and Grosshans related to a possible connection between the bound $F=\frac{2}{3}$ and violation of Bell's inequalities (here via the behavior of the CHSH quantity $\mathcal{B}$). However, in our view there is a conflict between these concepts, with an illustration of this point provided by the plot of the CHSH quantity $\mathcal{B}$ [Eq. (49)] versus fidelity F [Eq. (45)] in Fig. 4. For example, for $\eta=0.90$, $\mathcal{B}>2$ over the range $0.50<F\leqslant 0.66$, while $\mathcal{B}<2$ for larger values of F. Hence local hidden variables theories are excluded for modest values of fidelity $0.50<F\leqslant 0.66$, but not for larger values $F\gtrsim 0.66$. This leads to the strange conclusion that quantum resources are required for smaller values of fidelity but not for larger ones.

The point is that the nonseparable states that can enable quantum teleportation, can *in a different context* also be used to demonstrate a violation of local realism. Again, the juxtaposition of these concepts in this section is in response to the work of Ref. [14], which in any event offers no quantitative evidence in support of their association.

VI. BELL'S INEQUALITIES FOR SCALED CORRELATIONS

The conclusions reached in Sec. V about violations of the CHSH inequality by the EPR (mixed) state for modes (1,2) follow directly from the analysis of Banaszek and Wodkiewicz [21,22] as extended to account for losses in propagation. Toward the end of making these results more amenable to experimental investigation, recall that the more traditional versions of the Bell inequalities formulated for spin-$\frac{1}{2}$ particles or photon polarizations are based upon an analysis of the expectation value

$$E(\vec{a},\vec{b}) \tag{53}$$

for detection events at locations (1,2) with analyzer settings along directions $(\vec{a},\vec{b})$. As emphasized by Clauser and Shimony, actual experiments do not measure directly $E(\vec{a},\vec{b})$ but rather record a reduced version due to ''imperfections in the analyzers, detectors, and state preparation [23].'' Even after more than 30 years of experiments, no *direct* violation of the CHSH inequality has been recorded, where by *direct* we mean without the need for post-selection to compensate for propagation and detection efficiencies (also called *strong violations*) [25,26]. Rather, only subsets of events that give rise to coincidences are included for various polarization settings. This ''problem'' is the so-called detector efficiency loophole that several groups are actively working to close.

Motivated by these considerations, we point out that an observation of violation of a Bell-type inequality was recently reported [27], based in large measure upon the earlier proposal of Ref. [52], as well as that of Refs. [21,22]. This experiment was carried out in a pulsed mode, and utilized a source that generates an EPR state of the form given by Eq. (11) in the limit $r \ll 1$. Here the probability $P(\alpha_1,\alpha_2)$ of detecting a coincidence event between detectors (D_1,D_2) for the EPR beams (1,2) is given by

$$P(\alpha_1,\alpha_2)=M[1+V\cos(\phi_1-\phi_2+\theta)], \tag{54}$$

with then the correlation function E relevant to the construction of a CHSH inequality $-2 \leqslant S \leqslant 2$ given by

$$E(\phi_1,\phi_2)=V\cos(\phi_1-\phi_2+\theta), \tag{55}$$

where the various quantities are as defined in association with Eqs. (2) and (3) in Ref. [27]. Note that the quantity M represents an overall scaling that incorporates losses in propagation and detection. Significantly, Kuzmich *et al.* demonstrated a violation of a CHSH inequality ($S_{\mathrm{exp}}=2.46 \pm 0.06$) in the limit $r \ll 1$ and with inefficient propagation and detection $\eta \ll 1$, albeit with the so-called ''detection'' or ''fair-sampling'' loophole.

In terms of our current discussion, this experimental violation of a CHSH inequality is only just within the nonseparability domain $\Delta x^2+\Delta p^2<1$ (by an amount that goes as $\eta r \ll 1$), yet it generates a large violation of a CHSH inequality. If this same EPR state were employed for the teleportation of coherent states, the conditional fidelity obtained would likewise be only slightly beyond the quantum-classical boundary $F_{classical}=\frac{1}{2}$. It would be far from a boundary consistent with $F=\frac{2}{3}$ proposed in Refs. [12–15] as the point for ''useful entanglement'' or ''true entanglement,'' yet it would nonetheless provide an example of teleportation with fidelity $F>\frac{1}{2}$ and of a violation of a CHSH inequality. Of course, the caveat would be the aforementioned ''fair-sampling'' loophole, but this same restriction accompanies all previous experimental demonstrations of violations of Bell's inequalities.

VII. CONCLUSIONS

Beyond the initial analysis of Ref. [6], we have examined further the question of the appropriate point of demarcation between the classical and quantum domains for the teleportation of coherent states. In support of our previous result that fidelity $F_{classical}=\frac{1}{2}$ represents the bound attainable by Alice and Bob if they make use only of a classical channel, we have shown that the nonseparability criteria introduced in Refs. [17,18] are sufficient to ensure fidelity beyond this bound for teleportation with the EPR state of Eq. (11), which is in general a mixed state. Significantly, the threshold for entanglement for the EPR beams as quantified by these nonseparability criteria coincides with the standard boundary between classical and quantum domains employed in quantum optics, namely, that the Glauber-Sudarshan phase-space function takes on negative values [20].

Furthermore, we have investigated possible violations of Bell's inequalities, and have shown that the threshold for the onset of such violations again corresponds to $F_{classical}=\frac{1}{2}$. For thermal photon number $\bar{n}=0$ as appropriate to current experiments, direct violations of a CHSH inequality are obtained over a large domain in the degree of squeezing r and overall efficiency η. Significant, relative to the claims made in Refs. [12–15], is that there is a regime for nonseparability and violation of the CHSH inequality for which $F<\frac{2}{3}$ and for which there criteria are not satisfied. Moreover, the experiment of Ref. [27] demonstrated a violation of the CHSH inequality in this domain for $(r,\eta) \ll 1$ (i.e., F would be only slightly beyond $\frac{1}{2}$), albeit with the caveat of the ''fair-sampling'' loophole. We conclude that fidelity $F>\frac{2}{3}$ offers absolutely no warranty relative to the issue of violation of a Bell inequality (as might be desirable, for example, in quantum cryptography). Quite the contrary: larger r (and hence larger F) leads to an exponentially decreasing domain in allowed loss $(1-\eta)$ for violation of the CHSH inequality, as expressed by Eq. (50) [51].

Moreover, beyond the analysis that we have presented here, there are several other results that support $F_{classical}=\frac{1}{2}$ as being the appropriate boundary between quantum and classical domains. In particular, we note that any nonseparable state and hence also our mixed EPR state is always capable of teleporting perfect entanglement, i.e., one-half of a pure maximally entangled state. This also applies to those nonseparable states which lead to fidelities $\frac{1}{2}<F\leqslant\frac{2}{3}$ in coherent-state teleportation. According to Refs. [12–15], this would force the conclusion that there is entanglement that is capable of teleporting truly nonclassical features (i.e., entanglement), but which is not ''useful'' [14] for teleporting rather more classical states such as coherent states. Further, in Ref. [28] it was shown that entanglement swapping can be achieved with two pure EPR states for *any nonzero squeezing* in both initial states. Neither of the initial states has to exceed a certain amount of squeezing in order to enable successful entanglement swapping. This is another indication that $F=\frac{2}{3}$, which is exceedable in coherent-state teleportation only with more than 3-dB squeezing, is inappropriate in delineating the quantum-classical boundary.

We also point out that Giedke *et al.* showed that for all bipartite Gaussian states for a pair of oscillators, nonseparability implies distillability [53]. This result applies to the EPR (mixed) states considered here, and in particular to those nonseparable states for which $\frac{1}{2}<F\leqslant\frac{2}{3}$ in coherent state teleportation, which are otherwise dismissed as not exhibiting ''true EPR entanglement'' [13]. Conversely, entanglement distillation could be applied to the mixed EPR states employed for teleportation in this domain (and in general for $F>\frac{1}{2}$) [54], leading to enhanced teleportation fidelities and to expanded regions for violations of Bell's inequalities for the distilled subensemble.

However, having said this, we emphasize that there is no criterion for quantum teleportation that is sufficient to all tasks. For the special case of teleportation of coherent states, the boundary between classical and quantum teleportation is fidelity $F_{classical}=\frac{1}{2}$, as should by now be firmly established. Fidelity $F>\frac{2}{3}$ will indeed enable certain tasks to be accomplished that could not otherwise be done with $\frac{1}{2}<F\leqslant\frac{2}{3}$. However, $F=\frac{2}{3}$ is clearly not the relevant point of demarcation for entrance into the quantum domain. There is instead a hierarchy of fidelity thresholds that enable ever more remarkable tasks to be accomplished via teleportation within the quantum domain, with no one value being sufficient for all possible purposes.

For example, if we wish to teleport a nonclassical state of the electromagnetic field, then $\overline{\sigma_-^2}\geqslant 1$ is sufficient to guarantee that all nonclassical features will vanish [39]. This implies that a necessary condition for nonclassical features to be teleported is $\overline{\sigma_-^2}<1$, which leads to the requirement $F>\frac{1}{2}$ for the teleportation of coherent states. If the task is to teleport a perfectly squeezed state with variance $\langle\Delta x_{in}^2\rangle\to 0$, then the teleported state will also be squeezed so long as $\overline{\sigma_-^2}<\frac{1}{2}$ [39], implying that teleportation of coherent states could indeed attain $F>\frac{2}{3}$. If instead the demand is for teleportation in a domain where unconditional violation of a Bell inequality as in Sec. V is required, then the efficiency η must exceed $\frac{2}{3}$, leading to fidelity for teleportation of coherent states $F>\frac{3}{4}$. Much more challenging would be if the state to be teleported were some intermediate result from a large-scale quantum computation as for Shor's algorithm. Surely then, the relevant fidelity threshold would be well beyond any value currently accessible to experiment, $F\sim 1-\epsilon$, with $\epsilon\lesssim 10^{-4}$ to be compatible with current work in fault tolerant architecture. We have never claimed that $F>\frac{1}{2}$ endows special powers for all tasks such as these, only that it provides an unambiguous point of entry into the quantum realm for the teleportation of coherent states.

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APPENDIX

The expressions of Eqs. (42)-(45) are strictly applicable only for the case gain $g=1$ for teleportation of coherent states uniformly distributed over the entire complex plane. Here g specifies the gain employed by Bob in generating a coherent state based upon the information received from Alice. More generally, when working with a restricted alphabet of states (e.g., coherent amplitudes selected from a Gaussian distribution), the optimal gain is not unity when referenced to the fidelity averaged over the input alphabet. In fact as shown in Ref. [6], the optimal gain is $g=1/(1+\lambda)$ for an input alphabet of coherent states distributed according to $p(\beta)=(\lambda/\pi)\exp(-\lambda|\beta|^2)$. When incorporated into the current analysis, we show in this appendix that nonseparable EPR states are sufficient to achieve $F>(1+\lambda)/(2+\lambda)$ (again with an optimal gain $g\neq 1$), although F is now no longer a monotonic function of r as in Fig. 1. This result is in complete correspondence with the prior result of Ref. [6] that $F_{classical}^{\lambda}=(1+\lambda)/(2+\lambda)$ is the bound for teleportation when only a classical channel is employed. To simplify the discussion in the text, we have set $\lambda=0$ throughout, with then the optimal gain $g=1$ and $F_{classical}=\frac{1}{2}$.

In the more general case, we begin by recalling from Eqs. (1) and (2) in Ref. [4] that the fidelity F for teleportation of a coherent state $|v_{in}\rangle$ can be expressed in the current notation by

$$F=\frac{2}{\sigma_Q^2}\exp[-2|v_{in}|^2(1-g)^2/\sigma_Q^2]. \tag{A1}$$

Here the variance σ_Q^2 of the Q function of the teleported field is given by

$$\sigma_Q^2=1+g^2+\frac{\overline{\sigma_-^2}}{2}(g+1)^2+\frac{\overline{\sigma_+^2}}{2}(g-1)^2. \tag{A2}$$

Relative to Ref. [4], various efficiencies are here taken to unity for the sake of simplicity. With reference to the nota-

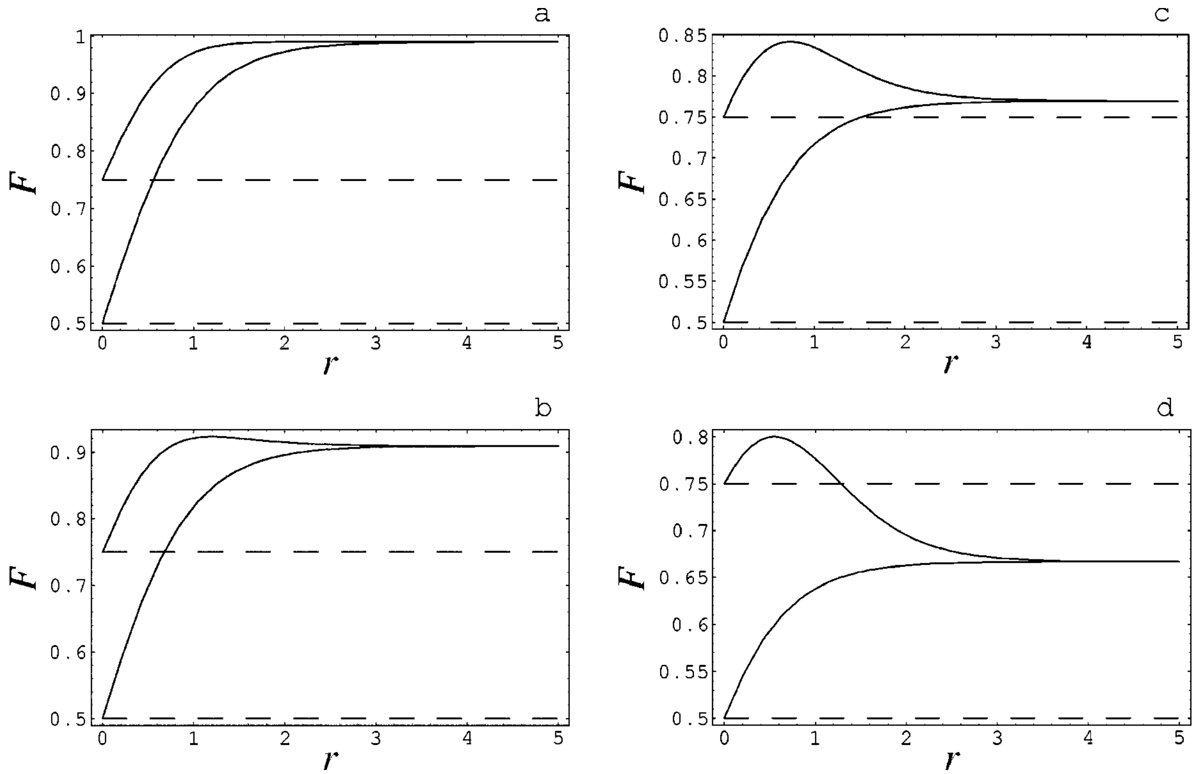

FIG. 5. Optimized fidelity $\overline{F}_0$ vs the degree of squeezing r, with $\lambda=2$ (upper curve) and $\lambda=0.001$ (lower curve). In (a)–(d) the efficiency $\eta=0.99$, 0.90, 0.70, and 0.50, respectively. The dashed lines give the limiting value $F=(1+\lambda)/(2+\lambda)$ for each case.

tion of Ref. [4], we assume that the EPR beams propagate and are detected with unit efficiency ($\xi_1=1=\xi_2$) and that Alice's detection efficiency η_A is likewise unity ($\eta_A=1$, where $\eta_A\equiv\eta$ in Ref. [4] is not to be confused with η here). Hence our model here is that each *ideal* squeezed beam goes through a beam splitter of transmission η to then produce a mixed (squeezed) state. These two squeezed states are then combined, to generate the EPR beams described by Eq. (11), with the resulting variances parametrized by (r,η). In effect, we consider the case where the only imperfection is in the squeezing beams that are combined to generate the EPR beams ($\overline{\sigma_+^2}\,\overline{\sigma_-^2}=1$ in the ideal case, but $\overline{\sigma_+^2}\,\overline{\sigma_-^2}\geqslant 1$ in the presence of loss).

We next proceed to average the fidelity as given in Eq. (A1) over a distribution of incident coherent states $\{|\beta\rangle\}$ of the form previously considered in Ref. [6], namely,

$$P(\beta)=\frac{\lambda}{\pi}\exp(-\lambda|\beta|^2). \tag{A3}$$

The calculation is a straightforward, and yields

$$\overline{F}=\frac{2\lambda}{\lambda\sigma_Q^2+2(1-g)^2}. \tag{A4}$$

Next we optimize this average fidelity $\overline{F}$ by choosing the best gain g, which is found from the relation

$$\frac{d}{dg}\overline{F}=0, \tag{A5}$$

remembering that σ_Q depends upon g. There results a solution for the optimal gain g_0 given by

$$g_0=\frac{1+\frac{\lambda}{4}(\overline{\sigma_+^2}-\overline{\sigma_-^2})}{1+\frac{\lambda}{4}(2+\overline{\sigma_+^2}+\overline{\sigma_-^2})}, \tag{A6}$$

which when substituted into Eq. (A2), gives the value $\sigma_{Q_0}^2$ of this variance at the optimal gain. Finally, $(g_0,\sigma_{Q_0}^2)$ together with Eq. (A4), leads to an expression for the optimum fidelity $\overline{F}_0$. Two limiting cases are worth checking straightaway.

(1) For vacuum inputs for the EPR beams, $\overline{\sigma_\pm^2}=1$ (no squeezing), so that

$$g_0=\frac{1}{1+\lambda},\qquad \overline{F}_0=\frac{1+\lambda}{2+\lambda}, \tag{A7}$$

which are in complete accord with the prior treatment of Ref. [6].

(2) For $\overline{\sigma_+^2}\rightarrow\infty$ (corresponding to very large parametric gain, $r\gg 1$), we have that

$$g_0 \to 1, \qquad \overline{F}_0 \to F, \tag{A8}$$

which is just the (unaveraged) fidelity given by Eq. (A1). The importance of this result is that it sets the limiting values of $(g_0, \overline{F}_0)$ for large r independent of λ, as will become apparent from the figures that follow.

Figure 5 show a series of plots, each of which contains two curves for the fidelity versus the squeezing parameter r for two values of λ. The upper trace is the optimized fidelity $\overline{F}_0$ from Eq. (A4) with the optimized values $(g_0, \sigma^2_{Q_0})$ for the particular choice $\lambda = 2$, while the lower trace is the fidelity $\overline{F}_0$ from Eq. (A4) for $\lambda = 0.001$ (and hence $g_0 \approx 1$). Also shown are two dashed lines corresponding to $F = (1+\lambda)/(2+\lambda)$ for the two values $\lambda = 2$ and 0.001.

As is apparent, $\overline{F}_0$ increases with $r \ll 1$ in all cases from its initial value $(1+\lambda)/(2+\lambda)$. However, if $(1+\lambda)/(2+\lambda) > F(r \gg 1)$ where F is the result for gain $g=1$ from Eq. (1), then $\overline{F}_0$ will rise to a maximum and then decrease (slope <0). Thus, although $r>0$ helps Alice and Bob initially, $\overline{F}$ is not monotonic in r. In many cases, there is an optimum value for the degree of squeezing r for given λ (the alphabet Victor has chosen) and η (the losses that Alice and Bob have to live with in generating and distributing their EPR beams). Further, if they nonetheless persist in increasing r past this optimum, in some cases they will do worse with r nonzero than with $r=0$.

While these results may at first sight seem strange, their interpretation is as follows. The initial value $\overline{F}_0(r=0) = (1+\lambda)/(2+\lambda)$ is artificially boosted in the sense that for $\lambda \gg 1$, Alice and Bob have to be less and less concerned about losses and squeezing. They simply increasingly bias their choice toward $\beta = 0$ as specified in Eq. (A3). Further, for increasing r, the spread of the Wigner function for the EPR beams at some point overtakes the spread associated with $P(\beta)$ so that the particular value of λ becomes irrelevant, and $\overline{F}$ reverts to the $g=1$ case.

Although we emphasize that the foregoing analysis is sufficient to demonstrate that nonseparable EPR states achieve fidelity $F > F^{\lambda}_{classical} = (1+\lambda)/(2+\lambda)$, we make no warranty that it provides the optimal strategy for Alice and Bob. The principal caveats are that we have assumed that Alice is always performing an Arthurs-Kelly measurement [41], and that Bob always generates a coherent state based upon the information from Alice, where this coherent state is given by $\alpha_{out} = g\alpha_{in}$, with g real and optimized, as discussed above.

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