

## Spin Relaxation in Quantum Hall Systems

W. Apel<sup>1</sup> and Yu. A. Bychkov<sup>2,1</sup>

<sup>1</sup>*Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany*

<sup>2</sup>*L. D. Landau Institute for Theoretical Physics, ul. Kosigina, 2, Moscow, Russia*

(Received 12 November 1998)

We study spin relaxation in an interacting two-dimensional electron gas in a strong magnetic field for the case where the electron density is close to filling just one Landau sublevel of one spin projection, i.e., for filling factor  $\nu \approx 1$ . Assuming the relaxation to be caused by scattering with phonons, we derive the kinetic equations for the electrons' spin density which replace the Bloch equations in our case. These equations are nonlinear and their solution depends crucially on the filling factor  $\nu$  and on the temperature  $T$  of the phonon bath. In the limit of  $T = 0$  and  $\nu = 1$ , the solution relaxes asymptotically with a power law inversely proportional to time, instead of following the conventional exponential behavior. [S0031-9007(99)08943-7]

PACS numbers: 73.40.Hm, 71.35.-y, 76.20.+q, 76.30.-v

Experiments in two-dimensional (2D) electronic structures in a strong magnetic field reveal a rich variety of features. Recently, many experiments have focused on properties related to the electrons' spins [1–8]. Optical measurements of the magnetization and transport measurements of the activation energy are interpreted with the concept of Skyrmion quasiparticles [9,10]. These can be described as extended spin textures containing a number of spins which are flipped with respect to the preferential direction set by the magnetic field. Their number is determined by the competition between Zeeman energy and Coulomb interaction of the electrons. While the equilibrium of systems containing Skyrmions has been studied in various ways—both experimentally and theoretically—the Skyrmion's dynamical properties are much less understood. There are indirect measurements of electron-spin resonance which use the magnetoresistivity to determine the spin splitting of the Landau levels [11–13] and which also yield a linewidth [12,13].

The first and fundamental question about the dynamics pertains to the formation of the Skyrmion. A Skyrmion–anti-Skyrmion pair has only half the energy of a particle-hole excitation, a spin exciton [9,10]; nevertheless, it needs for its formation a mechanism which flips the electrons' spins, since a Skyrmion may contain a large number of overturned spins. The mechanism must be provided from outside the bare electron system of Coulomb interaction and Zeeman energy, because the spin parallel to the magnetic field is a constant of motion. In this paper, we analyze the spin relaxation, originating from such a mechanism, specifically for a system of 2D interacting electrons in a strong magnetic field. As will be shown below, the result is intriguing and nontrivial—quite different from the well-known Bloch equations [14]. The reason can be related to the fact that the linear coefficient of the expansion of the relaxation equation around the stationary solution is small and can even vanish for this system. This already happens in the case of a uniform spin density. Thus, we will demonstrate our new

approach by studying the uniform case; this has to be seen as the first step towards a solution of the more general problem of the formation of the Skyrmion.

We consider, in the following, interacting electrons in two dimensions moving in a strong magnetic field under the following conditions: The electron density should be close to that of one filled spin-split Landau sublevel, i.e., the filling factor  $\nu \approx 1$ , and the Zeeman energy should be much less than the Coulomb energy. Both conditions are relevant for the Skyrmion case. The latter is realized in the experiment by a variation of the  $g$  factor with pressure [6]. In three-dimensional (3D) semiconductors, various mechanisms of spin relaxation have been studied experimentally (e.g., in optical measurements) and theoretically. Among them are scattering by holes due to the exchange energy, and scattering by phonons and impurities which becomes possible because of the spin-orbit interaction; for a comparison of these different mechanisms, see Ref. [15]. In a system in strong magnetic field, the Zeeman energy has to be taken up by the scatterer in an elementary spin flip. Therefore, we study here a general model of inelastic scattering by phonons. In deriving the kinetic equation which governs the relaxation of the spin density due to the scattering by phonons, the following three assumptions are made: (i) The 2D electrons are described by a nonequilibrium distribution which yields *uniform* expectation values for density and spin density. (ii) The phonons are in thermal equilibrium unperturbed by the electrons. (iii) The electron-phonon coupling is so weak that it can be treated in perturbation theory. The appropriate technique for performing perturbation theory in nonequilibrium systems was introduced by Keldysh [16,17]. Reference [17] shows how to derive the kinetic equation in the quasiclassical regime. The condition for the applicability of the quasiclassical approximation in the present context of the spin relaxation is given below.

How can phonons flip the spin? Consider the standard expression of the spin-orbit energy,

$$V_{\text{so}} = \frac{1}{2} \boldsymbol{\sigma} \cdot \frac{-e\hbar}{2m^2c^2} \mathbf{E} \times \mathbf{p}, \quad (1)$$

where  $\frac{1}{2} \boldsymbol{\sigma}$  and  $\mathbf{p}$  are the spin and momentum of the electron. The electric field  $\mathbf{E}$  has its origin in a piezoelectric distortion of the lattice and it is linear in phonon operators. Thus, the electrons are scattered and their spin is flipped by the phonons of the 3D lattice which contains the 2D electron system at  $z = 0$ . The orbital part of the electrons' single-particle states is confined to the lowest Landau level. We denote the projected spin density of the electrons at (2D) wave vector  $\mathbf{q}$  with  $\hat{\mathbf{S}}(\mathbf{q})$  [18]. Then, we get, for the electron-phonon part of the Hamiltonian,

$$H_{e\text{-ph}} = \sum_{\mathbf{Q},s} \hat{\mathbf{S}}(\mathbf{q}) [\boldsymbol{\lambda}_s(-\mathbf{Q}) b_{-\mathbf{Q},s} + \boldsymbol{\lambda}_s(\mathbf{Q})^* b_{\mathbf{Q},s}^\dagger]. \quad (2)$$

$b_{\mathbf{Q},s}^\dagger$  creates a phonon with (3D) wave vector  $\mathbf{Q} = (\mathbf{q}, Q^z)$  and polarization  $s$ . The coupling constant  $\boldsymbol{\lambda}_s(\mathbf{Q})$  can be derived from (1). It contains the common electronic wave function in the  $z$  direction. With a relation between electric field and phonons as in the model of Refs. [19,20],  $\boldsymbol{\lambda}_s(\mathbf{Q})$  is quadratic in momentum. The considerations below are valid for arbitrary  $\boldsymbol{\lambda}_s(\mathbf{Q})$ .

Having specified the model, we will now study first the kinetic equation for a strongly simplified case in which only two noninteracting electronic states are kept. In a second step, we will then come back to the full model of interacting electrons in two Landau sublevels coupled to a phonon bath.

*Two noninteracting states.*—The Hamiltonian is given by

$$H_{\text{toy}} = -\Delta(c_{\uparrow}^\dagger c_{\uparrow} - c_{\downarrow}^\dagger c_{\downarrow}) + \sum_{\mathbf{q}} \omega_{\mathbf{q}} \left( b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \frac{1}{2} \right) + \sum_{\mathbf{q}} \lambda_{\mathbf{q}} (c_{\uparrow}^\dagger c_{\uparrow} b_{\mathbf{q}} + c_{\downarrow}^\dagger c_{\downarrow} b_{\mathbf{q}}^\dagger), \quad (3)$$

where  $c_{\uparrow,\downarrow}^\dagger$  ( $b_{\mathbf{q}}^\dagger$ ) are electron (phonon) creation operators. Here, scattering from the  $\uparrow$  state to the  $\downarrow$  state occurs under absorption of a phonon with an arbitrary wave vector  $\mathbf{q}$ , and the same is true for the reverse process. The kinetic equation for the time-dependent average occupation numbers  $n_{\uparrow,\downarrow}(t)$  can be derived with the aid of Fermi's golden rule (here, time arguments are omitted):

$$\partial_t n_{\uparrow} = -\partial_t n_{\downarrow} = \frac{1}{\tau} [(1 - n_{\uparrow})n_{\downarrow}(1 + N) - n_{\uparrow}(1 - n_{\downarrow})N]. \quad (4)$$

Equation (4) is valid in the quasistationary limit  $\Delta \gg 1/\tau$ , and the relaxation time  $\tau$  is given by

$$\frac{1}{\tau} = \sum_{\mathbf{q}} \lambda_{\mathbf{q}}^2 2\pi \delta(2\Delta - \omega_{\mathbf{q}}). \quad (5)$$

$N = [e^{\beta 2\Delta} - 1]^{-1}$  is the occupation number of the only phonon ( $\omega_{\mathbf{q}} = 2\Delta$ ) which can be effective under energy conservation. The density  $\nu = n_{\uparrow}(t) + n_{\downarrow}(t)$  is fixed; thus it is clear that Eq. (4) has two stationary solutions for  $n_{\uparrow}$ . Only one of these is physical, the other is unphysical

( $n_{\uparrow} > 1$ ). The kinetic equation (4) is best discussed in terms of the depolarization, the deviation from the physical solution,  $\delta(t) = n_{\uparrow}(\infty) - n_{\uparrow}(t)$ :

$$\tau \partial_t \delta(t) = -\delta(t) [\epsilon + \delta(t)]. \quad (6)$$

The crucial parameter  $\epsilon$  is the difference between the two stationary solutions,  $\epsilon = [4N(1 + N) + (\nu - 1)^2]^{1/2}$ . It depends strongly on the temperature and the total density. For zero temperature  $1/\beta = 0$ ,  $\epsilon = |\nu - 1|$ . For  $\nu = 1$ , on the other hand,  $\epsilon \sim 2e^{-\beta\Delta}$  at low temperatures. Now it is obvious that, and under which conditions, linearization in the kinetic equation (6) can fail: Equation (6) shows two regimes depending on the relative size of  $\epsilon$  and  $\delta(t)$ . If  $\delta(t)$  is smaller than  $\epsilon$ , then one can linearize Eq. (6) in  $\delta(t)$ , and  $\delta(t)$  decays exponentially. If  $\delta(t)$  is larger than  $\epsilon$ , then Eq. (6) becomes quadratic in  $\delta(t)$ , and  $\delta(t)$  decays with  $1/t$  until it becomes so small that it crosses over into the first regime. This behavior is pictured in Fig. 1 for  $\delta(t=0) = 0.25$ . The solid line shows the depolarization for  $\epsilon = 0.05$ . Initially, the curve follows that of the quadratic equation ( $\epsilon = 0$ , dashed line), until it crosses over to exponential behavior. This limiting exponential behavior is indicated as a dotted line. We conclude that even the solution of the oversimplified model  $H_{\text{toy}}$  shows a nontrivial behavior, far from being exponential *per se*. It displays a strong dependence on temperature and density. The asymptotic exponential decay of  $\delta(t)$  can become arbitrarily slow ( $\epsilon \rightarrow 0$ ) as the temperature approaches 0 and  $\nu \rightarrow 1$ , and in the limit  $\delta(t) \propto 1/t$ .

*Electron-electron interaction.*—We now turn back to the case of interacting electrons in the two lowest Landau sublevels split by the Zeeman energy. The electrons are scattered by phonons [see  $H_{e\text{-ph}}$ , Eq. (2)]. The electronic part  $H_e$  of the total Hamiltonian  $H_e + H_{\text{ph}} + H_{e\text{-ph}}$

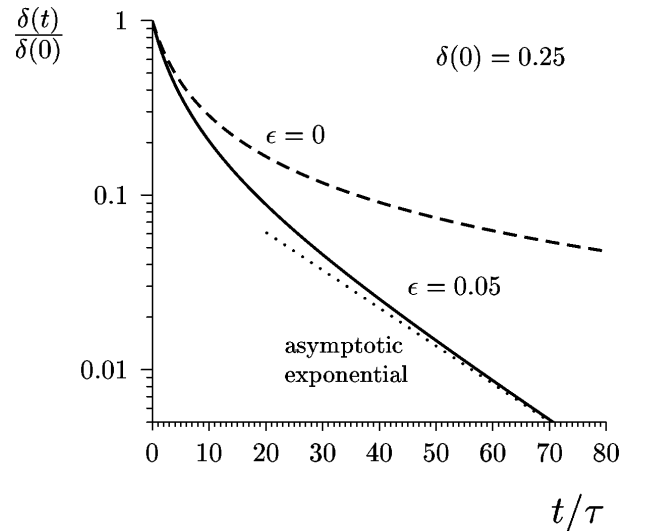


FIG. 1. Depolarization  $\delta(t)$  as a function of  $t/\tau$  for an initial value of  $\delta(0) = 0.25$ ; solid line:  $\epsilon = 0.05$ ; dashed line:  $\epsilon = 0$ .

is then

$$H_e = -\Delta \sum_p (c_{\uparrow p}^\dagger c_{\uparrow p} - c_{\downarrow p}^\dagger c_{\downarrow p}) + H_{\text{Coul}}. \quad (7)$$

Here,  $c_{\uparrow p}^\dagger$  ( $c_{\downarrow p}^\dagger$ ) creates a Landau state with linear momentum  $p$  and spin  $\uparrow$  ( $\downarrow$ ). The most important ingredient, which was missing in  $H_{\text{toy}}$ , is the Coulomb interaction  $H_{\text{Coul}}$  between the electrons (cf. [21]). As stated in assumption (i) above, the expectation values of density and spin density of the electrons are presumed to be uniform. The density is conserved and its value is given by the filling factor  $\nu \sim 1$ . We wish to study the uniform spin density,  $\mathbf{S}(t) = \langle \hat{\mathbf{S}}(\mathbf{q} = 0) \rangle$ ,

$$\mathbf{S}(t) = \frac{1}{N_\Phi} \sum_p \left\langle (c_{\uparrow p}^\dagger, c_{\downarrow p}^\dagger)(t) \frac{1}{2} \boldsymbol{\sigma} \begin{pmatrix} c_{\uparrow p} \\ c_{\downarrow p} \end{pmatrix} (t) \right\rangle. \quad (8)$$

$N_\Phi$  denotes the number of states in a Landau sublevel; if all  $\uparrow$  states are populated and all  $\downarrow$  states are empty,  $\mathbf{S}(t) = \frac{1}{2} \hat{e}_z$ . The derivation of the kinetic equation for  $\mathbf{S}(t)$  can be performed with the Keldysh method [16]. The kinetic equation contains a precession term, which is already present without any phonon scattering, and the main term resulting from the collisions with the phonons. In leading (second) order perturbation theory in the electron-phonon coupling, the collision integral becomes

$$\begin{aligned} \partial_t S^m(t)|_{\text{coll}} = & -i \int_{-\infty}^t dt' \sum_{\mathbf{Q}} \sum_{j,k,l} \epsilon^{mjk} \\ & \times [C^{kl}(\mathbf{q}; t, t') \mathcal{D}^{jl}(\mathbf{Q}; t - t') \\ & - C^{lk}(\mathbf{q}; t', t) \mathcal{D}^{lj}(\mathbf{Q}; t' - t)]. \quad (9) \end{aligned}$$

Only times  $t' < t$  contribute due to causality. The function  $C$  is the dynamical spin-spin correlation function,

$$C^{kl}(\mathbf{q}; t, t') = \frac{1}{N_\Phi} \langle \hat{S}^k(\mathbf{q}, t) \hat{S}^l(-\mathbf{q}, t') \rangle. \quad (10)$$

The spin density couples in  $H_{e\text{-ph}}$  to the phonons via

$$\hat{\Phi}(\mathbf{Q}) = \sum_s [\boldsymbol{\lambda}_s(-\mathbf{Q}) b_{-\mathbf{Q},s} + \boldsymbol{\lambda}_s(\mathbf{Q})^* b_{\mathbf{Q},s}^\dagger]; \quad (11)$$

thus, the phonon expectation value  $\mathcal{D}$  in (9) is given by

$$\mathcal{D}^{jl}(\mathbf{Q}; t - t') = \langle \hat{\Phi}^j(\mathbf{Q}, t) \hat{\Phi}^l(-\mathbf{Q}, t') \rangle. \quad (12)$$

$\mathcal{D}$  is easily calculated with the equilibrium Hamiltonian  $H_{\text{ph}}$  of the phonons.  $\epsilon^{ijk}$  is the antisymmetric tensor. Its origin is the time derivative in (9) which leads to a commutator of  $\hat{\mathbf{S}}$  with the spin density in  $H_{e\text{-ph}}$ .

It is apparent now that *collective modes* are responsible for the relaxation process. The question is whether one can express the two-particle (four-fermion) correlation function of the interacting system,  $C$ , again by  $\mathbf{S}(t)$  and thus derive a closed equation for  $\mathbf{S}(t)$ . Fortunately, this turns out to be possible in our case: The time dependence of  $C$  is determined by particle-hole excitations (e.g.,  $\hat{S}^+ = \hat{S}^x + i\hat{S}^y \sim c_{\uparrow}^\dagger c_{\downarrow}$ ). For  $H_e$ , the particle-hole excitations above the ground state are spin-excitons and

their dispersion  $E_{\text{ex}}(q)$  is rigorously known [22,23] at  $\nu = 1$ ,

$$E_{\text{ex}}(q) = 2\Delta + \frac{e^2}{\kappa l_B} \sqrt{\frac{\pi}{2}} \left[ 1 - e^{-q^2 l_B^2 / 4} I_0 \left( \frac{q^2 l_B^2}{4} \right) \right], \quad (13)$$

where  $\kappa$  is the dielectric constant,  $l_B$  is the magnetic length, and  $I_0(x)$  denotes the Bessel function. We now approximate, in the present case of a nonequilibrium state and  $\nu \sim 1$ , the time dependence of  $C$  with the above dispersion of the spin-exciton,  $E_{\text{ex}}(q)$ . Thus we neglect (i) the difference between the real energy of a single spin-exciton in a nonequilibrium state and  $E_{\text{ex}}(q)$ , and (ii) interactions between the spin excitations at  $\nu \neq 1$ . This yields, e.g.,

$$C^{k+}(\mathbf{q}; t, t') \simeq e^{iE_{\text{ex}}(q)(t-t')} C^{k+}(\mathbf{q}; t, t). \quad (14)$$

The remaining equal-time correlations are then calculated in the Hartree-Fock approximation. This is the weak-scattering approximation in which the effect of the phonons is included in the lowest order in the state, but neglected in the time dependence [17]. Since the characteristic energy for the time dependence is the Zeeman energy  $\Delta$ , the approximation corresponds to the condition  $\Delta \gg 1/\tau$ , where the scattering time  $\tau$  is defined below in Eq. (16). The other two approximations above regarding the exciton energy demand that the electronic temperature must be small compared to the Zeeman energy  $\Delta$  and also that the filling factor must be such that  $|\nu - 1| \ll 1$ . Now, collecting all terms, we obtain the resulting kinetic equations for the components of  $\mathbf{S}(t)$  ( $S^+ = S^x + iS^y$ ; details of the calculation are deferred to a forthcoming publication):

$$\begin{aligned} \partial_t S^z(t) = & \frac{1}{\tau} \left[ 1 - \frac{\nu}{2} - S^z(t) \right] \left[ \frac{\nu}{2} - S^z(t) \right] \\ & - \frac{2}{\tau} \bar{N} S^z(t), \quad (15) \end{aligned}$$

$$(\partial_t + i2\Delta) S^+(t) = -\frac{1}{\tau} \left[ \frac{1}{2} - S^z(t) + \bar{N} \right] S^+(t).$$

The relaxation time  $\tau$  and the average phonon number  $\bar{N}$  are defined by [here,  $N(\omega) = [e^{\beta\omega} - 1]^{-1}$ ]

$$1/\tau = \sum_{\substack{Q,s \\ j=x,y}} \frac{\pi}{2} |\lambda_s^j(\mathbf{Q})|^2 \delta[E_{\text{ex}}(q) - \omega_{\mathbf{Q},s}] \quad (16)$$

$$\bar{N}/\tau = \sum_{\substack{Q,s \\ j=x,y}} \frac{\pi}{2} |\lambda_s^j(\mathbf{Q})|^2 \delta[E_{\text{ex}}(q) - \omega_{\mathbf{Q},s}] N(\omega_{\mathbf{Q},s}).$$

It is quite instructive to formulate again the kinetic equation in terms of the depolarization vector  $\boldsymbol{\delta}(t)$ . We use  $\delta^z(t) = S^z(\infty) - S^z(t)$  and we split the precession

term off  $S^+(t)$  by redefining  $\delta^+(t) = \exp(i2\Delta t)S^+(t)$ . Then, the result is

$$\begin{aligned}\tau\partial_t\delta^z(t) &= -\delta^z(t)[\epsilon + \delta^z(t)], \\ \tau\partial_t\delta^+(t) &= -\delta^+(t)\left[\frac{\epsilon}{2} + \delta^z(t)\right].\end{aligned}\quad (17)$$

Here, the parameter  $\epsilon$  corresponding to the one used above in the case of the two-state model is defined as  $\epsilon = [4\bar{N}(1 + \bar{N}) + (\nu - 1)^2]^{1/2}$ . Obviously, Eqs. (17) are quite different from the standard Bloch equations [14]. They are nonlinear, and it is seen that the relaxation of the transverse component  $\delta^+(t)$  depends on the other component. Since the first of the two kinetic equations (17) is identical to (6), the same discussion applies. Observing  $\tau\partial_t \ln[\delta^z(t)/\delta^+(t)] = -\epsilon/2$ , Eqs. (17) can be explicitly solved with the result

$$\frac{\delta^z(t)}{\delta^z(0)} = e^{-\epsilon t/(2\tau)} \frac{\delta^+(t)}{\delta^+(0)} = \frac{e^{-\epsilon t/\tau}}{1 + \delta^z(0)(1 - e^{-\epsilon t/\tau})/\epsilon}.\quad (18)$$

For any finite value of the parameter  $\epsilon$ , the leading asymptotic behavior yields a ratio  $T_2 = 2T_1$  of the relaxation times. On the other hand, for zero temperature and  $\nu = 1$ , all normalized components of the depolarization follow *the same function* which is a power law  $\propto t^{-1}$ .

The relaxation time  $\tau$  is determined by those phonons, whose energy and in-plane momentum match the energy and the momentum of the spin-excitons [see Eq. (16)]. Using the model of D'yakonov and Perel' (cf. Refs. [19,20]), we calculate the electron phonon coupling parameter  $\lambda_s^j(\mathbf{Q})$  and perform the summations and integrations in (16) with the following result in the limit of  $\Delta = 0$ :

$$\frac{1}{\tau} = \frac{1}{8} \sqrt{\frac{2}{\pi}} \left(\frac{\nu}{s}\right)^2 \frac{\hbar}{\rho l_B^5} x_0 e^{-4x_0} W_{1,1}(8x_0).\quad (19)$$

Here,  $\nu$  parametrizes the electron-phonon coupling,  $s$  is the phonon velocity,  $\rho$  is the 3D density, and  $W_{1,1}(x)$  is the Whittaker function. The parameter  $x_0$  is given by  $x_0 = [\hbar s/(\epsilon_c l_B)]^2$ , where  $\epsilon_c$  is the Coulomb energy. An estimate with characteristic values for electrons in GaAs at a magnetic field of 10 T yields  $\tau \sim 10^{-10}$  sec. This order of magnitude is much smaller than the estimate in Ref. [20] and is in agreement with the experimental observation of the ESR linewidth in both Refs. [12] and [13].

We have studied the spin relaxation in a quantum Hall system for filling factors  $\nu \approx 1$ . The resulting kinetic equation (15) is nonlinear and quite different from a conventional Bloch equation. This difference is most pronounced for  $\nu \rightarrow 1$  and  $T \rightarrow 0$ ; there, a linearization of the kinetic equation fails, since the linear term vanishes

at  $\nu = 1$ ,  $T = 0$ . The relevant time  $\tau$  [(16) and (19)] in the kinetic equation is determined dominantly by the Coulomb interaction between the electrons.

We thank R. Haug for a careful reading of this manuscript. Yu.B. thanks the PTB and W.A. thanks the Landau Institute, respectively, for their hospitality. We acknowledge support by the U.S. Civilian Research and Development Foundation under Award No. RP1-273 and RFFI Grant No. 97-02-16042 (Yu.B.) and by the Deutsche Forschungsgemeinschaft, Ap 47/1-2 (W.A.).

- 
- [1] R.J. Nicholas, R.J. Haug, K. von Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).
  - [2] A. Usher, R.J. Nicholas, J.J. Harris, and C.T. Foxon, Phys. Rev. B **41**, 1129 (1990).
  - [3] S.E. Barrett *et al.*, Phys. Rev. Lett. **74**, 5112 (1995).
  - [4] A. Schmeller, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **75**, 4290 (1995).
  - [5] E.H. Aifer, B.B. Goldberg, and D.A. Broido, Phys. Rev. Lett. **76**, 680 (1996).
  - [6] D.K. Maude *et al.*, Phys. Rev. Lett. **77**, 4604 (1996).
  - [7] I.V. Kukushkin, K. von Klitzing, and K. Eberl, Phys. Rev. B **55**, 10 607 (1997).
  - [8] D.R. Leadley *et al.*, Phys. Rev. Lett. **79**, 4246 (1997).
  - [9] S.L. Sondhi, A. Karlhede, S.A. Kivelson, and E.H. Rezayi, Phys. Rev. B **47**, 16419 (1993).
  - [10] H.A. Fertig, L. Brey, R. Côté, and A.H. MacDonald, Phys. Rev. B **50**, 11 018 (1994).
  - [11] D. Stein, K. von Klitzing, and G. Weimann, Phys. Rev. Lett. **51**, 130 (1983).
  - [12] M. Döbers, K. von Klitzing, and G. Weimann, Phys. Rev. B **38**, 5453 (1988).
  - [13] M. Döbers *et al.*, Phys. Rev. Lett. **61**, 1650 (1988).
  - [14] C.P. Slichter, *Principles of Magnetic Resonance*, Springer Series in Solid-State Sciences Vol. 1 (Springer-Verlag, Berlin, 1980).
  - [15] A.G. Aronov, G.E. Pikus, and A.N. Titkov, Sov. Phys. JETP **57**, 680 (1983).
  - [16] L.V. Keldysh, Sov. Phys. JETP **20**, 1018 (1965).
  - [17] L.D. Landau and E.M. Lifshitz, *Physical Kinetics*, Course of Theoretical Physics Vol. X (Pergamon, Oxford, 1981), Chap. 91.
  - [18] Yu.A. Bychkov, T. Maniv, and I.D. Vagner, Phys. Rev. B **53**, 10 148 (1996).
  - [19] M.I. D'yakonov, V.A. Marushchak, V.I. Perel', and A.N. Titkov, Sov. Phys. JETP **63**, 655 (1986).
  - [20] D.M. Frenkel, Phys. Rev. B **43**, 14 228 (1991).
  - [21] W. Apel and Yu.A. Bychkov, Phys. Rev. Lett. **78**, 2188 (1997).
  - [22] Yu.A. Bychkov, S.V. Iordanskii, and G.M. Éliashberg, JETP Lett. **33**, 143 (1981).
  - [23] C. Kallin and B.I. Halperin, Phys. Rev. B **30**, 5655 (1984).