Noise Enhanced Stability in an Unstable System

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We experimentally detect noise enhanced stability in an unstable physical system. The average escape time from a metastable, periodically driven, system is measured in the stable and unstable regimes in a noisy environment. In the unstable regime, we measure that the average escape time has a maximum for a finite value of the noise intensity. The scaling properties of the average escape time and of the variance of escape times are compared with the predictions obtained for a system in a marginal state.

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Nonlinear systems in the presence of noise can show quite counterintuitive dynamics. Paradigmatic examples are stochastic resonance [1] and resonant activation [2]. In the presence of a strong deterministic modulation of the order parameter across a phase transition [3] or of the height of the well barrier in bistable [4,5] or monostable [6] dynamical systems, unexpected structures and/or new phenomena occur. In a theoretical study of an overdamped model system with a time dependent potential, Dayan, Gitterman, and Weiss [6] show that the stability of an unstable system is *enhanced* in the presence of a *finite* amount of noise intensity. We address this phenomenon as noise enhanced stability (NES) of an unstable system.

In this Letter, we show that the phenomenon of NES is experimentally observable in the transient dynamics of an unstable physical system. Moreover, we show that the observation of the phenomenon is not crucially dependent on the exact shape of the potential of the investigated system.

We observe NES of an unstable system by investigating the escape times from a time modulated metastable physical system. In our system, we periodically modulate the height of the well barrier. The modulation can be so intense that the height of the well barrier can be negligible or completely absent for a short interval of time; i.e., the system may be deterministically overall stable or overall unstable depending on the strength of the modulating signal. By investigating a deterministically overall-unstable system we observe that the maximum of the average escape time from the well is observed for a finite value of the noise intensity.

Our physical system is the series of a biasing resistor with a tunnel diode in parallel to a capacitor (in our case the sum of the diode and the input capacitor of the measuring instrument). The physics of this system in the presence of fluctuations has been extensively investigated since the pioneering work of Landauer [7]. In this system, stochastic resonance has been recently observed [8]. We apply across the network a signal $v(t) = V_b + V_s \sin(\omega_s t) + V_n(t)$, where V_b is a biasing voltage, $V_s \sin\omega_s t$ is a sinusoidal modulating signal, and $V_n(t)$ a noise voltage. Under this condition, the equation of motion for the voltage across the tunnel diode v_d is a Langevin equation, in the overdamped regime, of a system characterized by the time dependent generalized potential

$$U(v_d, t) = -\frac{V_b v_d}{RC} + \frac{v_d^2}{2RC} + \frac{1}{RC}\int_0^{v_d} RI(v)\,dv - \frac{V_s v_d}{RC}\sin(\omega_s t), \quad (1)$$

where R is the biasing resistor and $I(v)$ is the current-voltage characteristic of the tunnel diode. $I(v)$ is characterized by a rapid increase from $v = 0$ to $v = V_p$ (V_p is the peak voltage of the characteristic) followed by a region of negative differential resistance from $v = V_p$ to $v = V_v$ (valley voltage) where $I(v)$ decreases from the peak value I_p to the valley value I_v. For values of v higher than V_v the I-V characteristic is very close to the usual characteristic of a semiconductor diode. The generalized potential is a bistable potential. The asymmetry (in the height of the well barriers) of the bistable system can be easily controlled by selecting the values of V_b and R.

Our experiments are performed when the system presents a strongly asymmetric generalized bistable potential. The experimental setup allows us to control the initial condition of the system for each different realization. The system always starts in the minimum of the well characterized by the smaller barrier at $t = 0$. The difference between the barrier of the two wells is so relevant that the probability that the system goes back to the starting well after the first escape is totally negligible for the noise intensities used in these experiments. For all practical purposes, our system behaves as a metastable state.

We electronically modulate the height of the well barrier of the generalized potential by applying a periodic voltage $V_s \sin(\omega_s t)$ across the electrical network. By properly selecting V_b and V_s one can obtain the following: (i) An overall-stable system. A system with a time dependent generalized potential well which has a finite barrier at any time [Fig. 1(a)]. (ii) An overall-unstable system. A system with a time dependent generalized potential well which has no barrier for a short interval of time within each period of the modulating signal [Fig. 1(b)].

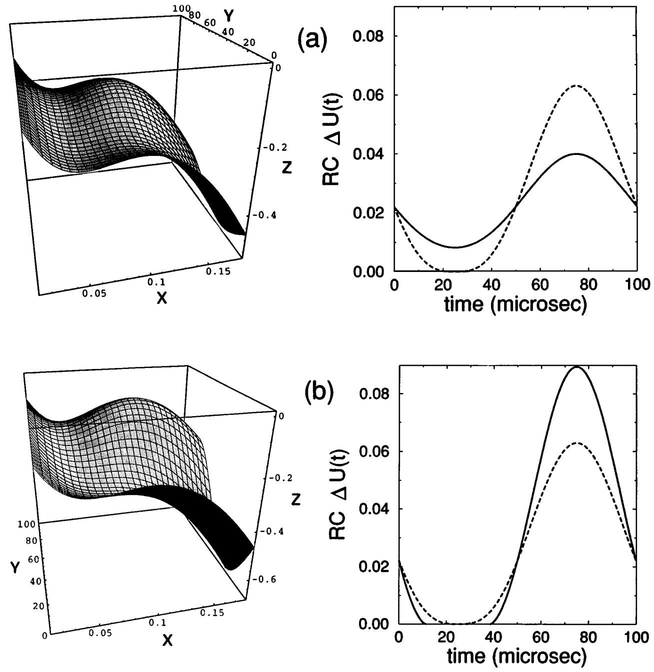

FIG. 1. (a) 3D plot [$x = v_d$, $y = t$, and $z = RC\,U(v_d, t)$] of the time dependent potential [Eq. (1)] for an overall-stable regime ($V_b = 9.59$ V, $V_{sm} = 0.77$ V, $V_s = 0.37$ V, $f_s = 10$ kHz, and $\delta = -0.52$) together with the time evolution of the height of the well barrier ΔU (solid line) during a period of modulation. ΔU is always greater than zero. The dashed line shows the ΔU observed in the overall-marginal regime, i.e., the regime where the system is marginal, $\Delta U = 0$, exactly at $t = T_s/4$ ($V_s = 0.77$ V, $\delta = 0$). (b) The same as in (a) for an overall-unstable system ($V_s = 1.17$ V, $\delta = +0.52$). ΔU is greater than zero for a large part of the modulation period, but a time window exists (close to $T_s/4$) where the height of the well barrier is completely absent. The values of $|\delta|$ are much greater than those used in the experiments for illustrative reason.

The internal noise of the system is minimized by properly selecting the electronic devices used in our network. We are able to experimentally control the transition from the overall-stable to the overall-unstable regime with an accuracy of 1 mV in the amplitude of the modulating signal V_s. To quantify the degree of stability of the time modulated system we introduce the variable $\delta \equiv (V_s - V_{sm})/V_{sm}$, where V_{sm} is the value of the amplitude of the modulating signal driving the system into the marginal state of the potential at $t = T_s/4$. The marginal state is the point at which the unstable and locally stable branches of the generalized potential coalesce [9].

Our experiments are performed with a germanium tunnel diode 1N3149A. This tunnel diode has nominal peak current $I_p = 10.0$ mA, peak voltage $V_p = 60$ mV, valley current $I_v = 1.3$ mA, valley voltage $V_v = 350$ mV, and the switching time is very short (less than 40 nsec). We perform our experiments by choosing $V_b = 9.59$ V (which implies with our diode $V_{sm} = 770$ mV), $R = 1050\ \Omega$, $C \approx 50$ pF, $f_s = 10$ kHz, and V_s ranging from 750 to 790 mV. The temperature of the apparatus is kept constant to 18 ± 1 °C. The analogic additive external noise is obtained starting from a digital pseudorandom generator. The noise voltage is a stochastic Gaussian process with a correlation time of 90 nsec. The root mean square voltage of the noise V_{rms} can be varied from 1 to 1000 mV. Under computer control, we measure, for each set of the control parameters, 10^4 escape times from the potential well measured in different statistical realizations of the process. In each realization, the time evolution of v_d is characterized by a periodic motion performed in the starting well which ends with an abrupt transition to the second well at a random time. The system remains in the second well until it is forced into the minimum of the starting well by the control electronic network which starts a new realization. We define the escape time T as the time interval measured between the setting of the system in the minimum of the well ($t = 0$) and the crossing of v_d of a voltage threshold which ensure that the system is quite far from the starting well (we choose $v_d = 0.4$ V). The exact value of the threshold is not a determinant parameter because the escape from the well is extremely fast (less than 40 nsec). From the set of measured escape times $\{T\}$, we determine the distribution $P(T)$, the average value $\langle T \rangle$, and the variance $\mathrm{Var}(T)$ of the escape time.

In Fig. 2we show two series of experimental distributions $P(T)$ measured in the overall-stable [$\delta = -0.0039$, Fig. 2(a)] and in the overall-unstable [$\delta = +0.0039$, Fig. 2(b)] regimes for different values of the noise amplitude V_{rms}. By comparing the experimental results observed in the overall-stable and overall-unstable regimes, we observe that $P(T)$ is almost the same in the two regimes for higher values of the noise amplitude while for lower V_{rms} a clear difference is observed. $P(T)$ has sharp peaks of decreasing amplitude in both regimes but the amplitude of the peaks decays much faster in the overall-unstable case. The difference in the decay constant obviously implies a difference in the average escape time. In Fig. 3 we show the average escape time measured for six different values of δ as a function of V_{rms}. The δ values refer to three overall-stable and three overall-unstable cases. As expected, for an overall-stable system ($\delta < 0$) the average escape time is a monotonic decreasing function of V_{rms} (top three sets of points in Fig. 3). More interestingly, for an overall-unstable system ($\delta > 0$), we observe a nonmonotonic behavior of the average escape time (bottom three sets of points in Fig. 3). For each investigated value of $\delta > 0$, we observe that it exists an interval of V_{rms}, bounded by two finite values, in which $\langle T \rangle$ is longer than $T_{det} \approx T_s/4$, the deterministic escape time observed in the absence of noise. Hence, within this interval of V_{rms}, the presence of the noise enhances the stability of an unstable modulated physical system. We observe a *noise enhanced stability* of an unstable system. The experimental results (Fig. 3) show that the interval of V_{rms} where NES is observed is quite wide (approximately 2 orders of

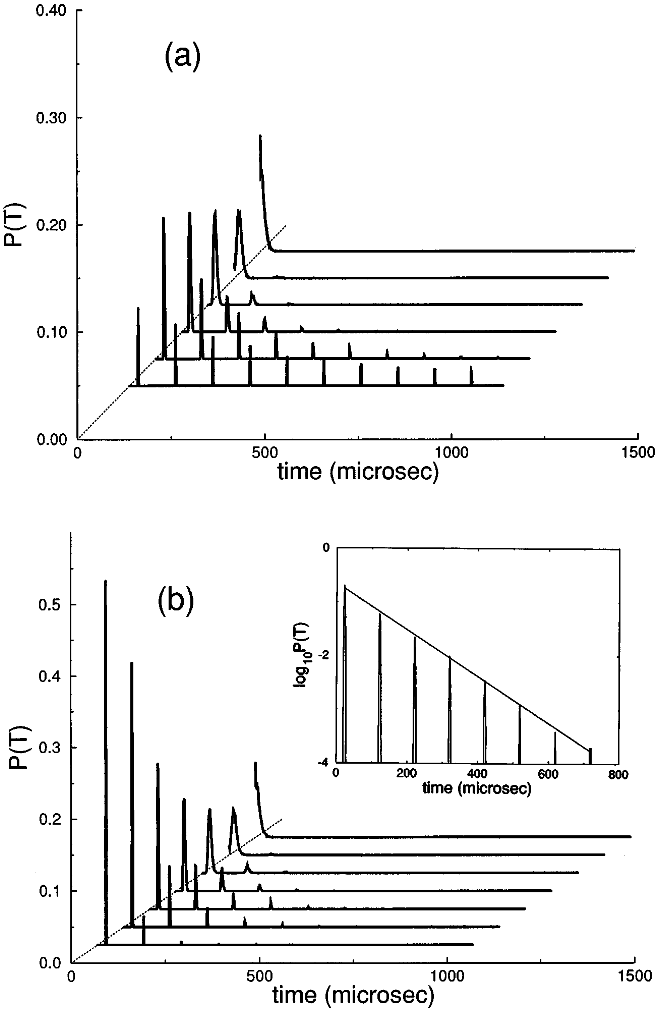

FIG. 2. (a) Escape time distributions measured in an overall-stable system ($\delta = -0.0039$). Different curves are obtained for different values of the noise amplitude. From top to bottom $V_{\rm rms} = 1000$, 300, 100, 30, 10, 3 mV. (b) The same as in (a) for an overall-unstable system ($\delta = +0.0039$). From top to bottom $V_{\rm rms} = 1000$, 300, 100, 30, 10, 3, 1 mV. Inset: Representative escape time distribution measured for $\delta = +0.0039$ and $V_{\rm rms} = 10$ mV. The dotted straight line is the exponential envelope with time constant τ_e of the multipeaked distribution. A similar behavior is observed for overall-stable systems ($\delta < 0$).

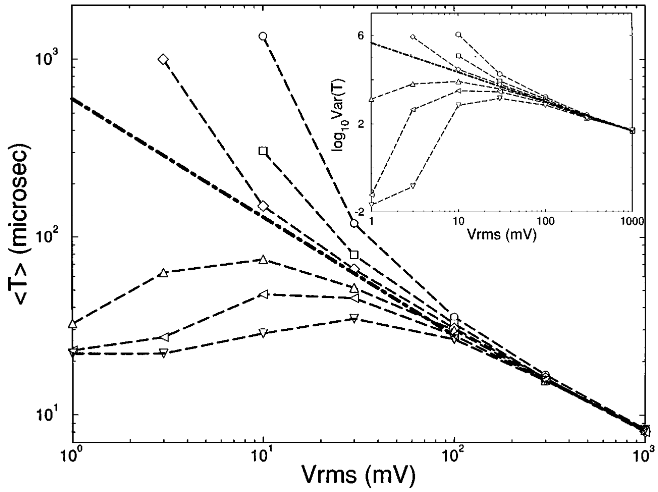

FIG. 3. Average escape time as a function of the noise amplitude for six different values of the stability parameter δ. The overall-stable cases $\delta = -0.0195$ (○), $\delta = -0.0091$ (□), and $\delta = -0.0039$ (◇) show a monotonic decrease of $\langle T \rangle$ while the overall-unstable cases $\delta = +0.0039$ (△), $\delta = +0.0091$ (◁), and $\delta = +0.0195$ (▽) show a nonmonotonic behavior with a maximum of the average escape time observed for a finite value of the noise amplitude. The dot-dashed line is the scaling behavior of Eq. (2). The dotted lines are joining experimental points with the same δ. The experimental errors are of the size of the used symbols. Inset: Variance of the escape time (in microsecond square) as a function of the noise amplitude for the same six values of δ as in Fig. 3. The dot-dashed line is the scaling behavior of Eq. (3).

magnitude for $\delta = +0.0039$) and that it exists, for each value of δ, an optimal value of the noise amplitude $V_{\rm rms}$ which maximizes the increase of stability of the system.

In both the stable and unstable cases, the experimental $P(T)$ is multipeaked with an exponentially decaying envelope. In the inset of Fig. 2(b), we show a typical $P(T)$ in a semilogarithmic plot. The exponential envelope, characterized by a time constant τ_e, is directly observable. Multipeaked distributions with an exponentially decaying envelope have also been experimentally observed and theoretically described in the stochastic resonance [10]. We also note that the shape of the nth peak in the $P(T)$ is well fitted by a Gaussian function with standard deviation σ_T independent of n. The same behavior has been observed in the experiments of stochastic resonance performed in the strong-forcing limit [5].

A complete theoretical description of the phenomenon of NES in unstable systems is not yet available. However, there are theoretical and numerical studies of noise induced phenomena which are related to our experimental study of a physical system. Relevant to our study are the studies of the escape of a periodically driven particle from a metastable state in a noisy system [6,11,12] and the studies of the relaxation from a marginal state in bistable systems [9,13]. By numerically investigating a prototype of an unstable periodically modulate nonlinear system Dayan, Gitterman, and Weiss [6] observed, for the first time to the best of our knowledge, NES in an unstable system. Their study is performed by keeping constant the amplitude of the driving signal modulating their particular potential. They control the degree of stability of the system by varying the frequency of the driving signal. It is worth pointing out that the same phenomenon is expected in their system if one keeps constant the frequency and controls the transition from a stable to unstable system by varying the amplitude of the driving signal [11]. A numerical study of the same physical system [12] has shown that the distribution of the escape times is a multipeaked distribution of decaying amplitude. By performing numerical simulations of the same system, we find that the amplitude of the peaks in the $P(T)$ distribution decays exponentially [14]. The same decay is observed in our experimental results [see the inset of Fig. 2(b)].

NES has been originally predicted for a particular potential [6,11,12] and experimentally observed in our quite different physical system. This fact allows us to conclude that this phenomenon is not crucially depending on the exact shape of the potential of the investigated system.

The scaling properties of the average escape time as a function of the noise amplitude provide an interesting way to characterize the statistical dynamics of the system. The relaxation from a cubic marginal state is characterized by [9]

$$\langle T \rangle \propto V_{\mathrm{rms}}^{-2/3} \quad (2)$$

and

$$\mathrm{Var}(T) \equiv \langle (T - \langle T \rangle)^2 \rangle \propto V_{\mathrm{rms}}^{-4/3} \quad (3)$$

In our experiments, for low-noise amplitude, the dynamics of the system is dominated by the noise in the time intervals when the system is close to its marginal state. For this reason, we expect that the boundary between the unstable and stable regimes in the average escape time and in its variance is well located by Eqs. (2) and (3), respectively.

In Fig. 3 we plot (as a dot-dashed line) the scaling behavior predicted by Eq. (2). The straight line separates the average escape time measured for stable systems from the one measured for unstable systems for lower values of V_{rms}. In the inset of Fig. 3, we plot the experimental values of $\mathrm{Var}(T)$ as a function of the noise amplitude together with the scaling behavior predicted by Eq. (3) for the variance of the escape times in a marginal state in the presence of noise (dot-dashed straight line). Once more, the scaling behavior predicted for a marginal state clearly separates the plot into two regions where we observe our experimental results obtained for stable (top) and unstable (bottom) systems. By comparing Fig. 3 and its inset one concludes that, for each value of δ, $\langle T \rangle$ and $\mathrm{Var}(T)$ have their maximum value for the same value of V_{rms}. This experimental result manifests that, in the low-noise regime, $\langle T \rangle$ and $\mathrm{Var}(T)$ are substantially controlled by the values of τ_e and τ_e^2, respectively. We experimentally verify that $\langle T \rangle$ and $\mathrm{Var}(T)$ have their maximum when τ_e is maximum as a function of V_{rms}.

The comparison of our experimental findings with the theoretical and numerical results on the strong driving of a metastable system across or at its marginal state confirms that we observe experimentally NES in an unstable system. This phenomenon provides a quite unexpected way to maximize the stability of an unstable system.

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