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2007 J. Phys. D: Appl. Phys. 40 2507

(http://iopscience.iop.org/0022-3727/40/8/014)

IOP Publishing Journal of Physics D: Applied Physics
J. Phys. D: Appl. Phys. **40** (2007) 2507–2512 doi:10.1088/0022-3727/40/8/014

Propagation of sound in glow discharge plasma

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Received 6 November 2006
Published 4 April 2007
Online at stacks.iop.org/JPhysD/40/2507

Abstract
Propagation of sound in a medium where the rate of local heat addition is a function of gas density is analysed theoretically and the results are applied for modelling the experimentally observed effect of amplification of acoustic waves by an extended glow discharge in air. The model adequately describes the experimental dependences of the gain on the wave frequency and discharge power density and predicts that the amplification of sound by an unconfined glow discharge in air increases with discharge current density but does not change noticeably with gas pressure when the current density is kept constant. Quantitative estimates indicate that a gain of as high as 1 m^{-1} (or 9 dB for a 60 dB wave passing through 1 m of plasma) could be realized using a discharge in air with a current density of 100 mA cm^{-2}.

1. Introduction

Advances in glow discharges have yielded a number of technical solutions that realized sizeable glow discharge plasmas at atmospheric pressures in air. One of the applications for such technologies is aerodynamic flow control and management of aeroacoustic noise. Experiments [1–3] demonstrated that acoustic waves propagating through a plasma formation may undergo significant attenuation, and the effect has been associated with reflection of the wave in the non-uniform regions of the plasma boundary [4]. With plasma dimensions increased, mechanisms that describe the interaction of the wave with uniform plasma should also be brought into account. On the other hand, in a number of experiments that studied propagation of sound in low-pressure cylindrical discharge tubes, amplification of sound waves has been observed [5–8]. The effect of sound amplification by glow discharge plasma was recently proposed as an enabling tool for stimulating precipitation in atmospheric clouds [9].

For practical purposes, discharges in air with high energy inputs in plasma are of particular interest while most theoretical work related to the problem dealt with low-energy plasmas of inert gases [5–8,10]. Interaction of acoustic waves with vibrationally excited molecular gas was analysed theoretically in [11] where a case of vibrational energy relaxation time significantly exceeding the wave period was discussed. When the latter condition is satisfied, the stationary plasma parameters are established in compressions and rarefactions of the wave and therefore the effect on the wave is determined by gas temperature variations that are caused by density variations. But for higher-power air plasmas, as well as for wave frequencies below approximately 1 kHz, the vibrational energy relaxation time is typically smaller than the period of the wave and the approach [11] is not adequate.

Summarizing the results of prior theoretical discussions, one may conclude that glow discharge plasma can be generally described as a medium with continuously distributed source of energy addition where the local rate of energy addition is a function of gas density. A mechanism of plasma–wave interaction in such a medium (referred here to, following our earlier publication [11], as Rayleigh mechanism and medium) may lead to either attenuation or amplification of a travelling acoustic wave, depending on the phase shift between the gas density profile and the heat addition rate. This sign of the effect (amplification or attenuation) is determined by mutual orientation of the plasma electric field and wave vectors: the wave amplitude increases if these vectors are orthogonal and it decreases when the vectors are parallel [12, 13].

The objective of this paper is to analyse a one-dimensional problem of acoustic wave propagation in a Rayleigh medium and to evaluate the effect for unconfined plasma in air.

2. Governing equation

Without a wave, the Rayleigh medium is a fluid where the distributed energy addition is balanced with similarly distributed energy loss; thus, the net energy addition to the media is zero. The appearance of a wave in such a medium leads to local non-uniformities where the energy balance shifts to one direction or the other, depending on how the rate of net energy addition $Q(x,t)$ changes with density. Assuming that the propagation of a disturbance in a Rayleigh medium is sufficiently described by Euler's equations, and following the approach reported elsewhere [4], one may obtain the following set of equations connecting mass velocity $U(x,t)$, speed of sound a and gas density ρ:

$$\frac{\partial F}{\partial t}+U\frac{\partial F}{\partial x}=\frac{\gamma(\gamma-1)}{2}\frac{F^{(1+\gamma)/(\gamma-1)}}{a^{\frac{2\gamma}{\gamma-1}}}Q,$$

$$\frac{\partial U}{\partial t}+(U+a)\frac{\partial U}{\partial x}=-\frac{2}{\gamma-1}\left\{\frac{\partial \phi}{\partial t}+U\frac{\partial \phi}{\partial x}\right\}-\frac{\gamma F^{2/(\gamma-1)}}{a^{(1+\gamma)/(\gamma-1)}}Q,$$

$$\frac{\partial \phi}{\partial t}+U\frac{\partial \phi}{\partial x}=a\frac{\partial \phi}{\partial x}-\frac{a^2}{\gamma F}\frac{\partial F}{\partial x}-\frac{\gamma(\gamma-1)F^{2/(\gamma-1)}}{2a^{(1+\gamma)/(\gamma-1)}}Q. \quad (1)$$

Here the disturbance is assumed to propagate in the direction of mass velocity and $\gamma = c_p/c_v$. Functions $\Phi(x,t)$ and $F(x,t)$ are related to a and ρ through $a = F(x,t)\rho^{\frac{\gamma-1}{2}}$ and $a = \Phi(x,t) \pm ((\gamma-1)/2)U$. Linearizing equations (1) and applying a variational approach described earlier in [4], the system is reduced to a single equation that governs the propagation of a disturbance in a Rayleigh medium:

$$a_0^2\frac{\partial^2 U}{\partial x^2}-(\gamma-1)a_0^2\frac{\partial g}{\partial x}=\frac{\partial^2 U}{\partial t^2}, \quad (2)$$

where $g = Q/\gamma p_0$ and a_0 and p_0 are the speed of sound and pressure in the undisturbed medium. Assuming that the rate of net heat addition is a function of a single variable $(\partial g/\partial x = (\mathrm{d}g/\mathrm{d}\rho)(\partial\rho/\partial x))$, and introducing dimensionless coordinates $\tau = \omega_0 t$; $z = k_0 x$ $(\omega_0/k_0 = a_0)$, the latter equation becomes

$$\frac{\partial}{\partial \tau}\left\{\frac{\partial^2 U}{\partial \tau^2}-\frac{\partial^2 U}{\partial z^2}\right\}=2b\frac{\partial^2 U}{\partial z^2}, \quad (3)$$

where $b = (\gamma-1)/2(\rho_0/\omega_0)(\mathrm{d}g/\mathrm{d}\rho)\,|_{\rho=\rho_0}$ is the parameter of the problem that can also be expressed in terms of a characteristic frequency $\nu_g = ((\gamma-1)\rho_0/2)(\mathrm{d}g/\mathrm{d}\rho)\,|_{\rho=\rho_0}$ (the heating frequency). And if the net heat addition rate is sufficiently low, so that for a wave with a frequency of $\nu_0 = \omega_0/2\pi$ and a wave number of k_0

$$b \ll \frac{1}{4\pi} \qquad \text{or} \qquad \nu_0 \gg 2\nu_g, \quad (4)$$

equation (3) can be further simplified to

$$\frac{\partial^2 U}{\partial \tau^2}-\frac{\partial^2 U}{\partial z^2}=2b\frac{\partial U}{\partial \tau}. \quad (5)$$

3. Acoustic wave in Rayleigh medium

A solution for equation (3) is

$$U(x,t)=\exp\left[(\beta+\mathrm{i}\mu)\,\tau-\mathrm{i}z\right], \quad (6)$$

where β, μ are arbitrary parameters and $\mathrm{Im}\beta = \mathrm{Im}\mu = 0$. The general solution for equation (3) is given by a linear combination of partial solutions, and the coefficients for the series are found from the initial and boundary conditions [14]. Substituting (6) in (3) results in two sets of cubic equations for parameters β and μ, with only one of those corresponding to a travelling harmonic wave. It yields

$$\mu=\pm\sqrt{1+3\beta^2},$$
$$\beta=\frac{1}{2}\left[\left(b+\sqrt{b^2+\frac{1}{27}}\right)^{\frac{1}{3}}+\left(b-\sqrt{b^2+\frac{1}{27}}\right)^{\frac{1}{3}}\right]. \quad (7)$$

Similarly, a travelling wave solution for equation (5) is

$$\mu_1=\pm\sqrt{1+3b^2},$$
$$\beta_1=b. \quad (8)$$

A linear dependence of β versus b, similar to that given by equation (8), was obtained earlier [5, 6].

4. Plasma effect

The effect of the medium on the wave is determined by the factor $\mathrm{d}g/\mathrm{d}\rho\,|_{\rho=\rho_0} = (1/\rho_0 a_0^2)(\mathrm{d}Q/\mathrm{d}\rho)\big|_{\rho=\rho_0}$. In air plasmas, most of the electrical energy deposited in the gas accumulates in the vibrational states of nitrogen molecules. This energy is released predominantly into the translational mode (the V–T process or heating of the gas) or on the boundaries confining the plasma [15]. The energy $\varepsilon_{\mathrm{VT}}$ that is stored in the vibrational continuum should satisfy the following equation:

$$\frac{\mathrm{d}\varepsilon_{\mathrm{VT}}}{\mathrm{d}\tau}+\frac{\varepsilon_{\mathrm{VT}}}{\tau_{\mathrm{VT}}(\rho(\tau))\omega_0}=\frac{g_H(\rho(\tau))}{\omega_0}, \quad (9)$$

where τ_{VT} is the V–T relaxation time and $g_H(\rho(\tau))$ is the rate of energy deposition into the vibrational states measured in the units of $\rho_0 a_0^2 = \gamma p_0$; $\varepsilon_{\mathrm{VT}}$ is normalized similarly. The solution is

$$\varepsilon_{\mathrm{VT}}(\tau)=\exp\left\{-\frac{1}{\omega_0}\int_0^{\tau}\frac{\mathrm{d}\tau'}{\tau_{\mathrm{VT}}}\right\}\left\{\varepsilon_{\mathrm{VT0}}+\frac{1}{\omega_0}\int_0^{\tau}\exp\left\{\frac{1}{\omega_0}\int_0^{\tau'}\frac{\mathrm{d}\tau''}{\tau_{\mathrm{VT}}}\right\}g_H(\tau')\mathrm{d}\tau'\right\}. \quad (10)$$

In this model, the rate of heat addition to the gas is simply the difference between the current rate of V–T relaxation and that in the undisturbed plasma: $g = (\varepsilon_{\mathrm{VT}}/\tau_{\mathrm{VT}}) - (\varepsilon_{\mathrm{VT0}}/\tau_{\mathrm{VT0}})$. Assuming that variations of plasma parameters in the wave are small, so that $(2\pi/\omega_0\tau_{\mathrm{VT0}})|\rho-\rho_0|/\rho_0 \ll 1$, $g_H \approx g_{H0}+\frac{\mathrm{d}g_H}{\mathrm{d}\rho}[\rho-\rho_0]$ and $\tau_{\mathrm{VT}} \approx \tau_{\mathrm{VT0}}+\frac{\mathrm{d}\tau_{\mathrm{VT}}}{\mathrm{d}\rho}[\rho-\rho_0]$, equation (10) yields

$$g(\tau) = -g_{H0}\frac{1}{\tau_{VT0}}\left.\frac{d\tau_{VT}}{d\rho}\right|_{\rho=\rho_0}(\rho-\rho_0) + \frac{g_{H0}}{\omega_0\tau_{VT0}}\exp\left\{-\frac{\tau}{\omega_0\tau_{VT0}}\right\} \times \int_0^{\tau}(\rho-\rho_0)\exp\left\{\frac{\tau'}{\omega_0\tau_{VT0}}\right\}d\tau'\left[\frac{1}{\tau_{VT0}}\left.\frac{d\tau_{VT}}{d\rho}\right|_{\rho=\rho_0} + \frac{1}{g_{H0}}\left.\frac{dg_H}{d\rho}\right|_{\rho=\rho_0}\right]. \quad (11)$$

Here we see that the original assumption of $g(\rho)$ being a function of a single argument is satisfied only in the peripheral regions of the parameter $\omega_0\tau_{VT0}$ domain:

$$\text{when } \omega_0\tau_{VT0} \gg 1, \qquad g(\rho) \approx -(\rho-\rho_0)g_{H0}\frac{1}{\tau_{VT0}}\left.\frac{d\tau_{VT}}{d\rho}\right|_{\rho=\rho_0}, \quad (12)$$

and

$$\text{when } \frac{2\pi}{\omega_0\tau_{VT0}} \gg 1, \qquad g(\rho) \approx (\rho-\rho_0)\left.\frac{dg_H}{d\rho}\right|_{\rho=\rho_0}. \quad (13)$$

In the interior of the domain, g is a function of both $\rho(\tau)$ and τ; therefore it would be more accurate for the propagation equation (3) to have a source term proportional to $\partial g(\rho(\tau),\tau)/\partial\tau|_{\rho=\text{const}}$. Instead of introducing such a correction, we will note that $g(\tau)$ is a monotonic function of $\omega_0\tau_{VT0}$; thus, an interpolation that satisfies $\partial g/\partial\tau_{\rho=\text{const}} = 0$ and asymptotically relaxes to (12) and (13), respectively, should be sufficient to describe $g(\tau)$ in the total domain of parameter $\omega_0\tau_{VT0}$.

In terms of parameter b, the asymptotic solutions (12) and (13) are written as

$$b = -\frac{\gamma-1}{2}\frac{\rho_0}{\omega_0}g_{H0}\frac{1}{\tau_{VT0}}\left.\frac{d\tau_{VT}}{d\rho}\right|_{\rho=\rho_0} \qquad \text{for } \omega_0\tau_{VT0} \gg 1, \quad (14)$$

$$b = \frac{\gamma-1}{2}\frac{\rho_0}{\omega_0}\frac{dg_H}{d\rho}\Big|_{\rho=\rho_0} \qquad \text{for } \frac{2\pi}{\omega_0\tau_{VT0}} \gg 1. \quad (15)$$

Change in the relaxation time with density occurs in compressions predominantly due to two factors: first, the increased gas density leads to higher collision rates between molecules and, second, compression leads to temperature rise, and the rate coefficient for V–T relaxation is a strong function of temperature; the relaxation time decreases due to both these factors. The opposite trend is expected in rarefactions.

At higher wave frequencies, the vibrational energy density does not change over the wave period, and the energy addition rate varies due to change in τ_{VT} as given by equation (14). When the wave frequency is low (15), the V–T relaxation is fast and the rate of energy addition into the gas is determined by the rate of energy deposition into the vibrational continuum. Change in the energy deposition rate with density occurs predominantly due to the change in the collision rate between molecules and electrons that affects the magnitude of the electric field in plasma and thus the energy deposition rate.

To find $\frac{1}{\tau_{VT0}}\left.\frac{d\tau_{VT0}}{d\rho}\right|_{\rho=\rho_0} \equiv y$, we will consider an energy equation:

$$\frac{dT}{d\rho} = (\gamma-1)\frac{T}{\rho} + \frac{1}{C_V\omega_0\rho}\frac{dQ}{d\rho} \quad (16)$$

with $\frac{dQ}{d\rho} = -Q_{H0}y$ and $Q_{HO} = \gamma g_{H0}p_0$. Assuming that the relaxation time is expressed as $\tau_{VT} \propto 1/\rho\exp\{B_0/T^{1/3}\}$ with $B_0 \approx 234.9$ [16], the parameter of interest is

$$y = -\frac{1}{\rho_0}\frac{1+\frac{(\gamma-1)B_0}{3T_0^{1/3}}}{1-\frac{B_0Q_{H0}}{3T_0^{3/4}\rho_0C_V\omega_0}}. \quad (17)$$

When the electron energy relaxation and ionization frequencies significantly exceed the frequency of the wave (such conditions are realized in air plasmas at $p_0 > 10$ Torr and for wave frequencies of $\omega_0 < 10$ kHz), compressions (or rarefactions) occur slowly enough for plasma parameters, such as electron density and electric field, to establish their stationary values. The value of $\left.\frac{dg_H}{d\rho}\right|_{\rho=\rho_0}$ can then be calculated from [5, 12, 13]

$$\left.\frac{dg_H}{d\rho}\right|_{\rho=\rho_0} = \pm\frac{g_{H0}}{\rho_0} = \pm\frac{jE}{\gamma P\rho_0}. \quad (18)$$

The rate of net energy addition increases in compressions and decreases in rarefactions if the density gradient and electric field vectors are collinear (upper sign in (18)). The opposite effect takes place when sound propagates in the direction perpendicular to the electric field [12].

5. Discussion

Relations (7) indicate that the speed of sound in plasma always exceeds that in the gas heated to the same temperature by a factor of $\mu > 1$. This effect has been qualitatively explained in [17]. Acoustic waves in plasma are amplified when $\beta > 0$ and attenuated when $\beta < 0$. The gain (that is negative for attenuation) per unit length of plasma is

$$K = \frac{\omega_0}{a_0}\beta. \quad (19)$$

This coefficient was estimated by other authors [5, 6] as

$$K_1 = \frac{\omega_0}{a_0}b = \frac{\nu_g}{a_0}; \quad (20)$$

however, the latter equation is only valid if the heating frequency is sufficiently low (criterion (4)).

While K_1 does not depend on the wave frequency, K increases with ω_0 at a given b and is always below its asymptotic value, K_1. Dependences of K and K_1 on the wave frequency are given in figure 1 for three different values of heating frequency ν_g. Dependence K versus ω_0 qualitatively agrees with experimental observations [7] where amplification of standing acoustic waves was studied in a low-pressure (below 80 Torr) argon plasma at wave frequencies of up to 1 kHz. Lack of experimental detail reported in [7] does not allow for a quantitative comparison; however, the observed increase

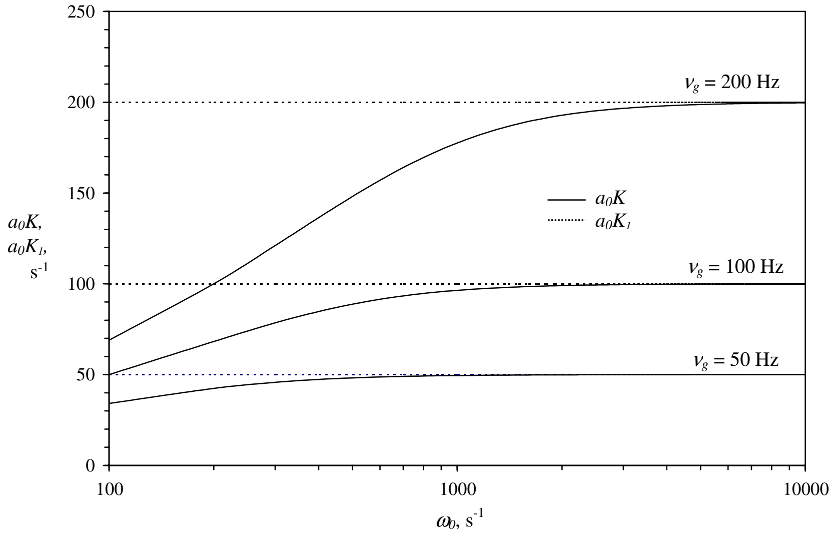

Figure 1. Gain versus acoustic wave frequency for a range of heating frequencies, ν_g .

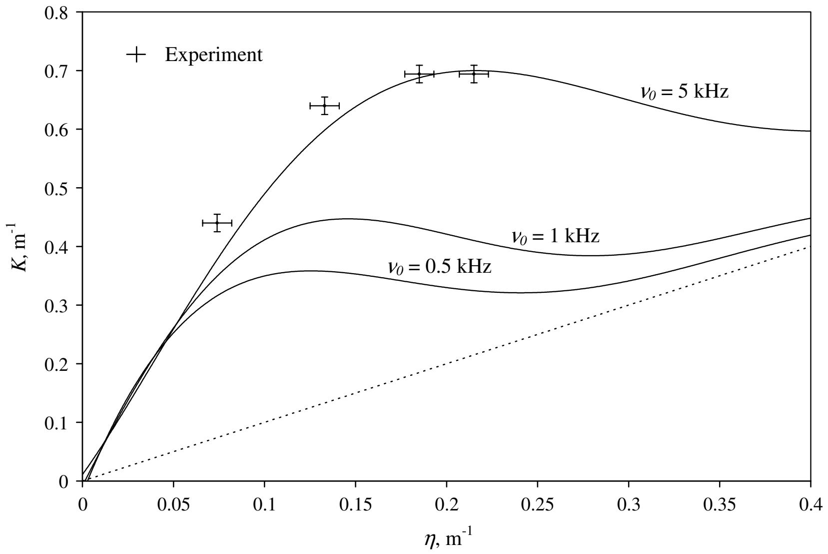

Figure 2. Gain versus discharge power density for a range of wave frequencies. Experimental data are taken from [7]. Calculations used plasma parameters of the experiment [7]. Dotted line indicates gain based solely on mechanism (15).

in the gain with frequency is consistent with the current result.

For evaluation of the efficiency of using a glow discharge plasma as a sound amplification (attenuation) medium, it is particularly important to know how the gain varies with the power input in plasma. Amplification of an acoustic wave in a cylindrical longitudinal discharge in air was measured at a gas pressure of $p_0 = 12.3$ Torr and a frequency of the wave of $\nu_0 = 5$ kHz [5]. In figure 2, these experimental data are compared with the predicted value of the gain. For relatively low discharge power inputs that were realized in experiment [5], criterion (4) was satisfied throughout the total range of plasma power density $\eta = (\gamma - 1)/2(jE/\gamma P a_0)$ (where j and E are the electric current density and field, respectively); therefore, $K \approx K_1$ and equation (20) is sufficient for the gain estimate. To calculate parameter b, one needs to analyse which of the two mechanisms discussed above ((14) and (15)) or both of them are responsible for the gain at a given value of η. The V–T relaxation time is sufficiently large for low power inputs where condition $\omega_0\tau_{VT0} \gg 1$ is satisfied and mechanism (14) is responsible for the plasma effect on the wave. With energy inputs into plasma increasing, the temperature of the undisturbed plasma rises, which causes τ_{VT0} and, for a given wave frequency, $\omega_0\tau_{VT0}$ to fall. Eventually, at sufficiently high energy inputs, mechanism (15) dominates.

To describe the dependence of K over the total range of plasma power density realized in experiment [5], we approximated parameter b with

$$b = b_1\bar{F}_1 + b_2\bar{F}_2, \qquad (21)$$

where b_1 and b_2 are found using relations (14) and (15), respectively, and $\bar{F}_1 = 1/(1 + f_1)$, $\bar{F}_2 = f_1/(1 + f_1)$, with $f_1 = (A_0/\omega_0)\exp\left\{25.89 - (234.9/T^{1/3})\right\}$. $A_0 = 1.7 \times 10^{-5}\,s^{-1}$ is a fitting coefficient. Gas temperature was measured in the experiment [7] only at $\eta \approx 0.23\,m^{-1}$ ($T_0 \approx 600$ K).

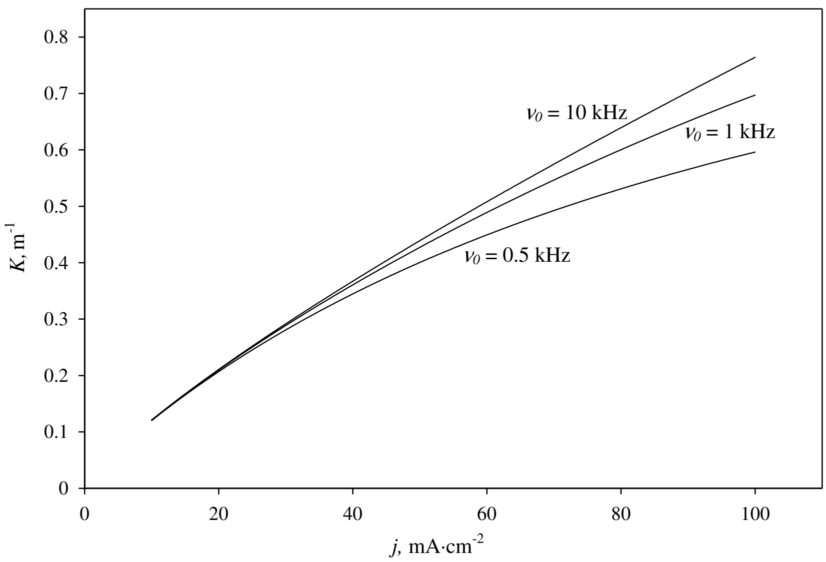

Figure 3. Gain versus discharge current density for an unconfined glow discharge in air. Gas pressure $p = 50$ Torr.

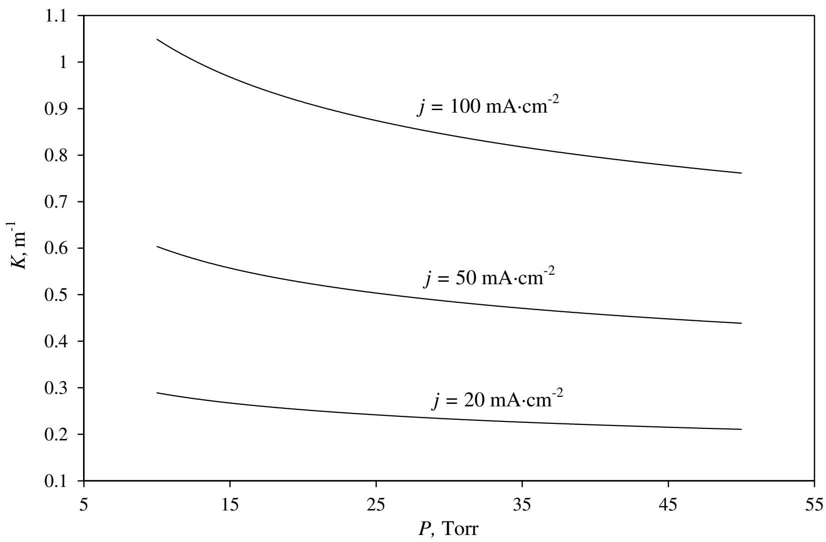

Figure 4. Gain versus gas pressure for an unconfined glow discharge in air. Wave frequency $\nu_0 = 5$ kHz.

Having assumed that thermal conductivity is proportional to the gas temperature, we approximated the temperature dependence with $T(\mathrm{K}) = 300\sqrt{13.333\eta + 1}$, which is a solution for the heat conduction equation on the axis of a cylindrical tube.

The model developed in [5] did not account for V–T relaxation, and theoretical estimates of the gain were carried out in [5] based solely on mechanism (15) that, for molecular gases, dominates only at sufficiently high plasma energy inputs. Under such an assumption, the gain is a linear function of η shown with a dotted line in figure 2. The conversion of the heat addition mechanism that occurs for acoustic frequencies, as illustrated in figure 2, over the range $0.05 < \eta < 0.5$ may lead to a local maximum in the dependence $K(\eta)$.

The above considerations did not account for energy loss on the walls which is typical in experiments with cylindrical plasma cells. Accounting for this effect will reduce the magnitude of the gain predicted theoretically by equation (19), albeit insignificantly. [5] The role of the walls decreases if the characteristic size of thermal conduction, which for a cylindrical tube is equal to its diameter, increases.

For practical applications, discharges at higher pressures ($p > 10$ Torr) and current densities ($j > 10\,\mathrm{mA\,cm^{-2}}$) are of interest. Figures 3 and 4 demonstrate the dependence of the gain on the discharge current density and pressure, respectively, for an unconfined cylindrically symmetric discharge in air. The gain for a wave propagating along the discharge axis was found using equations (19) and (21). The gas temperature on the axis of an unconfined discharge was calculated based on the approach developed in [18]. The gain grows steadily with discharge current density, almost independently of the frequency of the wave (figure 3). This effect is apparently related to the increase in energy input into the plasma. In contrast, the gain does not noticeably change with gas pressure if the discharge current density is kept constant (figure 4). The power density increases with pressure just slightly, due to an increase in gas temperature that leads to higher gas density and electric field in plasma. The temperature increase also leads to a greater magnitude of speed of sound that causes K to fall. A combination of these two factors results in incremental lessening of the gain with pressure.

In conclusion, the model for acoustic wave propagation in a medium where local energy addition is a function of gas density that was developed here adequately described the experimentally observed effect of sound amplification in low-pressure glow discharges in air and molecular gases. The analysis of a practically important case of glow discharges in air at pressures between 10 and 50 Torr demonstrates that gains (or attenuation coefficients) of as high as 1 m^{-1} are achievable. For a plasma column as long as 1 m and a 60 dB acoustic wave, this corresponds to amplitude amplification (reduction) of 9 dB.

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