

# Properties of the strongly correlated two-dimensional electron gas in Si MOSFET's

B. Spivak

*Physics Department, University of Washington, Seattle, Washington 98195*

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We discuss properties of the strongly correlated two-dimensional electron gas in Si metal-oxide-semiconductor field-effect transistor at low concentrations assuming that the electron liquid is close to crystallization. Analogy with the theory of  $^3\text{He}$  is emphasized.

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## I. INTRODUCTION

Recent experiments<sup>1-10</sup> in high-mobility two-dimensional electron systems indicate that in the presence of disorder, they cannot be described by the conventional single-particle localization theory.<sup>11</sup> A number of unusual features have been reported taking place in the dependence of kinetic coefficients on the magnetic field  $H$  and temperature  $T$ . They are observed at low  $T$  and relatively small electron concentration  $n$ . This regime corresponds to  $r_s = E_p/E_F \gg 1$ , where  $E_p = e^2 p_F / \epsilon \hbar$  is the potential energy per one electron,  $E_F = p_F^2 / 2m$  is the Fermi energy,  $\epsilon$  is the dielectric constant,  $m$  is the bare electron mass, and  $p_F$  is the Fermi momentum. In this paper we give a qualitative interpretation of the experimental results assuming that the electron liquid is close to Wigner crystallization. Our analysis is based on an analogy with the theory of  $^3\text{He}$ . We would like to explain the following experimentally observed facts.

(i) The electron system exhibits a "transition" as a function of  $n$  from a metallic phase, where the resistivity  $\rho$  of the system saturates at low temperatures, to an insulating phase, where the resistivity increases as  $T$  decreases. The value of the critical concentration  $n_c$  depends on the amount of disorder in the sample and corresponds to  $r_s = r_s^c \gg 1$ . On the metallic side of the transition at  $T=0$  and  $n \approx n_c$ , the value of the resistance is of order  $\hbar/e^2$ .

(ii) At  $T=0$  and electron concentration sufficiently close to critical, magnetic field parallel to the film  $H_{\parallel}$  suppresses the metallic phase and drives the system toward the insulating phase.<sup>3,5,12</sup> Thus the critical metal-insulator concentration  $n_c(H_{\parallel})$  increases with  $H_{\parallel}$ .

In the metallic phase [ $n > n_c(H_{\parallel}=0)$ ] and at small  $T$  the system exhibits a big positive magnetoresistance as a function of  $H_{\parallel}$ . This magnetoresistance saturates at  $H_{\parallel} \geq H_{\parallel}^c(n)$  and  $\rho(H_{\parallel}^c)/\rho(0) \gg 1$ .<sup>3,5</sup>

(iii) In the dielectric phase the system exhibits a big positive magnetoresistance as a function of  $H_{\parallel}$ , which saturates at  $H_{\parallel} > H_{\parallel}^c$ . However, at given  $H_{\parallel} > H_{\parallel}^c$  the conductance of the system exhibits a big negative magnetoresistance as a function of the component of the magnetic field  $H_{\perp}$  perpendicular to the film.<sup>4</sup>

(iv) In the metallic phase at  $H_{\parallel}=0$  and  $T < E_F$  the resistance  $\rho(T)$  significantly increases with temperature. The characteristic value of  $d \ln \rho / dT > E_F^{-1}$  at small  $T$  is large and depends on the value of  $n - n_c$ .

(v) If at  $H_{\parallel} > H_{\parallel}^c$  the system is still in the metallic phase

[ $n > n_c(H_{\parallel})$ ], the  $T$  dependence of the resistance is much smaller than in the  $H_{\parallel}=0$  case [ $d \ln \rho(H_{\parallel} > H_{\parallel}^c) / dT \ll E_F^{-1}$ ].<sup>8,9</sup>

(vi) In the metallic regime in magnetic fields of order  $H_{\parallel}^c$  or less, the system exhibits an anisotropy: The magnetoresistance in the parallel field is larger than the magnetoresistance in the perpendicular field.<sup>4</sup>

## II. A COMPARISON OF LOW-TEMPERATURE PROPERTIES OF $^3\text{He}$ AND THE ELECTRON LIQUIDS

We believe that the physics of the strongly correlated electrons in Si metal-oxide-semiconductor field-effect transistors (MOSFET's) at  $r_s \gg 1$  is quite similar to the low-temperature physics of  $^3\text{He}$ , despite the difference in interactions. Indeed, in the absence of disorder, both systems exhibit a quantum zero-temperature liquid-crystal transition. In both cases the transition is believed to be of the first order. The liquid phase near the transition in both systems is characterized by a large ratio of the potential to kinetic energy. In the  $^3\text{He}$  liquid the ratio between the Debye energy  $\Omega_D$  and the Fermi energy reaches 80 near the melting pressure.<sup>13</sup> On the other hand, it is known, from numerical simulations,<sup>14</sup> that in the two-dimensional case, the Fermi liquid Wigner crystal transition takes place at a relatively large  $r_s = r_s^c \sim 38$ . Therefore, in both systems, there are wide regions of concentrations where the systems are in the liquid phases, though the potential energy is already much larger than the kinetic one.

The only qualitative difference between these systems is that  $^3\text{He}$  crystallizes upon increasing the atomic concentration, while the electron system crystallizes when the concentration is lowered.

A model that describes properties of liquid  $^3\text{He}$  near the crystallization pressure has been introduced in Refs. 13,15,16. It is based on the existence of the strong inequality  $\hbar \Omega_D \gg E_{F(\text{He})}$  which indicates that the  $^3\text{He}$  liquid can be regarded as a system that is close to a solid.<sup>13,15,17</sup> Since the system has strong short-range crystalline order correlations, on a fast time scale,  $\hbar/\Omega_D$ , the atoms vibrate near certain positions in their own cages. The system is locally rigid and can resist stress. On the much slower time scale  $\tau_{(\text{He})} \gg \Omega_D^{-1}, \hbar/E_{F(\text{He})}$ , the cages start to drift around each other, leading to a plastic deformation expected in a liquid. Here  $E_{F(\text{He})}$  is the Fermi energy of liquid  $^3\text{He}$ . The characteristic time  $\tau_{(\text{He})}$  is associated with a collective rearrangement of "equilibrium" particle configurations due to many-particle

quantum tunneling.<sup>15,16</sup> It determines all low-temperature Fermi-liquid parameters of the systems. At  $t > \tau_{(\text{He})}$ , the quantum-mechanical indistinguishability of particles plays a decisive role in determining the ground state of the system. The energy uncertainty  $\hbar/\tau_{(\text{He})} = E_{d(\text{He})} \ll E_{F(\text{He})}$  in this case, determines the order of magnitude of the temperature of the quantum degeneracy  $E_{d(\text{He})}$ . The quasiparticle effective mass  $m_{(\text{He})}^*$  can be introduced in the usual way  $E_{d(\text{He})} = p_{F(\text{He})}^2/2m_{(\text{He})}^*$ . It can be extracted from the temperature dependence of the electron heat capacity. Accordingly, at  $T = 0$ , both the effective mass  $m_{(\text{He})}^* \sim \hbar \tau_{(\text{He})} n \gg m_{(\text{He})}$ , and the magnetic susceptibility of the liquid  $\chi_{L(\text{He})} \sim n_{(\text{He})} \mu_N^2 \tau_{(\text{He})} / \hbar \gg \chi_{0(\text{He})}$  are significantly enhanced as compared to their values  $\chi_{0(\text{He})}$  and  $m_{(\text{He})}$  in a noninteracting liquid.<sup>15,16</sup> Here  $\mu_N$  and  $n_{(\text{He})}$  are the nuclear Bohr magneton and the concentration, respectively. The latter formula corresponds to the Curie susceptibility with  $T = \hbar / \tau_{(\text{He})}$ . It reflects the fact the particles change their equilibrium positions on the time scale  $\tau_{(\text{He})}$ .

According to the picture of the liquid, which is nearly solid,<sup>13</sup> not only the linear spin susceptibility, but also all nonlinear ones should be large. It means that the complete polarization of the electron liquid can be achieved at relatively low magnetic fields  $\mu_N H^c \sim E_{d(\text{He})} < E_{F(\text{He})}$ .

Furthermore, near the crystallization point, the magnetic susceptibility of the liquid  ${}^3\text{He}$  is much smaller than the susceptibility of the solid  ${}^3\text{He}$ . Therefore the energy density of the solid decreases with  $H$  faster than the energy density of the liquid. This means that the magnetic field drives the system toward the crystallization.<sup>13,17</sup> By itself it does not mean that the effective mass of quasiparticles  $m_{(\text{He})}'(H)$  increases with  $H$ . However, the calculations in the framework of the Hubbard model showed<sup>18</sup> that it is the case and  $m_{(\text{He})}^*(H) - m_{(\text{He})}^*(0) > 0$ , which means that  $\tau_{(\text{He})}(H)$  is an increasing function of  $H$ .

Let us now discuss the  $T$  dependence of the viscosity of the  ${}^3\text{He}$  liquid  $\eta_{(\text{He})}(T)$ . At low temperatures  $T \ll E_{d(\text{He})}$  the viscosity  $\eta_{(\text{He})}(T)$  of liquid  ${}^3\text{He}$  is given by the Fermi-liquid theory  $\eta_{(\text{He})} \sim T^{-2}$ .<sup>19</sup> There is, however, a wide temperature interval  $E_d < T < \Omega_D$  where the liquid is nondegenerate already, but still it is strongly correlated. Surprisingly enough the experimental data for the  $T$  dependence of  $\eta_{(\text{He})}$  in this temperature interval are unavailable. Theoretically this dependence was considered in Refs. 15,16 where it was argued that in this temperature interval  $\eta \sim T^{-1}$ .

Let us turn now to the case of the electron liquid. If  $r_s \gg 1$ , we have  $\Omega_p \gg E_F$ , where  $\Omega_p = (4\pi e^2 n^{3/2}/m)^{1/2}$  is the plasma frequency at the wavelength of order of the interelectron distance. Thus the discussed above model of strongly correlated liquid can be applied to the electron liquid as well. Namely, one can view the electron liquid as a system that is close to the Wigner crystallization. Due to existence of the short-range crystalline order, effective electron mass  $m^* = \hbar n \tau$  and the spin susceptibility  $\chi = \mu^2 \tau n / \hbar$  are enhanced compared to the noninteracting Fermi gas case, where  $\mu$  is the Bohr magneton. The enhancement of both quantities is governed by the parameter  $\tau \gg \Omega_p^{-1}$ . Furthermore, since zero-temperature spin susceptibility of the Wigner crystal  $\chi_S$

is larger than the susceptibility of the Fermi liquid  $\chi_L$ , the energy density of the Wigner crystal  $E_S(H_{\parallel}) = E_S(0) - \chi_S H_{\parallel}^2$  decreases with  $H_{\parallel}$  faster than the energy density of the Fermi liquid  $E_L(H_{\parallel}) = E_L(0) - \chi_L H_{\parallel}^2$ . This means that the magnetic field parallel to the film, drives the electron system toward the crystallization.<sup>13,17</sup> An estimate for the change of the critical electron concentration is

$$n_c(H_{\parallel}) - n_c(0) = \frac{(\chi_S - \chi_L) H_{\parallel}^2}{\nu_L - \nu_S} > 0. \quad (1)$$

Here  $\nu_L$  and  $\nu_S$  are chemical potentials of the liquid and crystal, respectively. By analogy with the theory of  ${}^3\text{He}$ , we can assume that near the crystallization point the effective quasiparticle mass in the liquid  $m^*(H)$  and the parameter  $\tau(H)$  are increasing functions of  $H$ .

In concluding this section, we would like to remind that the existence of two different energy scales is not a unique property of the quantum liquids near the solidification point. It has been known since Frenkel<sup>20</sup> that melting of ordinary classical liquids occurs at temperatures much smaller than the interaction energy between atoms. As a result, following Ref. 20, the viscosity of classical liquids is governed by thermal activation of the over-the-barrier processes and decreases with  $T$  exponentially.

To describe the temperature and the magnetic-field dependence of the resistance of the system we have to consider the electron system in the presence of random elastic scattering potential. Below we consider two limiting cases: a model of a potential slowly varying in space, and a case when the potential is modeled by short-range randomly distributed scatterers.

### III. THE CASE OF A SMOOTH SCATTERING POTENTIAL

Let us consider a model where the fluctuations of the external potential of a relatively small amplitude are smooth functions of coordinates. If the electron concentration  $n$  is close to the critical  $n_c$ , the system can get split into the regions of a Fermi liquid and a Wigner crystal. This model can explain the following experimental facts.

(i) The fractions of volume occupied by the Fermi liquid and the Wigner crystal depend on  $n$  and therefore the system should exhibit a percolation type zero-temperature metal-insulator transition as  $n$  decreases, and the area occupied by the Wigner crystal grows.

(ii) Since  $\chi_S \gg \chi_L$ , the magnetic field parallel to the film, drives the electron system toward the crystallization<sup>13,17</sup> and the fraction of volume occupied by the Wigner crystal increases with increasing  $H_{\parallel}$ . This leads to a big positive magnetoresistance as a function of  $H_{\parallel}$ . The magnetoresistance should saturate when  $H_{\parallel} > H_{\parallel}^c$ , and the electron Fermi liquid is polarized. The saturation field  $H_{\parallel}^c \sim E_d / \mu < E_F$  decreases with decreasing  $n$ . This conclusion is in agreement with the experiment.<sup>10</sup>

Depending on the value of  $n$  and the amplitude of the scattering potential at  $H_{\parallel} > H_{\parallel}^c$  the system could be either in the metallic or in the dielectric regime. If  $n < n^c(H_{\parallel}^c)$ , the system at  $H_{\parallel} > H_{\parallel}^c$  is in the insulating regime. It exhibits a

giant positive magnetoresistance, which corresponds to decreasing of the localization radius with increasing  $H_{\parallel}$ .

(iii) Consider the case  $n < n_c(H_{\parallel}^c)$ , when at  $H_{\parallel} > H_{\parallel}^c$  the system is in the insulating phase. The model presented above can also explain why, in the presence of the parallel magnetic field  $H_{\parallel} > H_{\parallel}^c$ , the system exhibits a big negative magnetoresistance as a function of the component of the magnetic field  $H_{\perp}$  perpendicular to the film. Indeed at  $H_{\parallel} > H_{\parallel}^c$ , electron spins are polarized and the problem of the magnetoresistance is of the single-particle nature. It was shown that at relatively small magnetic field when  $L_{H_{\perp}} \gg \xi$ , it is dominated by an interference of direct tunneling paths.<sup>21–23</sup> Here  $L_{H_{\perp}} = \sqrt{c\hbar/eH_{\perp}}$  is the magnetic length and  $\xi$  is the localization radius. In this case the magnetoresistance is shown to be negative and the  $H_{\perp}$  dependence of the resistance corresponds to a correction to the localization radius  $[\xi(H_{\perp}) - \xi(0)] \sim \xi^2(0)L_{H_{\perp}}^{-1}$ . Existence of such an interference effect depends on the spin structure of the system ground state.<sup>22,24</sup> At  $H_{\parallel} = 0$ , the tunneling is a collective process that involves an interchange of positions of electrons with different spins. As a result, the final states of the system, which correspond to different tunneling paths, have different spin configurations and, therefore, they are orthogonal. As a result, the single-particle interference mechanism of the negative magnetoresistance can be significantly suppressed in the case where  $H_{\parallel} = 0$ .

(iv) The significant increase of the resistance as a function of temperature can also be explained naturally in the framework of the presented above model as a consequence of the Pomeranchuk effect. Indeed, if temperature is not significantly smaller than the spin exchange energy in the Wigner crystal, the spin entropy of the Wigner crystal is larger than the entropy of the Fermi liquid. This means that the Wigner crystal regions grow with increasing temperature.

(v) The Pomeranchuk effect disappears when  $H_{\parallel} > H_{\parallel}^c$  and electron spins are polarized. In this case the entropies of both the liquid and the solid are much smaller than the spin entropy of the crystal at  $H_{\parallel} = 0$ . This means that in the first approximation the areas occupied by the crystal and the liquid are  $T$  independent. This explains the fact that in the metallic state at  $H_{\parallel} > H_{\parallel}^c$ , the  $T$  dependence of the resistance is much smaller than in the case  $H_{\parallel} = 0$ .<sup>8,9</sup>

(vi) The question about the origin of the anisotropy of the magnetoresistance in the metallic phase is open. The fact that the magnetoresistance in the parallel field is larger than the magnetoresistance in the perpendicular field, implies that the orbital effects are insignificant and the magnetoresistance is determined by the spin magnetization of the system. A possible explanation of the large anisotropy of the spin magnetization induced by the magnetic field can be related to the existence of the Rashba term in a one-particle spectrum of electrons

$$\alpha \mathbf{p}[\boldsymbol{\sigma} \times \mathbf{n}]. \quad (2)$$

Here  $\mathbf{n}$  is a unit vector normal to the conducting plane and  $\boldsymbol{\sigma}$  is the vector of Pauli matrices and  $\mathbf{p}$  is the electron momentum. A reliable estimate for the value of  $\alpha$  in Si-MOSFET's

is unavailable at the moment. A rough estimate<sup>25</sup> implies that the magnitude of Eq. (2) can be just several times less than the electron Fermi energy. We would like to note, however, that (a) in the presence of the electron-electron interaction, the degeneracy energy is renormalized ( $E_d < E_F$ ), and (b) the electron-electron interaction renormalizes the factor  $\alpha$  in Eq. (2).<sup>26</sup> It is natural to expect that the latter renormalization is determined by the parameter  $\tau\Omega_p \gg 1$ . Both effects lead to increasing of the relative amplitude of the Rashba term. We also would like to mention that the spin-orbit interaction can manifest itself also in the in-plane anisotropy when the resistance depends on the relative orientation of the current and the magnetic field parallel to the plane.<sup>12,27</sup>

At last we would like to mention that the resistance of order  $\hbar/e^2$  is a typical feature of the percolation transition. (See, for example, Ref. 28.) The difference between the model presented above and Ref. 28 is in the many-particle nature of the transition which, for example, makes it sensitive to the parallel magnetic field.

A difficulty associated with the model of smooth potential is the following. In the case of a small amplitude of the fluctuations of the scattering potential at  $T = 0$ , the transition should take place at  $r_s = r_s^c \sim 38$ , which is significantly larger than the experimental value  $r_s^c = 10 - 20$ . On the other hand, if the amplitude of the potential is large, in principle, a percolation type transition can take place at any  $r_s$ . In the latter case, however, the system breaks into three phases: insulator, Wigner crystal, and the Fermi liquid. One can neglect the difference in compressibilities of the Fermi liquid and the Wigner crystal. As a result, the electrostatic analysis shows that the electron density should increase as a square root of the distance from the boundary.<sup>29</sup> Then the assumption that the liquid-crystal transition takes place at  $r_s = 38$ , leads to a conclusion that the fraction of the area occupied by the Wigner crystal is numerically small and, therefore, at  $r_s \sim 10 - 20$ , the discussed above  $T$  and  $H$  dependencies of the resistance should not be pronounced. A possibility to have  $r_s^c$  less than 38 in the presence of a short-range scattering potential, is considered in the next section.

We would also like to mention that even in the framework of the model, presented above the question whether there is a well-defined transition or just a sharp crossover between metallic and insulating phases, is open. Indeed, in the two-dimensional case an arbitrarily small disorder destroys the first-order phase transition.<sup>30</sup> The question of whether in this case the first-order Fermi liquid Wigner crystal transition is transformed into a second-order one, or is destroyed completely, is open. (See a discussion of this question in Ref. 31.)

The question about possible localization in the “metallic phase” in the presence of a weak disorder is also open. All existing theories of the localization are perturbative in electron-electron interaction. At  $r_s \gg 1$  the potential energy is much larger than the kinetic energy and these theories are not reliable especially in the case  $r_s \gg 1$  when the renormalized mass is big, which means that the size of electron packets that carry current should be large as well. It may be that the localization length in the metallic phase is larger than the sample size.

#### IV. SHORT-RANGE SCATTERING POTENTIAL

In this section we show that (a) in the presence of a short-range potential the Wigner crystallization takes place at  $r_s < 38$  and (b) one can explain the above-mentioned experiments using the fact that the electron liquid is nearly solid, but without involving the Wigner crystal state.

Suppose that there is an impurity with a short-range potential of a radius  $a \sim n^{-1/2}$ , which is embedded into the metal with  $r_s \gg 1$ . Let us now discuss the  $r_s$  and  $H_{\parallel}$  dependencies of the two-dimensional cross section of the quasiparticle elastic scattering on impurities  $A(T=0)$  at zero temperature. It has been pointed out<sup>32</sup> that due to the existence of the short-range crystalline order in the Fermi liquid with  $r_s \gg 1$ , a large characteristic length  $\tau\Omega_p n^{-1/2} \gg n^{-1/2}$  should exist, where the liquid behaves like a solid. Since the impurity pins the liquid, the value of  $A$  should increase with  $\tau$  as well

$$A(T=0) \sim a \frac{m^*}{m} \gg a. \quad (3)$$

Therefore, it is plausible that the transition takes place when the impurity concentration  $N_i$  equals to a critical one

$$N_{ic} = \frac{1}{A^2} \sim \frac{1}{a^2} \left( \frac{m}{m^*} \right)^2 \quad (4)$$

and strongly correlated regions near impurities overlap. Thus, in disordered samples, the transition takes place at  $r_s \ll 38$  and the critical electron concentration decreases with increasing the amount of disorder. This is in agreement with the experimental observation<sup>6,7</sup> that the critical concentration  $n_c$  is lower in samples with higher mobility. We would like to note that numerical simulations in the presence of disorder give a critical value of  $r_s$  for the Fermi liquid Wigner crystal transition, which is significantly smaller than 38.<sup>33,34</sup>

As it has been mentioned above that both  $\tau(H_{\parallel})$  and  $m^*$  are increasing functions of  $H_{\parallel}$ . As a result, the zero-temperature resistance of the system

$$\rho(T=0) = \frac{\hbar}{e^2} \frac{N_i A(T=0)}{n^{1/2}} \quad (5)$$

increases with  $H_{\parallel}$  and saturates at  $H_{\parallel} > H_{\parallel}^c$ .

Temperature corrections to Eq. (1) should be small as long as  $T \ll T_d$ . Since processes of electron-electron scattering in semiconductors conserve the total momentum, the  $T$  dependence of the resistivity of the system can be only due to  $T$  dependence of  $A(T)$ .

It has been shown<sup>35,36,38</sup> that at low temperatures the resistance of the system increases linearly with  $T$ . This effect is due to the temperature dependence of the Friedel oscillations induced by impurities. If  $r_s > 1$ , the value of the scattering cross section turns out to be an increasing function of  $H_{\parallel}$  as well.<sup>35</sup> This mechanism, in principle, could explain qualitatively  $T$  and  $H_{\parallel}$  dependencies of the resistance of the metallic state. It is hard, however, to explain in the framework of<sup>35-37</sup> why at  $H_{\parallel} > H_{\parallel}^c$  the temperature dependence of the resistance

in the metallic state is much smaller than at  $H=0$ , and why the magnetoresistance in the dielectric state with respect to  $H_{\perp}$  is negative. It may be, though, that the mechanism of the  $T$  dependence considered in Refs. 35-37,38 is relevant in the case of GaAs samples.

Let us now discuss the  $T$  dependence of the resistance in the temperature interval  $T_d < T < \Omega_p$ . Though, in this case, the liquid is not degenerate, it is strongly correlated. Therefore the electron-electron scattering in the liquid is very effective and the local equilibrium is reached in a short time on a spatial scale of order  $n^{-1/2}$ . As a result, the flow of the liquid near an impurity can be considered in the framework of hydrodynamics. In the two-dimensional case the electron liquid exerts a force on an impurity given by the Stokes formula  $F \sim \eta u / \ln(\eta/nua)$ , where  $u$  and  $\eta$  are the liquid hydrodynamic velocity and viscosity of the electron liquid, respectively. In a system with a finite concentration of impurities, the logarithmic factor in the equation for  $F$  should be substituted for  $\ln(1/aN_i^{1/2})$ . Thus the resistance of the system is

$$\rho(T) \sim \frac{N_i \eta(T)}{e^2 n^2} \ln^{-1} \frac{1}{N_i^{1/2} a}. \quad (6)$$

Strictly speaking the hydrodynamic approach is valid when  $a > l_{ee}$ . Here  $l_{ee}$  is the electron-electron mean-free-path. However, since  $a$  dependence in Eq. (6) is only logarithmic, the estimate Eq. (6) is also valid in the case of semiquantum liquid when  $a \sim l_{ee} \sim n^{-1/2}$ . The logarithmic factors in Eq. (6) is associated with the Stokes paradox in the two-dimensional case.<sup>39</sup> It is not universal. For example, electron-phonon scattering will change it significantly. We will neglect this factor estimating the resistance of the system by the order of magnitude.

The essential ingredient of the picture that leads to the Stokes formula for  $F$  and to Eq. (6) is that the hydrodynamic velocity near impurities is significantly reduced as compared to its bulk value. This is the reason why the conventional description of the two-dimensional electron system with the help of the Boltzmann kinetic equation containing the electron-electron and the electron-impurity scattering integrals<sup>40</sup> would give a result completely different from Eq. (6). In the former case the electron distribution function is spatially uniform and, therefore, the resistance of the system is independent of  $l_{ee}$  and proportional to the electron-impurity scattering rate.

The viscosity of the electron liquid in the semiclassical regime can be estimated in a way similar to Refs. 15,16. On a time scale smaller than  $\tau$ , the structure of the low-energy excitations in the liquid is similar to the structure of excitations in glasses.<sup>41</sup> At  $T \ll \hbar\Omega$ , the liquid excitations are two-level systems with the density-of-states per particle  $\nu_0 \sim U^{-1}$ . In the case of the electron liquid,  $U \sim e^2 \epsilon^{-1} n^{1/2}$  is the typical interelectron interaction energy. As a result,<sup>15</sup> contrary to the gas case, the viscosity of the liquid  $\eta \sim \hbar n \nu_0 V^2 / T$  decreases with increasing temperature. Here  $V$  is the typical matrix element of the transition between the states in the two level systems. Making a natural assumption that  $V \sim \hbar\Omega_p$  we get

$$\eta(T) \sim \frac{T_d}{T} \frac{m^*}{m} \hbar n. \quad (7)$$

Thus, we arrive at the conclusion that at  $T > T_d$  the resistance associated with the impurity scattering

$$\rho(T) \sim \frac{\hbar}{e^2} \frac{N_i}{n} \frac{m^*}{m} \frac{T_d}{T} \quad (8)$$

should decrease with increasing  $T$ . Therefore,  $\rho(T)$  may have a maximum at  $T \sim T_d$ . At  $T=0$  and in the “metal” regime [ $N_i A^2(T=0) \ll 1$ ] contributions from different impurities into  $\rho(T)$  are independent and

$$\rho(T=0) = \frac{\hbar}{e^2} \frac{N_i A(T=0)}{n^{1/2}}. \quad (9)$$

Thus at  $T=T_d$ , Eq. (4) matches the zero-temperature value of the impurity scattering cross section [Eq. (3)].

## V. CONCLUSION

We would like to mention several consequences of the model presented above that can be checked experimentally.

(i) In the metallic phase at  $r_s \gg 1$  the thermopower, normalized by  $\rho(T=0)$ , should be relatively large due to the strong enhancement of the effective mass  $m^*/m \gg 1$ . Also, the thermopower should be a strongly increasing function of  $H_{\parallel}$  because of the corresponding increase in  $m^*(H_{\parallel})$ . In principle, large thermopower in a metal can also occur due to localized spins present in the sample and the Kondo effect associated with them. However, in the latter case, the ther-

mopower would be a decreasing function of  $H_{\parallel}$ .

(ii) The tunneling density-of-states in the metallic regime with  $r_s \gg 1$  is reduced by the factor  $m^*(H_{\parallel})/m \gg 1$ , compared to that of noninteracting electrons. The tunneling density-of-states will be additionally suppressed by the magnetic field parallel to the film.

(iii) In the Wigner crystal state, as well as in the liquid with  $r_s \gg 1$ , the electron compressibility is negative, whereas at  $r_s \ll 1$ , it is positive.<sup>42</sup> The magnetic field applied parallel to the plane drives the system toward the Wigner crystal state, and thus it should make the compressibility of the “metallic state” at  $38 \gg r_s \gg 1$  even more negative.

(iv) Following the picture presented above the saturation of the magnetoresistance in the metallic phase in the parallel magnetic field takes place at  $H_{\parallel} > H_{\parallel}^c$  when the electron Fermi liquid gets polarized. On the other hand, the Pomeranchuk effect disappears when the Wigner crystal is polarized. This means that the significant temperature dependence of the metallic state may disappear in a much smaller magnetic field  $H_{\parallel} \sim T/\mu \ll H_{\parallel}^c$ .

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