Structure of a superconducting vortex pinned by a screw dislocation

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Spatial dependence of the magnetic field and the superconducting current in a flux line pinned by a screw dislocation are computed. Interaction of a superconducting vortex with the chiral-symmetry breaking elastic strain of a screw dislocation results in a helical current along the axis of the dislocation. It is argued that screw dislocations make impossible a force-free arrangement of flux lines in the presence of a transport current.

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Pinning of vortices by screw dislocations can be of interest because of the experimental evidence of this effect in Y–Ba–Cu–O.[1–4] Studies of pinning by dislocations are almost as old as the Abrikosov's idea of superconducting vortices (see, e.g., the review by Campbell and Evetts[5] and references therein). More recently, these studies have been revived in connection with high-temperature superconductors.[6] They are normally based upon the expansion of scalar parameters of a superconductor, such as, e.g., T_c, in terms of the deformation tensor and its trace.

A superconducting vortex coupled to a screw dislocation is an interesting theoretical problem because of the broken chiral symmetry of the deformation field produced by such a dislocation. This makes possible a linear relation between the polar vector of the current density $\mathbf{j}$ and the axial vector of the magnetic field $\mathbf{B}$: $\mathbf{j}\propto\mathbf{B}$, the relation that would be otherwise prohibited by the invariance with respect to reflections.

A closely related problem of a vortex coupled to a spiral defect has been studied by Ivlev and Thompson[7,8] for an extreme case of a layered superconductor with a Josephson coupling between the layers. In such a case, a spiral defect, running perpendicular to the layers, geometrically connects them by a continuous helical path around the defect. Ivlev and Thompson elegantly solved this problem in spiral coordinates and, in accordance with the above symmetry arguments, demonstrated the existence of a ''fountainlike'' current along the axis of the defect.

The purpose of this paper is to solve the problem in the opposite extreme case of a flux line coupled to a screw dislocation in an isotropic three-dimensional superconductor. In such a case the existence of longitudinal currents parallel to the dislocation core is somewhat less obvious. The model we suggest is complementary to the model of Ivlev and Thompson. It is based upon the description of dislocations within continuous elastic theory. We find that superconducting currents do flow along screw dislocations in three-dimensional superconductors, although the spatial dependence of these currents is different from that found in Ref. 7. Our results should be directly relevant to MgB_2 and conventional isotropic superconductors. Y–Ba–Cu–O, where the coupling between vortices and screw dislocations has been experimentally observed, falls somewhere between the two models.

The free energy of the system is $\mathcal{F}=\mathcal{F}_D+\mathcal{F}_N+\mathcal{F}_{GL}$, where $\mathcal{F}_D$ is the energy of the dislocation, $\mathcal{F}_N$ is the energy of the normal electron liquid in the absence of the magnetic field, and $\mathcal{F}_{GL}$ is the Ginzburg–Landau free energy,

$$\mathcal{F}_{GL}=\int d^3r\left[\frac{(\boldsymbol{\nabla}\times\mathbf{A})^2}{8\pi}\right]+\int d^3r\left[a|\psi|^2+\frac{b}{2}|\psi|^4+\frac{\hbar^2}{4}(D_i\psi)m_{ij}^{-1}(D_j\psi)^*\right]. \quad (1)$$

Here $\mathbf{A}$ is the vector potential ($\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$), $\psi=|\psi|\exp(i\phi)$ is the complex order parameter of the superconducting phase, a and b are constants, D_i is the gauge-invariant derivative,

$$D_i=\nabla_i-\frac{2ie}{\hbar c}A_i, \quad (2)$$

and m_{ij} is the tensor of effective masses. For an isotropic superconductor, in the absence of crystal defects, $m_{ik}=m\delta_{ik}$.

The presence of a dislocation results in a nonzero elastic strain,

$$u_{ij}=\tfrac{1}{2}(\nabla_i u_j+\nabla_j u_i). \quad (3)$$

At distances exceeding a few lattice spacings from the dislocation core, the components of the dimensionless tensor u_{ij} are small[9] and the parameters of the superconductor, such as a,b, and m_{ij}^{-1}, can be expanded into the power series of u_{ij}. We shall see that for a screw dislocation $\mathrm{Tr}(u_{ij})=0$, that is, screw deformations change the symmetry of the crystal but not the local density. Thus to the lowest order in u_{ij}, the interaction of the screw dislocation with the Ginzburg–Landau order parameter can be introduced by the substitution

$$m_{ij}^{-1}\rightarrow\frac{1}{m}(\delta_{ij}+gu_{ij}), \quad (4)$$

where g is a dimensionless parameter. From the physics of the electron states in crystals, it is clear that the effect of the lattice deformations on the tensor of effective masses is small as long as u_{ij} are small. If some of u_{ij} are large, the effect should be also large. Consequently, g must be of order unity.

Since crystal defects are insensitive to superconductivity, $\mathcal{F}_{\rm GL}$ must have a very weak dependence on ψ. We will neglect that dependence. Then the variation of $\mathcal{F}_{\mathrm{GL}}$ with respect to ψ and $\mathbf{A}$ gives the following two Ginzburg–Landau equations:

$$-\frac{\hbar^2}{4m}D_i g_{ij} D_j \psi + a\psi + b|\psi|^2\psi = 0 \tag{5}$$

and the Maxwell equation $\nabla\times\mathbf{B}=(4\pi/c)\mathbf{j}$ with

$$j_i = -g_{ij}\left[\frac{ie\hbar}{2m}(\psi^*\nabla_j\psi - \psi\nabla_j\psi^*) + \frac{2e^2}{mc}A_j|\psi|^2\right], \tag{6}$$

where $g_{ij}=\delta_{ij}+gu_{ij}$.

Equation (5) describes the vortex core where the concentration of Cooper pairs, $|\psi|^2$, changes from zero at the center of the vortex to a constant value, $|a|/b$, at distances exceeding the coherence length $\xi=\hbar/2(m|a|)^{1/2}$. At such distances the second Ginzburg–Landau equation becomes

$$\lambda_L^2 g_{ij}^{-1}(\nabla\times\mathbf{B})_j + A_i = \frac{\Phi_0}{2\pi}\nabla_i\phi, \tag{7}$$

where ϕ is the phase of ψ, $\Phi_0=hc/2e$ is the flux quantum, and $\lambda_L=(mc^2/8\pi e^2|\psi|^2)^{1/2}$ is the London penetration depth. We shall consider the case of $\lambda_L\gg\xi$ which is relevant to high-temperature superconductors. At a large distance from the vortex core, where $\mathbf{B}$ exponentially goes to zero, Eq. (7) reduces to $\mathbf{A}=(\Phi_0/2\pi)\nabla\phi$. After the integration over a closed distant contour enclosing the vortex, it produces the conventional condition of the quantization of the magnetic flux. This condition remains unchanged by the deformations. Applying curl to both sides of Eq. (7), one obtains a modified London equation

$$\lambda_L^{-2}B_i + \epsilon_{ijk}\nabla_j g_{kl}^{-1}(\nabla\times\mathbf{B})_l = 0 \tag{8}$$

that is valid outside the vortex core.

We can now turn to the problem of a flux line centered at a screw dislocation parallel to the Z-axis of the crystal. It should be naturally studied in circular cylindrical coordinates: z,r,φ. At distances, r, exceeding a few lattice spacings from the core of the screw dislocation, the only nonzero component of the displacement field $\mathbf{u}$ is[9]

$$u_z = pb\frac{\varphi}{2\pi}, \tag{9}$$

where $p=\pm1$ is the chirality of the screw dislocation and b is the Burgers vector that coincides with the lattice spacing in the Z-direction. Consequently, the only nonzero components of the elastic strain are

$$u_{z\varphi} = u_{\varphi z} = p\frac{b}{4\pi r}. \tag{10}$$

Although the linear elastic theory fails near the axis of the dislocation, for the purpose of our study the above formulas are exact as long as the coherence length ξ and the London penetration depth λ_L are greater than b. As shown below, the significant effect of the dislocation on the superconducting vortex comes from the fact that the elastic strain given by Eq. (10) is long range.

It is convenient to introduce a dimensionless coordinate $\rho=r/\lambda_L$ and a dimensionless small parameter $\beta=gb/4\pi\lambda_L$. Substituting Eq. (10) into Eq. (8), and retaining terms of the lowest order in β, one obtains the following two equations for nonzero components of the magnetic field $B_z(\rho)$ and $B_\varphi(\rho)$:

$$B_z = \frac{1}{\rho}\frac{d}{d\rho}\left[\rho\frac{dB_z}{d\rho} + \frac{p\beta}{\rho}\frac{d}{d\rho}(\rho B_\varphi)\right], \tag{11}$$

$$B_\varphi = \frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}(\rho B_\varphi) + \frac{p\beta}{\rho}\frac{dB_z}{d\rho}\right]. \tag{12}$$

For a flux line parallel to Z, the condition $\beta\ll1$ results in $B_\varphi\ll B_z$ for all $r>\xi$. Consequently, the effect of the dislocation on Eq. (11) can be neglected and the system of equations for $B_z(\rho)$ and $B_\varphi(\rho)$ can be solved by iteration. Putting $\beta=0$ in Eq.(11) one obtains a conventional solution for the Z-component of the magnetic field inside the flux line,

$$B_z(\rho) = \frac{\Phi_0}{2\pi\lambda_L^2}K_0(\rho), \tag{13}$$

where K_0 is the modified Bessel function and the coefficient in front of K_0 is such that the total magnetic flux through the XY-plane equals Φ_0. Then Eq. (12) can be reduced to the following form:

$$\frac{d}{d\rho}\left(\rho\frac{dB_\varphi}{d\rho}\right) - \left(\rho + \frac{1}{\rho}\right)B_\varphi = -\frac{p\beta\Phi_0}{2\pi\lambda_L^2}K_2(\rho). \tag{14}$$

The solution of this equation, that goes to zero at $r\to\infty$, is

$$B_\varphi(\rho) = \frac{p\beta\Phi_0}{4\pi\lambda_L^2}\left[CK_1(\rho) + K_1(\rho)\ln(\rho) - \frac{1}{\rho}K_0(\rho)\right], \tag{15}$$

where C is a constant of integration (to be computed later).

The current density in the flux line is given by $\mathbf{j}=(c/4\pi)\boldsymbol{\nabla}\times\mathbf{B}$. The φ-component of $\mathbf{j}$ is the conventional vortex current

$$j_\varphi(\rho) = -\frac{c}{4\pi\lambda_L}\frac{dB_z}{d\rho} = \frac{c\Phi_0}{8\pi^2\lambda_L^3}K_1(\rho). \tag{16}$$

The unusual feature of the problem, as in Ref. 7, is the presence of the Z-component of the current

$$j_z(\rho) = \frac{c}{4\pi\lambda_L}\frac{1}{\rho}\frac{d}{d\rho}(\rho B_\varphi). \tag{17}$$

Substituting here B_φ of Eq. (15), we obtain

$$j_z(\rho) = \frac{p\beta c\Phi_0}{16\pi^2\lambda_L^3}\left[\frac{2}{\rho}K_1(\rho) - K_0(\rho)\ln(\rho) - CK_0(\rho)\right]. \tag{18}$$

Note that the dependence of j_z on ρ, given by Eq. (18), differs from the one obtained by Ivlev and Thompson.[7]

The exact value of the constant C depends on the structure of the vortex core. This problem is more involved as it requires solution of the full Ginzburg–Landau theory for $\psi(\rho)$ and $\mathbf{A}(\rho)$, as well as the knowledge of $m_{ij}(\rho)$ near the core of the dislocation. The essential role of the vortex core, however, is that it makes B_z, B_φ, j_z, and j_φ nondivergent functions of ρ at $\rho\to 0$. This provides zero total superconducting current flowing parallel to the dislocation, $I_z=0$. Assuming a normal core of radius $r=\xi$, and introducing $\kappa=\lambda_L/\xi$, an estimate of C can be obtained from the condition $B_\varphi(1/\kappa)=0$, which is equivalent to

$$I_z=\int_{1/\kappa}^{\infty} j_z(\rho)2\pi\rho d\rho=0. \tag{19}$$

This condition gives

$$C=\ln(\kappa)+\kappa K_0(1/\kappa)K_1^{-1}(1/\kappa). \tag{20}$$

A few observations are in order. The dependence of C on the core cutoff radius is rather weak. This can be seen from the fact that C changes from $C=0.1916$ at $\kappa=1/\sqrt{2}$ (which is the boundary of type-II superconductivity) to the asymptotic form: $C=2\ln(\kappa)$, at $\kappa\to\infty$. Consequently, the κ dependence of $j_z(\rho)$ is also weak.

For $\kappa=100$ and $p=1$, $j_z(r)$ is shown in Fig. 1. The maximal longitudinal current, $j_z\sim(b/\xi)j_0$ (with $j_0=c\Phi_0/12\sqrt{3}\pi^2\xi\lambda_L^2$ being the Ginzburg–Landau critical current) occurs at $r\sim\xi$ near the core of the vortex. The longitudinal current j_z changes sign at r of order λ_L. At $r\gg\lambda_L$ it becomes exponentially small. For a screw dislocation of the opposite chirality, the current is in the opposite direction.

A screw dislocation either ends at the surface of the crystal or forms a loop inside the crystal. In the first case, the boundary condition, $n_i g_{ij} D_j \psi=0$ (with $\mathbf{n}$ being the unit vector normal to the boundary), prohibits currents through the surface of the superconductor. This results in a surface current that flows outward from the vortex core at $\xi<r<\lambda_L$. In the case of a dislocation loop, the flux line will follow the loop.

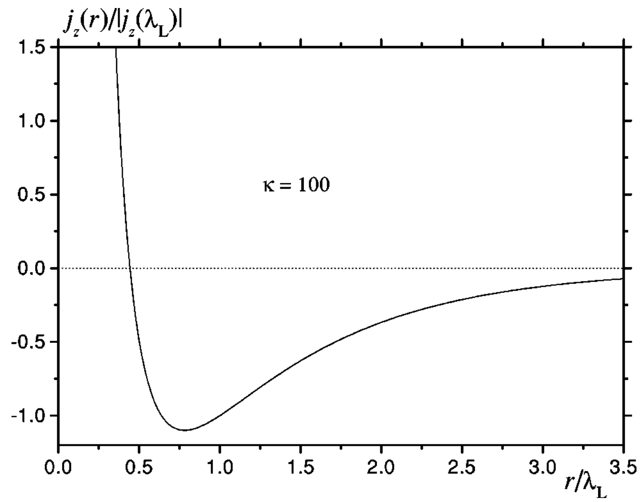

FIG. 1. Radial dependence of the longitudinal current density in a superconducting vortex coupled to a screw dislocation.

The coupling of the flux line to a screw dislocation is the $(b/\xi)^2$ fraction of the vortex-core energy available for pinning. In high-temperature superconductors, where the coherence length can be of order of the Burgers parameter, dislocations can provide a rather strong pinning,[6] in accordance with observations.[1–4] One should keep in mind, however, that the presence of the B_φ component of the field in the flux line pinned by a screw dislocation makes impossible the ''force-free'' situation in which the field is parallel to the transport current. In that sense columnar pins with no chirality have the advantage over screw dislocations.

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