

Angular dependence of nonclassical magnetic quantum oscillations in a quasi-two-dimensional multiband Fermi liquid with impurities

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(Received 24 April 2001; published 2 January 2002)

The semiclassical Lifshitz-Kosevich-type description is given for the angular dependence of quantum oscillations with combination frequencies in a multiband quasi-two-dimensional Fermi liquid with a constant number of electrons. The analytical expressions are found for the Dingle, thermal, spin, and amplitude (Yamaji) reduction factors of the nonclassical combination harmonics, where the latter two strongly oscillate with the direction of the field. At the “magic” angles those factors reduce to the purely two-dimensional expressions given earlier. The combination harmonics are suppressed in the presence of the nonquantized (“background”) states, and they decay exponentially faster with temperature and/or disorder compared to the standard harmonics, providing an additional tool for electronic structure determination. The theory is applied to Sr₂RuO₄.

DOI: 10.1103/PhysRevB.65.035418

PACS number(s): 71.18.+y, 71.27.+a, 73.21.-b, 73.90.+f

The low-dimensional organic conductors exhibit very unusual electronic properties, like magnetic-field induced superconductivity, and are the focus of solid-state research.¹ Magnetic oscillations of susceptibility and resistivity are the main source of detailed information about their electronic structure, yet there is no complete theory of these phenomena for quasi-two-dimensional (quasi-2D) metals. The analytical semiclassical (Lifshitz-Kosevich type^{2,3}) theory of the de Haas–van Alphen (dHvA) combination harmonics in the multiband two-dimensional (2D) metals has been suggested recently.⁴ Even earlier it was shown,⁵ for a system like Sr₂RuO₄,⁶ that the magnetic quantum oscillations in a multiband 2D metal with a fixed electron density [canonical ensemble (CE)] are qualitatively different from those in an open system where the chemical potential is fixed [grand canonical ensemble (GCE)]. The chemical potential oscillates with the field in closed systems and this provides a mechanism for different bands to communicate with each other in CE producing a dHvA signal with the “sum”⁵ and the “difference” combination frequencies^{7–9} in addition to the ordinary dHvA frequencies. The combination frequencies only appear in the *multiband* 2D metals with constant number of electrons (canonical ensemble),^{5,4} they do *not* show up in a standard canonical single band 2D metal extensively treated in the literature.^{10,11} Those nonclassical combination oscillations have been observed in quantum well structures¹² and Sr₂RuO₄.^{13,14} Their amplitudes are comparable with the standard components, and they are robust with respect to any *background* (nonquantized) density of states at low temperatures,⁷ but fall off exponentially faster with the temperature.⁴ Note that in systems with magnetic breakdown (MB) there are the electronic orbits corresponding to combination frequencies. Some of those (“difference”) frequencies are approximately forbidden in semiclassical theory, but are very pronounced in canonical systems due to chemical potential oscillations.⁸ Weak traces of these “forbidden” oscillations are also found in exact diagonalization of grand canonical MB systems.¹⁵ This simply illustrates an approximate nature of the semiclassical rule, which excludes the semiclassical trajectories with abrupt turns. We focus the

present discussion on systems (Sr₂RuO₄, quantum wells) where the MB is not important^{6,12–14} but the difference in thermodynamic ensembles is.

Many layered systems, including Sr₂RuO₄, are actually *quasi*-2D. Weak coupling between conducting layers, which introduces a dispersion of the Fermi surface in the direction perpendicular to the layers, leads to a strong dependence of magnetic oscillations on the angle θ between the normal to the conducting planes and the magnetic field \vec{B} .¹⁶ This is related to the fact that the energy spectrum in the field becomes almost 2D (the width of the Landau minibands almost vanishes) at some “magic” (Yamaji) angles,¹⁷ where the amplitude of the oscillations is strongly enhanced. The theory⁴ applies only to the clean systems at the magic angles. The full analytical semiclassical theory of the quasi-2D multiband canonical metals at finite temperature with full account for the angular dependence of the Landau miniband widths and the spin factors, the collision broadening of the minibands (Dingle factor), and the background density of states is given in the present paper and applied to a generic case of Sr₂RuO₄.

First, we derive a convenient expression for a multiband two-dimensional thermodynamic potential in magnetic field B , mainly in the units $\hbar = |e| = c = 1$,

$$\Omega = -T \int d\epsilon \mathcal{N}(\epsilon, B) \ln \left(1 + \exp \frac{\mu - \epsilon}{T} \right). \quad (1)$$

Consider the multiband quasi-2D system with a general dispersion law,¹⁸

$$\epsilon = \Delta_\alpha + \frac{k_x^2 + k_y^2}{2m_\alpha} - 2t_\alpha \cos jk_z d, \quad (2)$$

where k_x, k_y are the momenta in the conducting plane, t_α is the hopping between the (conducting) layers, d the distance between the layers, j the integer number. We are mainly interested in the situation where the kinetic energy of electrons is much larger than the hopping between layers, $\mu - \Delta_\alpha$

$\gg 2t_\alpha$. The density of states (DOS) with an account for collision broadening of the Landau levels (Dingle factor)^{19,20} can be written as

$$\mathcal{N}(\epsilon, B) = \sum_{\alpha, bg}^{\text{bands, spin}} \mathcal{N}_\alpha(\epsilon, B), \quad (3)$$

$$\mathcal{N}_\alpha(\epsilon, B) = -\frac{1}{\pi} \text{Im} \sum_{n=0}^{\infty} \frac{\Delta S}{(2\pi)^3} \int_{-\pi/d}^{\pi/d} \frac{dk_z}{\epsilon - \epsilon_{\alpha, nk_z} + i\Gamma_\alpha}, \quad (4)$$

where $\Delta S = 2\pi|e|B/\hbar c$ is the cross-sectional area in k space between the two successive Landau orbits,

$$\epsilon_{\alpha, nk_z} = \Delta_{\alpha 0} + \omega_\alpha(n + 1/2) + D_\alpha \cos jk_z d + g_\alpha \sigma \mu_B B, \quad (5)$$

the energy dispersion for a present geometry of the Landau orbits, where $D_\alpha = 2t_\alpha J_0(jk_f d \tan \theta)$ with $J_0(x)$ the zeroth-order Bessel function, $\Gamma_\alpha = \pi/2\tau_\alpha$, τ_α the scattering mean free time in zero field²⁰ in the band α , and the background (bg) is included in the DOS (3) too.⁷ After integration we obtain an important exact result

$$\mathcal{N}_\alpha(\epsilon, B) = \sum_{n=0}^{\infty} \rho_\alpha \omega_\alpha \nu(\epsilon - \epsilon_{\alpha, n}), \quad (6)$$

$$\nu(\epsilon - \epsilon_{\alpha, n}) = \text{Im} \frac{i}{\pi \sqrt{D_\alpha^2 - (\epsilon - \epsilon_{\alpha, n} + i\Gamma_\alpha)^2}}, \quad (7)$$

where $k_{f\alpha}^2/2m_\alpha = \mu - \Delta_\alpha$, $\Delta_\alpha = \Delta_{\alpha 0} + g_\alpha \sigma \mu_B B$,

$$\epsilon_{\alpha, n} = \Delta_{\alpha 0} + \omega_\alpha(n + 1/2) + g_\alpha \sigma \mu_B B, \quad (8)$$

$\omega_\alpha = |B \cos \theta / m_\alpha c|$ the cyclotron frequency with the cyclotron mass m_α , $\Delta_{\alpha 0}$ the band edge in zero magnetic field, μ the chemical potential, g_α the electron g factor, $\sigma = \pm 1/2$, and μ_B the Bohr magneton. The index $\alpha \equiv b\sigma$ includes the band index b and the spin index σ . There ρ_α is the zero-field density of states in the band α . For the energies of interest, $|\epsilon - \Delta_\alpha| > 2t_\alpha$, $\rho_\alpha = m_\alpha/2\pi\hbar^2 d$ per energy and unit volume, otherwise $\rho_\alpha = (m_\alpha/2\pi^2\hbar^2 d) \arccos[(\epsilon - \Delta_\alpha)/2t_\alpha]$. Note that in a clean system at the magic angles, where $D_\alpha = \Gamma_\alpha = 0$, the expression for ν reduces to $\nu(\epsilon - \epsilon_{\alpha, n}) = \delta(\epsilon - \epsilon_{\alpha, n})$, meaning that the spectrum becomes 2D, the Landau minibands reduce to the Landau levels, and the previous expressions⁴ fully apply. The background density of states, ρ_{bg} , corresponding to possible nonquantized (or largely broadened) bands, can be included in Eq. (6) as the formal limit $\omega_{bg} \rightarrow 0$. Obviously, the non-quantized background will contribute to the nonoscillating characteristics of the system. In a clean limit the density of states (7) takes a standard form with the one-dimensional square-root singularities $\mathcal{N} \sim 1/\sqrt{D_\alpha^2 - (\epsilon - \epsilon_{\alpha, n})^2}$.²¹

By applying the Poisson formula³ to the sum over n in the thermodynamic potential

$$\Omega = -T \sum_{n=0}^{\infty} \int d\epsilon \sum_{\alpha, bg} \rho_\alpha \omega_\alpha \nu(\epsilon - \epsilon_{\alpha, n}) \ln \left(1 + \exp \frac{\mu - \epsilon}{T} \right) \quad (9)$$

with $\mu_\alpha = \mu - \Delta_\alpha$ it can be written as

$$\Omega = \Omega_0 + \tilde{\Omega}, \quad (10)$$

where, after substituting $x = (\epsilon - \epsilon_{\alpha, n})/D_\alpha$,

$$\Omega_0 = -T \int_0^\infty d\epsilon \sum_{\alpha, bg} \rho_\alpha \int_{-\infty}^\infty \frac{dx}{\pi} \text{Im} \frac{i}{\sqrt{1 - (x + i\Gamma'_\alpha)^2}} \times \ln \left(1 + \exp \frac{\mu_\alpha - \epsilon - D_\alpha x}{T} \right) \quad (11)$$

is the ‘‘classical’’ part of the thermodynamic potential, with $\Gamma'_\alpha \equiv \Gamma_\alpha/D_\alpha$. In the GCE Ω_0 does not oscillate as a function of $1/B$, and contains the contribution due to spin susceptibility (Pauli paramagnetism). At low temperatures one finds

$$\Omega_0 = - \sum_{\alpha, bg} \frac{1}{2} \rho_\alpha \left(\mu_\alpha^2 + \frac{1}{2} D_\alpha^2 \right). \quad (12)$$

The oscillating part of the thermodynamic potential includes only the Landau quantized bands,

$$\tilde{\Omega} = -2T \sum_\alpha \rho_\alpha \sum_{r=1}^{\infty} \int_0^\infty d\epsilon \int_{-\infty}^\infty \frac{dx}{\pi} \text{Im} \frac{i}{\sqrt{1 - (x + i\Gamma'_\alpha)^2}} \times \ln \left(1 + \exp \frac{\mu_\alpha - \epsilon - D_\alpha x}{T} \right) \cos 2\pi r \left(\frac{\epsilon}{\omega_\alpha} - \frac{1}{2} \right). \quad (13)$$

This expression, after integrating over ϵ and x , reduces to

$$\tilde{\Omega} = \frac{1}{24} \sum_\alpha \rho_\alpha \omega_\alpha^2 + 2 \sum_\alpha \sum_{r=1}^{\infty} A'_r \cos 2\pi r \left(\frac{F_\alpha}{B} - \frac{1}{2} - \psi_\alpha \right) = \frac{1}{24} \sum_\alpha \rho_\alpha \omega_\alpha^2 + 4 \sum_b^{\text{bands}} \sum_{r=1}^{\infty} A_b^r \cos 2\pi r \left(\frac{F_b}{B} - \frac{1}{2} \right), \quad (14)$$

where $F_b = (\mu - \Delta_{b0})|m_b|/(2\mu_B m \cos \theta) \equiv \hbar c S_{fb}/2\pi e$, S_{fb} is the mean Fermi surface zero-field cross section (in standard units), and the spin-related phase $\psi_\alpha = \sigma \gamma_b$, $\gamma_b = g_b|m_b|/(2m \cos \theta)$. If the scattering time were not dependent on the spin projection, the phase would be the only quantity explicitly depending on spin in Eq. (14), hence one can perform a summation over σ in the second term to reveal the standard spin reduction factor $\cos \pi r \gamma_b$.³

The amplitudes of the Fourier harmonics in Eq. (14) are explicitly given by

$$A_b^r = \frac{\rho_b \omega_b^2}{4\pi^2 r^2} R_T \left(\frac{2\pi^2 r T}{\omega_b} \right) R_Y \left(\frac{2\pi r D_b}{\omega_b} \right) R_D R_s, \quad (15)$$

where $R_T(z) = z/\sinh z$ is the usual temperature reduction factor,³ $R_Y(z) = J_0(z)$ the orientation (Yamaji) factor noticed in Ref. 9, $R_D = e^{-2\pi r \Gamma_b/\omega_b}$ the Dingle exponential damping due to collision broadening of the Landau minibands, $R_s = \cos \pi r \gamma_b$ the spin reduction factor, which all strongly depend on the orientation of the field, angle θ (Fig. 1). As usual, one can view the Dingle factor as corresponding to the effective temperature $T + T_D$ in a clean system, where the

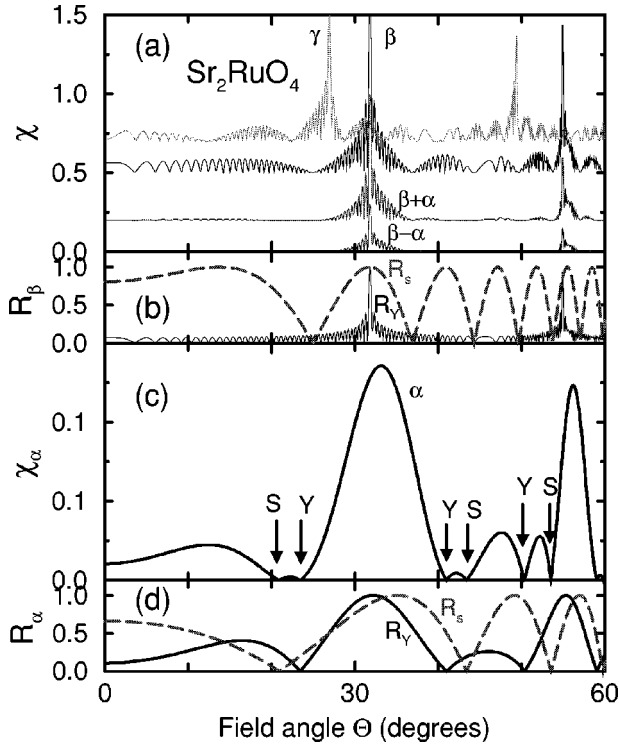


FIG. 1. The amplitudes of magnetic susceptibility harmonics χ_α [panel (c)], χ_β , χ_γ , and $\chi_{\beta\pm\alpha}$ [panel (a)] in Sr_2RuO_4 for $T=T_D=0$. Parameters used in Eq. (5) are $d=6.37$ Å, $j=2$ for α , $j=1$ for β and γ bands (Ref. 10). The corresponding reduction factors R_Y and R_S are shown in panels (b) for β and (d) for α bands. The arrows mark the dips in the harmonic χ_α , corresponding to zeros of the Yamaji R_Y (Y) and the spin R_S (S) factors. The maximal amplitude of α , β , and $\beta\pm\alpha$ bands is found at the first Yamaji angle 33° (see text).

Dingle temperature $T_D=\Gamma_b/\pi$. For the magic angles, where $D_\alpha=0$, one has $R_Y=1$ and hence a pure 2D situation is recovered.⁴ However, generally in a multiband case one should expect that only a spectrum of a particular band will become 2D at some particular magnetic-field tilt angle.

The expression for $\tilde{\Omega}$, Eq. (14), contains the (first) term, responsible for the Landau diamagnetism and the oscillating term, which is responsible for the de Haas–van Alphen effect. It is small compared with the “classical” part, since $\tilde{\Omega}/\Omega_0\sim(\omega/\mu)^2\ll 1$ for the present semiclassical situation. The Fourier components appear with the frequencies rF_α . We recover from Ω_0 [Eq. (12)] and the first term in $\tilde{\Omega}$ [Eq. (14)] the Pauli $\chi_P=\frac{1}{4}\mu_B^2\sum_\alpha\rho_\alpha g_\alpha^2$ and the Landau $\chi_L=-\frac{1}{3}\mu_B^2\cos^2\theta\sum_\alpha\rho_\alpha(m/m_\alpha)^2$ susceptibilities, respectively. Since usually $g_\alpha=2$, those are almost standard, apart from the $\cos^2\theta$ factor characteristic of the present geometry with the tilted magnetic field.

Note that the *chemical potential oscillates* in a closed system and, unlike in GCE the classical part of Ω contributes to oscillations as well.⁴ The relevant thermodynamic potential of the closed system (CE) is the free energy, $F=\Omega+\mu N$, for a fixed number of electrons, $N=-\partial\Omega/\partial\mu$. The chemical potential is⁴

$$\mu=\mu_0+\tilde{\mu}, \quad (16)$$

$$\tilde{\mu}=-\frac{\tilde{N}}{\rho}\equiv-\frac{\tilde{N}}{\rho_q+\rho_{bg}}, \quad (17)$$

where μ_0 is the nonoscillating, while $\tilde{\mu}$ is the oscillating, part of the chemical potential, $\tilde{N}=-\partial\tilde{\Omega}/\partial\mu$ is the oscillating part of the density of electrons, $\rho=\sum_\alpha\rho_\alpha\equiv\rho_q+\rho_{bg}$ is the total density of states, including the quantized ρ_q and the background ρ_{bg} DOS. Substituting this expression into Ω_0 , Eq. (12), we obtain $F=F_0+\tilde{F}$, where the oscillating part is⁴

$$\tilde{F}=\tilde{\Omega}-\frac{\tilde{N}^2}{2\rho}, \quad (18)$$

while F_0 is the nonoscillating part. One sees that the difference between the free energies of the ensembles is directly proportional to the fluctuation of the particle density, as it should, and is suppressed by the background density of states ρ_{bg} , $\sim\tilde{N}^2/(\rho_q+\rho_{bg})$. The oscillating part of the particle density is small, but both terms in Eq. (18) give comparable contribution to the magnetization oscillations. Indeed, $\tilde{N}=\sum_\alpha\tilde{N}_\alpha$, and at low temperatures

$$\frac{\tilde{N}_\alpha}{N_0}\sim\frac{\rho_\alpha}{\rho}\frac{B}{F_\alpha}\ll 1, \quad T<\omega_\alpha, \quad (19)$$

since $F_\alpha\gg B$, and it falls off exponentially with temperature, $\tilde{N}_\alpha/N_0\sim(T/\mu)\exp(-2\pi^2T/\omega_\alpha)$, at $T>\omega_\alpha$. Since the density oscillations are small, sometimes they can be screened out by the back electrode in, e.g., quantum well structures.¹² In a more explicit form one obtains

$$\begin{aligned} \tilde{F} &= \frac{1}{24}\sum_\alpha\rho_\alpha\omega_\alpha^2+4\sum_b\sum_{r=1}^{\infty}A_b^r\cos 2\pi r\left(\frac{F_b}{B}-\frac{1}{2}\right) \\ &\quad -16\sum_{b,b'}\sum_{r,r'=1}^{\infty}C_{bb'}^{rr'}\sin 2\pi r\left(\frac{F_b}{B}-\frac{1}{2}\right) \\ &\quad \times\sin 2\pi r'\left(\frac{F_{b'}}{B}-\frac{1}{2}\right). \end{aligned} \quad (20)$$

It is the last term, which yields the combination Fourier harmonics with the frequencies $F=rF_b\pm r'F_{b'}$. Their amplitudes,

$$C_{bb'}^{rr'}=\frac{2\pi^2rr'A_b^rA_{b'}^{r'}}{(\rho_q+\rho_{bg})\omega_b\omega_{b'}}, \quad (21)$$

are comparable with the standard single-band harmonics at low temperatures, $T<\omega_\alpha/2\pi^2r$, as found earlier⁴ and confirmed experimentally.^{12–14} The combination harmonics are suppressed in the presence of the background density of states. The spin factor depends on the tilt angle, and this results in a strong angular dependence of the corresponding dHvA amplitudes.^{13,14} Incidentally, if one of the angular dependent factors vanishes for some particular harmonic,

$r_0 F_b$, it would *not* mix up with other bands to produce combination harmonics, whereas $r F_b$ ($r \neq r_0$) would (cf. numerical results²²).

Similar to the usual 3D situation, the oscillations produce the (partial) contributions to magnetization M_α and susceptibility χ_α , which are much larger than the nonoscillating contributions. Indeed, the ratio of corresponding amplitudes at low temperatures is $|\chi'_\alpha|/\chi_0 \sim (F_\alpha/B)^2 (\rho_\alpha m/\pi^2 \rho |m_\alpha| r) J_0(2\pi r D_\alpha/\omega_\alpha) e^{-2\pi r \Gamma_\alpha/\omega_\alpha} \gg 1$, since $F_\alpha/B \gg 1$. Proportionality to $(F_\alpha/B)^2$ is the property of the two-dimensional geometry.³ The amplitudes of the standard χ^μ and the combination χ^c harmonics can be easily found from the expressions given above. The ratio of susceptibilities is

$$\frac{\chi_{rF_b, \pm r'F_{b'}}^c}{\chi_{rF_b}^\mu} = \frac{8\pi^2 r r' A_{b'}^{r'} \cos \pi r' \gamma_{b'}}{\rho \omega_b \omega_{b'}} \left(\frac{F}{rF_b} \right)^2 \quad (22)$$

with $F = rF_b \pm r'F_{b'}$. Thus for $r=r'=1$ we have, for a warped two-band cylindrical Fermi surface without the background DOS,

$$\begin{aligned} \frac{\chi_{F_b, \pm F_{b'}}^c}{\chi_{F_b}^\mu} &= \frac{4\pi^2 m_{b'}}{m_b + m_{b'}} \frac{T J_0(2\pi D_{b'}/\omega_{b'}) \cos \pi \gamma_{b'} e^{-2\pi \Gamma_{b'}/\omega_{b'}}}{\omega_{b'} \sinh(2\pi^2 T/\omega_{b'})} \\ &\times \left(\frac{F_b \pm F_{b'}}{F_b} \right)^2, \end{aligned} \quad (23)$$

where χ^μ denotes the amplitude of the standard dHvA harmonic for an open system, with χ^c the amplitude of the combination harmonics. We see that the combination harmonics are suppressed exponentially with respect to temperature⁴ and/or disorder compared to the standard harmonics, Fig. 2. In addition, they contain the product of the angular dependent reduction factors and therefore vary rapidly with the angle θ , their frequency being dominated by the band with the largest ratio $2t_b/\omega_b$ (e.g., β band in $\beta \pm \alpha$ harmonics in Sr_2RuO_4 , Fig. 1).

It is important to note that even in a simplest case of a *one-band* system the standard harmonics rF_b are modified by the second quadratic term, therefore

$$\chi_{rF_b}^N = \chi_{rF_b}^\mu + \sum_{p=1}^{r-1} \chi_{(r-p)F_b, pF_b}^c + \sum_{p=1}^{\infty} \chi_{(r+p)F_b, -pF_b}^c, \quad (24)$$

$$\chi_{rF_b \pm r'F_{b'}}^N = \chi_{rF_b, \pm r'F_{b'}}^c, \quad b \neq b', \quad (25)$$

and the wave front of magnetization of the main harmonics in the closed system with the carrier density $N = \text{const}$ is inverted compared to the open system, $\mu = \text{const}$, as it should.^{3,5,7}

The present formalism is applied to Sr_2RuO_4 with the results shown in Figs. 1 and 2, calculated with the parameters from Refs. 13 and 6 and the g factor $g_b = 2$ for all the bands. The main *maxima* in the Fourier harmonics

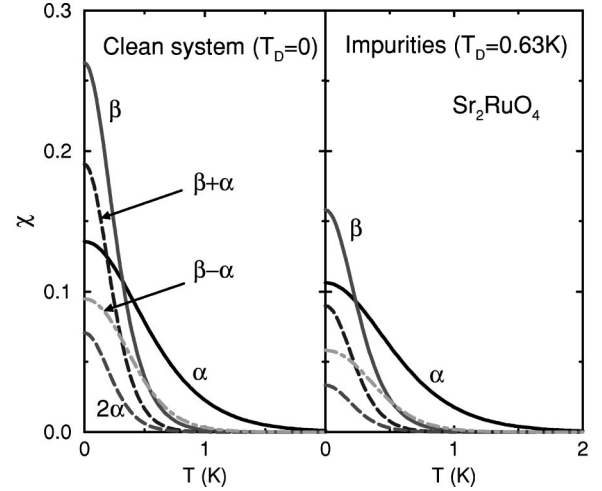


FIG. 2. The amplitudes of the harmonics in Fig. 1, including also 2α , versus the temperature in a clean (left panel) and a disordered (right panel) Sr_2RuO_4 at the (Yamaji) angle $\theta = 33^\circ$. The Dingle temperature $T_D = 0.63$ K (Ref. 13).

of susceptibility χ for the band α are found at $\theta_{\alpha 1(2)} = 33^\circ (55^\circ)$, for the band β at $\theta_\beta \approx \theta_\alpha$, and for band γ at $\theta_{\gamma 1(2)} = 27^\circ (49^\circ)$, which is in fair agreement with the data $\theta_{\alpha 1} = 30.6^\circ$, $\theta_{\beta 1} = 30^\circ$ and $\theta_{\gamma 1} = 15.3^\circ$,¹³ and $\theta_{\alpha 1} = 30^\circ$, $\theta_{\beta 1(2)} = 26^\circ (56^\circ)$.¹⁴ Those *maxima* in the harmonics apparently coincide with the points where the Yamaji factor is unity, $R_Y = 1$, and the *system effectively becomes 2D*, Figs. 1(b) and 1(d). The *minima*, observed in α band at 24° (Ref. 13) [25° (Ref. 14)] and 40° , are very close to the points where *both* the Yamaji R_Y and the spin R_s factors vanish: $R_Y = 0$ at 23° and 41° , and $R_s = 0$ at 21° and 43° , Fig. 1(d).

The conspicuous rapid variation with the *field angle* θ of the main, χ_β, χ_γ , and the combination, $\chi_{\beta+\alpha}, \chi_{\beta-\alpha}$, harmonics, Fig. 1(a), is explained by the large factor $2t_b/\omega_b$ in the argument of the Bessel function in the Yamaji factor, Eqs. (15) and (23): $2t_\beta/\omega_\beta = 7.5$, $2t_\gamma/\omega_\gamma = 5$. This necessarily leads to the high frequency variations of the amplitudes, and apparently only the envelope of those oscillations has been sampled experimentally in Refs. 13 and 14 (see also a numerical study⁹). Those rapid variations with angle θ might be observable, since the condition for the Yamaji approximation, $\pi k_{fb}^2 (t_b/\epsilon_f)^2 \ll \Delta S$ (Ref. 17) seems to hold for the β band [$\pi k_{f\beta}^2 (t_\beta/\epsilon_f)^2 : \Delta S \approx 1:5$], while for the γ band the ratio is only about 1:3. Interestingly, the spin factor R_s defines the envelope of the β amplitude, cf. Figs. 1(a) and 1(b), so the studies of the minima and maxima on the $\chi(\theta)$ (Ref. 13) should allow for accurate determination of the g factors g_b .

The combination harmonics contain *extra temperature and the Dingle reduction factors*, so they are falling off with either temperature or disorder, or both, faster than the standard harmonics do, see Eqs. (21), (15) and Fig. 2. The temperature dependence of the combination harmonics generally cannot be characterized by some effective mass $m_{rb, \pm r'b'}$, although at higher temperatures the relation $m_{rb, \pm r'b'} \approx r m_b + r' m_{b'}$ holds approximately, as follows from Eqs. (20), (21), and (15). It was suggested on the basis of numeri-

cal studies⁹ that there is no simple relation between the effective mass corresponding to a combination frequency and the partial effective masses of the mixing harmonics. The present results show that such a relation generally does not exist, and they should allow for correct interpretation of the temperature dependence of the combination harmonics and the corresponding effective masses. The amplitudes of all the harmonics decay quickly with T , especially in the presence of even weak disorder [the Dingle temperature $T_D = 0.63$ K (Ref. 13)], Fig. 2(b), which is in very good agreement with experiment (cf. Fig. 2 in Ref. 6).

Importantly, in the first experiments on Sr_2RuO_4 the disorder was actually larger, since the elastic mean free path was $l \sim 10^3$ Å,⁶ compared to $l \sim 2050\text{--}5000$ Å in the later experiments.^{13,14} The higher disorder in the first samples, and relatively small magnitude of the combination peaks at the field B directed along the c axis, $\theta=0$, instead of the “magic” angle $\theta=30^\circ$, where *all* the amplitudes are enhanced, Fig. 1, has possibly prevented the discovery of the combination harmonics, predicted in Ref. 5, in the first experiments on Sr_2RuO_4 .⁶ The combination harmonics have been detected in Refs. 12–14. The dependence of the dHvA combination amplitudes on angle, temperature, and disorder provide an additional valuable tool for studying the effective carrier masses, their densities, etc., in the multiband quasi-two-dimensional metals.

Note added in proof. To avoid a misunderstanding and misrepresentation of our analytical results,⁴ Eqs. (18) and (20), like that expressed in Ref. 23, we would like to emphasize that these equations are *exact* (in the semiclassical regime) as long as the chemical potential μ is exact. The chemical potential comprises, of course, an oscillating part $\tilde{\mu}$ in CE. In the limit $T=0$ one obtains from the present results a “saw-tooth” behavior,⁵

$\tilde{\mu} \approx \sum_b [\rho_b \omega_b] / \rho (1/2 - \{F_b/B - 1/2\})$, where $\{z\}$ is the fractional part of z . Let us check how this will change our expressions for the amplitudes of combination harmonics by substituting the first Fourier harmonic of $\tilde{\mu}$ into the trigonometric arguments of Eq. (20). We find that the amplitude $C_{bb'}^{rr'}$ changes by a factor of

$$\Pi_{b''} J_0 \left(2 \frac{r m_b + r' m_{b'}}{m_{b''}} \frac{\rho_{b''}}{\rho} \cos \pi \gamma_{b''} \right).$$

This factor is of the order unity for two bands with comparable masses for the first harmonics even at $T=0$. The second term in \tilde{F} (20), which produces the standard harmonics in GCE, will also produce the combination harmonics in CE if the oscillating part of the chemical potential is taken into account. However, such harmonics with frequencies $F = rF_b \pm r'F_{b'}$ appear with very small prefactor $J_{r',r}(2r\rho_{b'}/\rho m_{b'})$, so they are negligible compared with the last term in Eq. (20). Thus, while analyzing the temperature and damping dependence of the combination amplitudes, one can safely neglect the oscillating part of the chemical potential in Eq. (20) even in the most unfavorable case of zero temperature. Note that the difference between $\tilde{\Omega}$ and \tilde{F} is *small* but nonzero, $\propto \tilde{N}^2$, Eq. (18). At the same time, the contribution to the response functions acquires the powers of a large factor $F_b/B \gg 1$, and this makes the combination harmonics observable. Our analytical results are confirmed by our original numerical calculations,⁵ and those of Ref. 9. In a multiband case even at $T=0$ and/or with increasing temperature the corrections to our expressions for the combination harmonics due to quantum oscillations of the chemical potential become even smaller, the facts being appreciated in Ref. 23, where some of the results⁴ are rederived.

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