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Phase control of spontaneously generated coherence induced bistability

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Abstract

The steady-state nonlinear behavior in a ring cavity based on a Λ-type atomic model is investigated. It is shown that, due to the existence of strong spontaneously generated coherence, optical bistability still can be realized even if both coherent fields in resonance with the medium. Moreover, the appearance or disappearance of bistability can be controlled by adjusting the relative phase between the two coherent fields. When the probe field is not resonant, it is found that the threshold intensity for producing bistability is very sensitive to the relative phase, and bistability and multistablity can transform mutually by adjusting the relative phases.

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Keywords: Optical bistability; Spontaneously generated coherence; Relative phase

1. Introduction

In recent years, there has been considerable interest in the quantum interference and atomic coherence in a multilevel atom system induced by coherent electromagnetic fields. One of the most important examples is the modification of the absorption lineshape. For a three-level Λ-type system when the detunings of the probe and control fields from corresponding atomic transitions are equal, i.e., two-photon resonance, the atoms are trapped in the dark state, thus the absorption of the probe is eliminated and electromagnetically induced transparency occurs (see [1] for a review). When the absorption is used to obtain optical bistability, the threshold intensity is controlled by utilizing quantum interference. Harshawardhan and Agarwal [2] investigated the role of atomic coherence

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and interference in optical bistability and found that these effects result in a considerable lowering of the threshold intensity. Gong et al. [3] showed that the phase fluctuation of the control field lead to bistability even both fields are in resonance with the medium, and the threshold intensity is adjusted by changing the bandwidth of the control field. Hu et al. [4] have suggested a method of achieving phase control of the amplitude-fluctuation-induced bistability with two-photon resonance in such Λ atoms. On the other hand, it is now well-understood how the decay of a system with closely lying states induced by interaction with a common bath leads to one new type of coherence, generally called as spontaneously generated coherence [5]. This new kind of coherence has attracted more and more attentions [6]. We investigated the effect of spontaneously generated coherence (SGC) on the pump–probe response of a nearly generate Λ system by taking into account the dephasing of the low-frequency coherence and found that the optical property of the medium can be modified significantly, electromagnetically induced absorption (EIA) occurs instead of EIT, moreover, EIA and EIT can transform mutually by adjusting the relative phase [7]. Recently, Joshi et al. [8] demonstrated the controllability of SGC to optical bistability by using the theoretical model of three-level atoms in Λ-configuration in the presence of two arbitrary coherent fields and also discussed the possibilities of obtaining optical multistability. Here, we also consider the spontaneously generated coherence, but mainly focus on the phase control of bistability due to SGC. From the following discussions, we can see that the relative phase is a convenient parameter to modify the transition properties of the system.

In this paper, we investigate the steady-state nonlinear behavior in a unidirectional cavity (Fig. 1) by taking account into spontaneously generated coherence. For simplicity, we assume that mirrors 3 and 4 have 100% reflectivity, and call R and T (with R + T = 1) the reflection and transmission coefficient of mirrors 1 and 2. We describe the dynamics of the coupled system (atoms plus radiation fields) by the well-known Maxwell–Bloch equations. We find that even if both fields are in resonance, optical bistability induced by spontaneously generated coherence can still be realized. Moreover, the appearance or disappearance of bistability can be easily controlled by adjusting the relative phase between the two coherent fields. When the probe field is not resonant, we find the relative phase has an important effect on the threshold intensity to produce bistability and can make multistablity and bistability transform mutually.

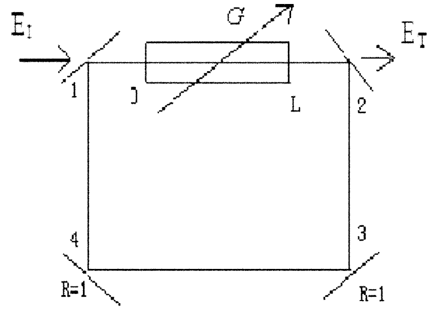

Fig. 1. Unidirectional ring cavity. E_I and E_T are the incident and transmitted fields, respectively; G represents the control field.

2. Equations of dynamics

Considering a closed Λ-type medium with excited state $|1\rangle$ and closely lying lower states, $|2\rangle$ and $|3\rangle$, as illustrated in Fig. 2. The excited state $|1\rangle$ decays to $|3\rangle$ and $|2\rangle$ with decay rates $2\gamma_1$

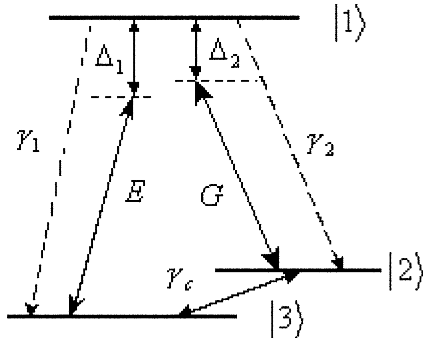

Fig. 2. Λ-type three-level atomic system. $E(G)$ is the Rabi frequency of a coherent field (control field); $2\gamma_1$ and $2\gamma_2$ are the decay rates, whereas collisional phase decay of the two to three polarization occurs at the rate $2\gamma_c$; Δ_1, Δ_2 are the detunings of both fields.

and $2\gamma_2$, respectively. A coherent probe field with Rabi frequency $2E$ couples the transition between states $|3\rangle$ and $|1\rangle$, and a coherent control field with Rabi frequency $2G$ is applied to the transition $|1\rangle$ and $|2\rangle$. The control field does not circulate in the unidirectional cavity, and thus its dynamical evolution can be ignored, so we only need considering the propagation of the coherent field. Under the rotating wave approximation the density-matrix equation and Maxwell equation can be derived as:

$$\mathrm{d}\rho_{11}/\mathrm{d}t = -2(\gamma_1+\gamma_2)\rho_{11} + \mathrm{i}E\rho_{31} + \mathrm{i}G\rho_{21} - \mathrm{i}G^*\rho_{12} - \mathrm{i}E^*\rho_{13}, \tag{1a}$$

$$\mathrm{d}\rho_{22}/\mathrm{d}t = 2\gamma_2\rho_{11} - \mathrm{i}G\rho_{21} + \mathrm{i}G^*\rho_{12}, \tag{1b}$$

$$\mathrm{d}\rho_{33}/\mathrm{d}t = 2\gamma_1\rho_{11} - \mathrm{i}E\rho_{31} + \mathrm{i}E^*\rho_{13}, \tag{1c}$$

$$\mathrm{d}\rho_{12}/\mathrm{d}t = -(\gamma_1+\gamma_2+\gamma_c+\mathrm{i}\Delta_2)\rho_{12} + \mathrm{i}E\rho_{32} - \mathrm{i}G(\rho_{11}-\rho_{22}), \tag{1d}$$

$$\mathrm{d}\rho_{13}/\mathrm{d}t = -(\gamma_1+\gamma_2+\mathrm{i}\Delta_1)\rho_{13} + \mathrm{i}G\rho_{23} - \mathrm{i}E(\rho_{11}-\rho_{33}), \tag{1e}$$

$$\mathrm{d}\rho_{23}/\mathrm{d}t = -[\gamma_c+\mathrm{i}(\Delta_1-\Delta_2)]\rho_{23} + 2p \times \sqrt{\gamma_1\gamma_2}\rho_{11} + \mathrm{i}G^*\rho_{13} - \mathrm{i}E\rho_{21}, \tag{1f}$$

$$c\frac{\partial E(z,t)}{\partial z} + \frac{\partial E(z,t)}{\partial t} = 2\pi\mathrm{i}\omega_g\mu_{31}P(\omega_g), \tag{1g}$$

with the closure relation $\rho_{11}+\rho_{22}+\rho_{33}=1$. $P(\omega_g)$ is the slowly oscillating term of the induced polarization, $P(\omega_g)=N\mu_{31}\rho_{13}$, N is the atomic density of the medium. μ_{13} is the transition matrix element of the atomic dipole moment. Here ω_g is the frequency of the probe field. Note that in Eq. (1) we have included one collision-induced perturbation $2\gamma_c$ of the energy of level $|2\rangle$ leading to a dephasing of the polarizations of $|1\rangle\leftrightarrow|2\rangle$ and $|2\rangle\leftrightarrow|3\rangle$. If there is not this dephasing term, Menon and Agarwal [9] investigated the effect of spontaneously generated coherence on this system and found that such coherence preserves both electromagnetically induced transparency (EIT) and coherent population trapping (CPT) phenomena. In practice, the dephasing is present in realistic systems [10] and the dephasing term can alter the property of the medium. Tao et al. [11] found, due to the dephasing between the two ground states, the most effective enhancement of Kerr nonlinearity occurs. Friedmann and Wilson Gordon [12] investigated the properties of the gain, the refractive index and the noise for a degenerate Λ system interacting with a single pump and weak probe and found that they have the same linear dependence on the rate of the collisional relaxation rate between the two lower lying states due to the collisional transfer of population between the ground states leading to a deviation form exact CPT, so that there is a strong correlation between gain without inversion, enhanced refractive index and noise in these systems. The parameter p denotes the alignment of the two transition matrix elements determining the strength of the interference in spontaneous emission and is defined as $p=\vec{u}_{12}\cdot\vec{u}_{13}/\mid\vec{u}_{12}\cdot\vec{u}_{13}\mid=\cos\theta$, where θ is the angle between the two induced dipole moments $\vec{u}_{12}$ and $\vec{u}_{13}$. The parameter p plays an important role in the creation of coherence and has significant effect on the dynamics of systems, which we will show. It is known that the usual EIT experiments with well-separated ground levels in a Λ system do not depend on the relative phase between the two applied fields. However, in the case of closely spaced levels, SGC makes the system quite sensitive to the relative phase between the two applied fields. The p dependent terms are always accompanied by a phase dependent term $\exp(\pm\mathrm{i}\Phi)$ where Φ denotes the relative phase between the two laser fields. The steady-state solutions of Eq. (1a)–(1f) can be found by setting all the time derivatives in Eq. (1) to zero and reducing it to a set of coupled 9×9 algebraic equations after splitting into real and imaginary parts. In numerical calculations, we will use computation package Maple and choose the parameters to be dimensionless units by scaling with γ (and setting $\gamma=1$).

3. Optical bistability

For a perfectly tuned cavity, the boundary conditions in the steady-state limit are [13]:

$$E(0) = \sqrt{T}E_I + RE(L), \tag{2a}$$

$$E_T = \sqrt{T}E(L), \tag{2b}$$

where E_I and E_T are the incident and the transmitted fields, respectively.

In the steady-state case, we can obtain the field amplitude $E(z,t)$ from Eq. (1g) as follows

$$c\frac{\partial E(z)}{\partial z} = 2\pi i\omega_g \mu_{31}^2 \rho_{13}(E(z)), \tag{3}$$

In the mean-field limit [14], using the boundary condition (2a) and (2b), we obtain the mean-field state equation

$$Y = X + 2CX\gamma\rho_{13}(X), \tag{4}$$

where $Y = E_I/\sqrt{T}$, $X = E_T/\sqrt{T}$ and C is the usual cooperation parameter [3,13]. The nonlinear term in above equation is essential to give rise to bistability. In following numerical analysis, we set the atomic decay rate $\gamma_2 = \gamma_1 = \gamma$ and the cooperation parameter $C = 200\gamma$ fixed only for simplicity.

3.1. Two-photon resonance case

Setting the Rabi frequency of the control field $G = 10\sin\theta\gamma$, $\Delta_1 = \Delta_2 = 0$, we obtain the plots of transmitted light versus incident light for three different cases, as shown in Fig. 3. Curves 1 and 2 correspond to $p = 0.0$ and $p = 0.99$, respectively, and curve 3 corresponds to $p = 0.99$ but in this case, the relative phase is π. This figure shows that under suitable parameters, optimal spontaneously generated coherence can induce bistability. Moreover, by adjusting the relative phase of the two coherent fields from 0 to π, bistability disappears again. Why this occurs? It is shown in [7], that when enough small γ_c exists (setting $\gamma_c = 0.05\gamma$), there exists very small absorption (good EIT is still preserved), so E_I is approximately direct proportional to E_T (curve 1). In this case, we can say, optical bistability is impossible. While when the spontaneously generated coherence is strong enough ($p = 0.99$), a larger nonlinear absorption occurs, which leads to bistability shown by curve 2.

Menon and Agarwal [9] pointed out that EIT is preserved regardless of what the relative phase is under the two-photon resonance. But this is correct only in the limit when the relaxation of the low-frequency coherence vanishes. It is found in [7] that under optimal SGC, EIA appears when Φ is 0 or 2π; While Φ is π, the system takes on good EIT. Due to this reason tuning the relative phase between the two applied fields from 0 to π, the medium becomes from EIA to EIT, bistability disappears.

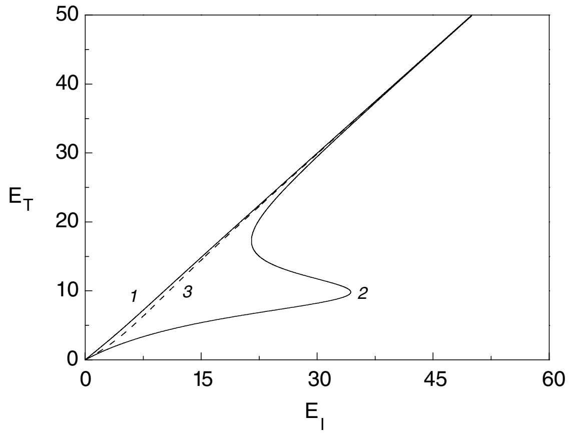

Fig. 3. Plots of transmitted light versus incident light for different cases. Curves 1 and 2 correspond to $p = 0.0$ and $p = 0.99$, respectively, and curve 3 corresponds to $p = 0.99$ but with the relative phase $\Phi = \pi$. The parameters are $\Delta_1 = \Delta_2 = 0$, $G = 10\sin\theta\gamma$.

3.2. One-photon resonance

Fixing $\Delta_1 = 5$, $\Delta_2 = 0$ (that is, the control field is still resonant), $p = 0.99$, and the Rabi frequency of the control field $G = 20\ \sin\theta\gamma$, by varying the relative phase of the two coherent fields, we obtain the plots of transmitted light versus incident light for four different relative phase shown in Fig. 4. Curves 1–4 correspond to $\Phi = \pi/4$, $\Phi = \pi/2$, $\Phi = 3\pi/4$ and $\Phi = \pi$. From this figure, we can easily see that the relative phase has an obvious effect on the threshold intensity to produce bistability. We know that multistability can be observed for an appropriate choice of the control field strength and detunings. In Fig. 5, under optimal SGC, choosing suitable parameters ($\Delta_1 = 3$ and $G = 25\ \sin\theta\gamma$), we also can obtain multistability corresponding to curve 1. We now change the relative phase, multistability

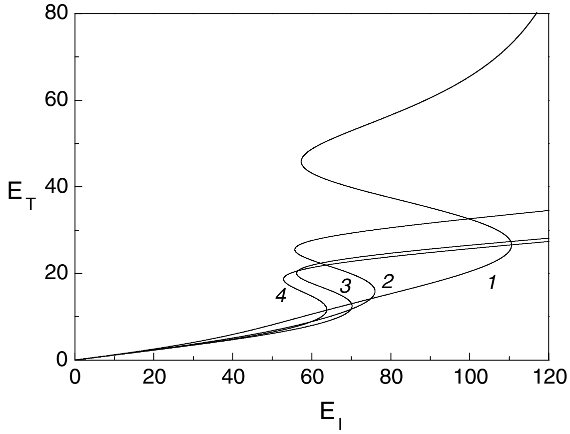

Fig. 4. Plots of transmitted light versus incident light for various relative phases. Curves 1–4 correspond to $\Phi=\pi/4$, $\Phi=\pi/2$, $\Phi=3\pi/4$ and $\Phi=\pi$. $\Delta_1=5$, $\Delta_2=0$, and $G=20\sin\theta\gamma$.

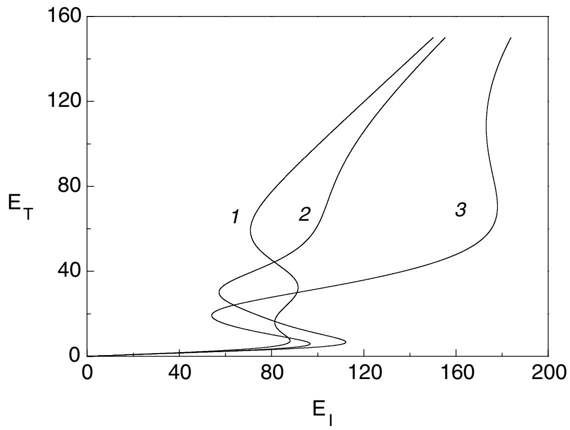

Fig. 5. Plots of transmitted light versus incident light for relative phases. Curves 1–3 correspond to $\Phi=0$, $\Phi=\pi/4$ $\Phi=\pi/2$, respectively. $\Delta_1=3$, $\Delta_2=0$, and $G=25\sin\theta\gamma$.

disappears. For curve 2, the relative phase is $\pi/4$, bistability occurs. For curve 3, the relative phase increases to $\pi/2$, discontinuous bistabilities occur. In other words, we can get bistability or multibility only simply tuning the relative phases.

4. Summary

In this paper, we investigated the steady-state nonlinear behavior in a ring cavity in a Λ-type atomic system and found that optical bistability still can be realized even if both fields in resonance due to the existence of spontaneously generated coherence. We can control the appearance or disappearance of bistability by adjusting the relative phase between the two coherent fields for two-photon resonance case. For one-photon resonance case, we investigated the effect of the relative phase and found that it has an important effect on the threshold intensity to give rise to bistability, so we can modulate the bistability or multibility just by changing the relative phase, which cannot be realized in a conventional Λ system. We know that the existence of SGC depends on the nonorthogonality of the two dipoles transition matrix elements. The nonorthogonality can be obtained from the mixing of the levels arising from internal fields or external microwave fields [15,16]. On the other hand, Joshi and Xiao [17] have experimentally demonstrated optical multistability in an optical ring cavity filled with a collection of three-level Λ-type rubidium atoms where the observed multistability is very sensitive to the induced atomic coherence in the system and can evolve from a normal bistable behavior with the change of the coupling field as well as the atomic number density. So these results in our paper also can be experimentally observed easily.

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