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Effects of spontaneous emission-induced coherence on population inversion in a ladder-type atomic system

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Abstract

Spontaneous emission can create coherence in a ladder-type three-level atom with equispaced levels, subject to the condition that the atomic dipole moments are nonorthogonal. We study the effects of this kind of coherence on the steady-state population inversion in the atomic system. We show that the population inversion can be greatly enhanced on one of the optical transitions due to the spontaneous emission-induced coherence. Furthermore, we find, within suitable parameters regions, that such coherence can also lead to unexpected population inversion on both of the optical transitions.

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1. Introduction

Quantum interference can lead to many interesting effects such as lasing without population inversion (LWI) [1,2], electromagnetically induced transparency (EIT) [3,4], subluminal and superluminal light [5–7], fluorescence quenching [8,9] and population inversion [10–13]. In an early work [10], Whitley and Stroud considered a ladder-type three-level atomic system driven by two coherent fields. They found that the steady-state population inversion can be achieved on one of the optical transitions and predicted that the population in the higher excited state may be larger than 0.5. In a $\vee$-type atomic system, Meduri et al. [11] showed the population inversion of a maximum value of 10% on one of the optical transitions. In [13], Hu and Peng found that a $\wedge$-type system can also lead to population inversion. However, they [10–13] found that the population inversion cannot occur simultaneously on both of the optical transitions in these systems.

There has been increasing interests in the coherence arising from spontaneous emission, subject to the condition that the atomic dipole moments must be nonorthogonal. This kind of

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coherence is created by the interference of spontaneous emission of either two close lying atomic levels to a common atomic level (∨-type atom) [14], or by a single excited level to two close lying atomic levels (∧-type atom) [15]. In a ladder-type atomic system [10,16], it can also be created in a nearly equispaced atomic levels case. In 1996, Xia et al. [17] carried out the first experimental investigation of constructive and destructive interference effects in spontaneous emission. They observed that the fluorescence spectrum exhibited two-peak or three-peak features according to parallel or anti-parallel dipole moments. The observed variation of the number of peaks with the mutual polarization of the dipole moments gave compelling evidence for coherence in spontaneous emission. However, in a later paper it was reported that a repeat of the experiment had failed to observe the same features [18]. Wang et al. [19] gave a theoretical investigation of this experiment and showed that the numbers of peaks depend on the excitation process. Nevertheless this experiment showed some interesting effects induced by spontaneous emission-induced coherence. Considering the effects of spontaneous emission-induced coherence, Zhou and Swain [20] studied the absorption of a weak probe beam for a ∨-type atom and demonstrated narrow resonances, transparency and gain without inversion. In [21], Gong et al. investigated that the coherence from spontaneous emission could greatly enhance the population inversion on one of the optical transitions in a ∨-type atomic system. Also, using this kind of coherence, Evers et al. [22] discussed the resonance fluorescence spectrum in a ∧-type atomic system and associated the dark state suppression and narrow fluorescent feature. In [23], Gong et al. found that a large and unexpected population inversion can be achieved on both of the optical transitions due to spontaneous emission-induced coherence effects in ∧-type system.

Ficek et al. [16] discussed the effects of spontaneous emission-induced coherence on the steady-state intensity and squeezing properties of the fluorescent light from a three-level atomic system in a ladder-type configuration. They found that in addition to the well-known resonant behavior of the upper-state population centered on zero two-photon detuning, the intermediate state also exhibits the same resonance, which leads to unusual three-level squeezing properties near two-photon resonance. In this paper, we investigate the effects of spontaneous emission-induced coherence on population inversion in a nearly equispaced ladder-type system driven by two coherent fields. Working with the master equation of the system, we find that the steady-state population inversion on one of the optical transitions can be greatly enhanced due to the effects of spontaneous emission-induced coherence. Choosing suitable parameters regions, we show that this kind of coherence can also lead to unexpected population inversion on both of the optical transitions.

This paper is organized as follows: in Section 2, we present the density-matrix equations for the ladder-type atomic model. In Sections 3 and 4, the population inversion on one of the optical transitions and on both of the optical transitions are investigated, respectively. Finally, we make some conclusions in Section 5.

2. Basic equation

A three-level ladder-type system which is in the case of nearly equispaced levels is shown in Fig. 1. Two coherent fields of frequencies ω_a and ω_b with real-valued Rabi frequencies $\Omega_1(=\vec{\mu}_{12}\cdot\vec{\varepsilon}_a)$ and $\Omega_2(=\vec{\mu}_{23}\cdot\vec{\varepsilon}_b)$ drive the transitions $|2\rangle \leftrightarrow |1\rangle$ and $|3\rangle \leftrightarrow |2\rangle$, respectively. The excited state $|2\rangle$ ($|3\rangle$)

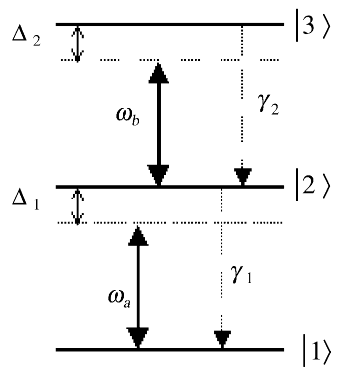

Fig. 1. Three-level ladder-type atom interacting with the two coherent fields ω_a and ω_b. γ_1 and γ_2 are the decay rates, and Δ_1 and Δ_2 are the coherent fields frequency detunings.

exhibits spontaneous decay to the state $|1\rangle$ ($|2\rangle$) at the rate γ_1 (γ_2). In such a case, the density-matrix equations in a rotating frame can be written as

$$\dot{\rho}_{33} = -2\gamma_2\rho_{33} - \Omega_2(\rho_{23} + \rho_{32}), \tag{1}$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{33} + \Omega_2(\rho_{23} + \rho_{32}) - \Omega_1(\rho_{12} + \rho_{21}) - 2\gamma_1\rho_{22}, \tag{2}$$

$$\dot{\rho}_{11} = 2\gamma_1\rho_{22} + \Omega_1(\rho_{21} + \rho_{12}), \tag{3}$$

$$\dot{\rho}_{31} = -\Omega_2\rho_{21} + \Omega_1\rho_{32} + (\mathrm{i}\Delta_1 + \mathrm{i}\Delta_2 - \gamma_2)\rho_{31}, \tag{4}$$

$$\dot{\rho}_{32} = \Omega_2(\rho_{33} - \rho_{22}) + (\mathrm{i}\Delta_2 - \gamma_1 - \gamma_2)\rho_{32} - \Omega_1\rho_{31}, \tag{5}$$

$$\dot{\rho}_{21} = \Omega_2\rho_{31} + \Omega_1(\rho_{22} - \rho_{11}) + (\mathrm{i}\Delta_1 - \gamma_1)\rho_{21} + 2p\sqrt{\gamma_1\gamma_2}\rho_{32}, \tag{6}$$

constrained by $\rho_{11} + \rho_{22} + \rho_{33} = 1$ and $\rho_{nm} = \rho^*_{mn}$. Here, in the case of nearly equispaced levels, the inclusion of two driving fields of different frequencies, both of which couple both the $|2\rangle \leftrightarrow |1\rangle$ and the $|3\rangle \leftrightarrow |2\rangle$ transitions, would lead to the optical Bloch equations with additional term ($2p\sqrt{\gamma_1\gamma_2}\rho_{32}$), which presents the effects of spontaneous emission-induced coherence. Note that, in the nonequispaced levels case, such a term of time dependence which includes an oscillation at the frequency $|\omega_a - \omega_b|$ can be neglected. Because $\hbar|\omega_a - \omega_b|$ has been assumed to be much larger than either the interaction energies $\hbar\Omega_1$ and $\hbar\Omega_2$ or $\hbar\gamma_1$ and $\hbar\gamma_2$, such terms are rapidly oscillating and will average out [10]. However, for the case of nearly equispaced levels, i.e., $\omega_a - \omega_b \approx 0$, the effects arising from spontaneous emission-induced coherence must be taken into account. The parameter p denotes the alignment of the two matrix elements which is nonparallel as well as nonorthogonal [21,24], and is defined as $p = \vec{\mu}_{12} \cdot \vec{\mu}_{23}/|\vec{\mu}_{12} \cdot \vec{\mu}_{23}| = \cos\theta$, where θ is the angle between the two induced dipole moments $\vec{\mu}_{12}$ and $\vec{\mu}_{23}$. The parameter p plays a very important role in the creation of spontaneous emission-induced coherence as we will show later. The steady-state solutions can be found by setting the time derivatives to zero and reducing Eqs. (1)–(6) to a set of coupled 9×9 algebraic equations after splitting into real and imaginary parts. In numerical calculations, we will use computation package Maple and choose the parameters to be dimensionless (Ω/γ and $\gamma = 1$, etc.).

3. Population inversion on one of the optical transitions

In this section, we examine the effects of spontaneous emission-induced coherence in population inversion on one of the optical transitions. Setting the parameters $\Omega_1 = 15\gamma$, $\Omega_2 = 10\gamma$, $\gamma_1 = 6\gamma$, $\gamma_2 = 1.5\gamma$ and $\Delta_1 = 20\gamma$, the steady-state population inversion $W_{23}(= \rho_{33} - \rho_{22})$ as a function of the laser detuning Δ_2 for three different values of p is shown in Fig. 2(a). There is a population inversion of the values of 20% for $p = 0$, which is similar

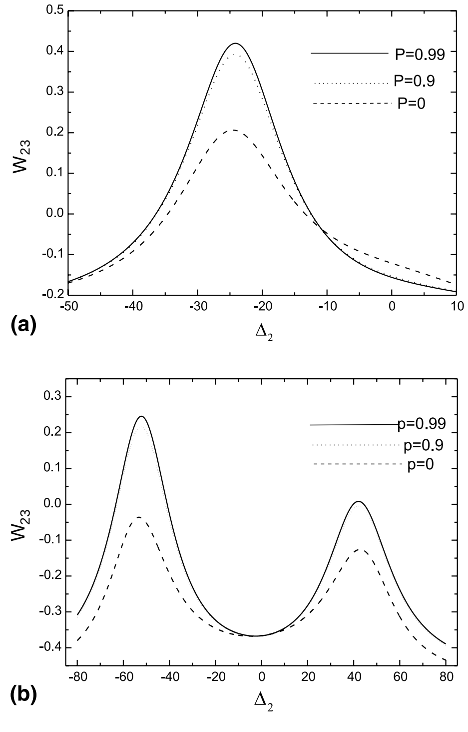

Fig. 2. The population inversion W_{23} against the detuning Δ_2 for three different parameters p. Solid line, dotted line, dashed line correspond to $p = 0.99$, 0.9, 0.0 respectively. The parameters are $\Omega_1 = 15\gamma$, $\Omega_2 = 10\gamma$, $\gamma_1 = 6\gamma$, $\gamma_2 = 1.5\gamma$, $\Delta_1 = 20\gamma$ in (a). In (b), $\Omega_1 = 50\gamma$, $\Delta_1 = 10\gamma$, the other parameters are the same with (a).

with [10]. The behavior of the system for small values of p has no much changes. However, by increasing the p parameter to 0.99, the maximum population inversion increases to approximately 42%. So, we can see that the population inversion in the case of large p becomes much larger than the case of $p = 0$ due to the effects of spontaneous emission-induced coherence. If we detune the interaction energy for the first transition and the frequency detuning, so that $\Omega_1 = 50\gamma$, $\Delta_1 = 10\gamma$, and at the same time let the decay rate and the second energy transition be the same with Fig. 2(a), we find the population inversion has a maximum value of 25% in the case of $p = 0.99$ in Fig. 2(b). But with these parameters, no population inversion occurs for $p = 0$. After taking the coherence effects into account, the population inversion can occur and reach a large value. Therefore, we can say that in this case, physical origin for generating population inversion can be considered as the spontaneous emission-induced coherence.

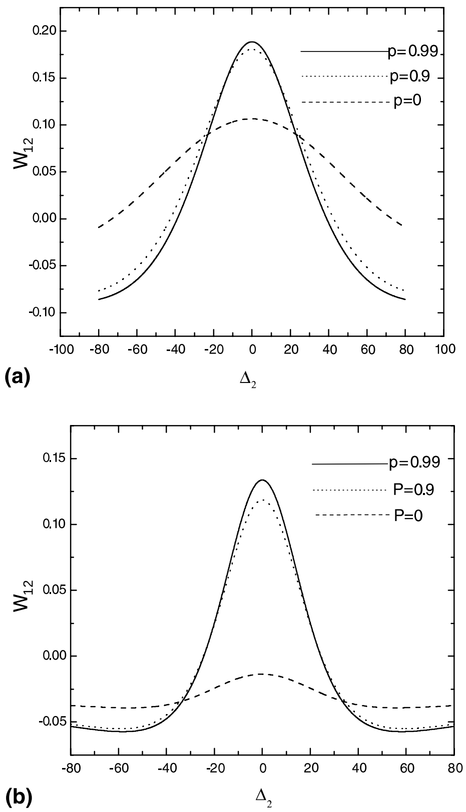

Fig. 3. The similar curves as Fig. 2, but for W_{12}. Values of the parameters are $\Omega_1 = 50\gamma$, $\Omega_2 = 40\gamma$, $\gamma_1 = 6\gamma$, $\gamma_2 = 30\gamma$, $\Delta_1 = 0$ in (a). In (b), the parameters are $\Omega_1 = 20\gamma$, $\Omega_2 = 10\gamma$, $\gamma_1 = 5\gamma$, $\gamma_2 = 30\gamma$, $\Delta_1 = 0$.

Similar results are presented in Fig. 3, which plots the steady-state population inversion $W_{12} = (\rho_{22} - \rho_{11})$ as a function of the laser detuning Δ_2 for three different parameter p. With the parameters $\Omega_1 = 50\gamma$, $\Omega_2 = 40\gamma$, $\gamma_1 = 6\gamma$, $\gamma_2 = 30\gamma$, $\Delta_1 = 0$, Fig. 3(a) shows a maximum population inversion of the value of 0.19. Also, Fig. 3(b) shows the large value of approximately 0.13 with $\Omega_1 = 20\gamma$, $\Omega_2 = 10\gamma$, $\gamma_1 = 5\gamma$, $\gamma_2 = 30\gamma$, $\Delta_1 = 0$.

4. Population inversion on both of the optical transitions

In [10], Whitley and Stroud investigated the population inversion of the ladder-type atomic system and showed that it is possible to have $\rho_{22} > \rho_{11}$ or $\rho_{33} > \rho_{22}$, but the two population inversions cannot be simultaneously reached. That is, the population inversion can be only achieved on one of the optical transitions. However, we will show that the unexpected population inversion on both of the optical transitions can be established in this system due to the effects of spontaneous emission-induced coherence. Choosing the same parameters $\Omega_1 = 2\Omega_2 = 50\gamma$, $\gamma_1 = \gamma_2 = 8\gamma$, $\Delta_1 = 10\gamma$, we plot the steady-state population inversion W_{23} and W_{12} as a function of Δ_2 for different parameter p in Fig. 4. From Fig. 4(a), we can see that the population inversion W_{23} reaches a large inversion of approximately 12% for $p = 0.99$. At the same time, we can obtain the steady-state population inversion W_{12} of values of approximately 4.5% in Fig. 4(b). Thus, by just tuning the coherent field frequency detuning to the corresponding regions, the steady-state population inversion on both of the optical transitions can occur due to the effects of spontaneous emission-induced coherence.

5. Conclusions

In this paper, we investigated the effects of spontaneous emission-induced coherence on pop-

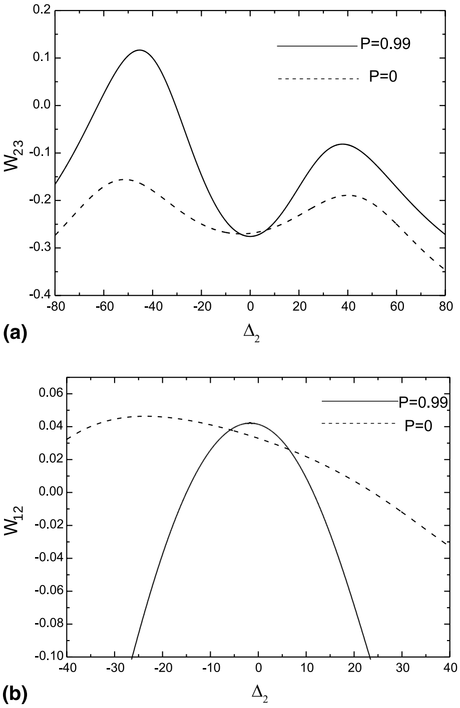

Fig. 4. Plots of W_{23} and W_{12} against Δ_2 in the case of $p = 0$ (dashed line), $p = 0.99$ (solid line) in (a) and the parameters are $\Omega_1 = 2\Omega_2 = 50\gamma$ and $\Delta_1 = 10\gamma,\ \gamma_1 = \gamma_2 = 8\gamma$.

ulation inversion in the ladder-type three-level system. We found that, even if there is no inversion in the absence of spontaneous emission-induced coherence, a large inversion on one of the optical transitions can be attained when the effects of this kind of coherence become more obvious. Furthermore, we also found that, within suitable parameters regions, this kind of coherence can lead to unexpected population inversion on both of the optical transitions.

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