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Aspects of light propagation in anisotropic dielectric media

V.A. De Lorenci *, J.M. Salim

[a] Instituto de Ciências Exatas, Universidade Federal de Itajubá, Av. BPS 1303 Pinheirinho, 37500-903 Itajubá, MG, Brazil
[b] Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150 Urca, 22290-180 Rio de Janeiro, RJ, Brazil

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Abstract

Some aspects of light propagation in local anisotropic nonlinear dielectric media at rest in the limit of geometrical optics are investigated. Natural and artificially induced anisotropies in dielectric materials are discussed. Analogies are proposed in such way that, as far as light is considered, kinematic aspects of some cosmological models are recovered. Particularly, analogue models for isotropic and anisotropic cosmologies are presented.

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1. Introduction

Inside material media electrodynamics becomes nonlinear. In such situations the Maxwell equations must be supplemented with constitutive relations which, in general, are nonlinear and depend on the physical properties of the medium under the action of external fields. As a consequence, several effects (nonusual in the context of linear Maxwell theory) are predicted. Of actual interest is the phenomenon of artificial birefringence: when an external field is applied in a medium with nonlinear dielectric properties, an artificial optical axis may appear [1–5].

The development of analogies in order to test kinematic aspects of general relativity in laboratory have been performed in several branches of physics [2,3,5–12]. Particularly, nonlinear electrodynamics has been considered as a possible scenario to construct analogue models for general relativity, either in the context of nonlinear Lagrangian or nonlinear material media. This is based in the fact that the trajectory of photons can be described by a null geodesic in an effective metric $g_{\mu\nu}$. In this work, homogeneous dielectric media at rest with the dielectric coefficients $\varepsilon^{\mu}{}_{\nu}(\vec{E})$ and constant μ, in the limit of geometrical optics, are used to construct analogue models for cosmology. The analysis is restricted to local electrodynamics. In order to avoid ambiguities with the wave velocity, dispersive effects were neglected by considering only monochromatic waves. It is shown that naturally uniaxial media presenting nonlinear dielectric properties can be operated by external fields in such way to induce anisotropy in the optical metric.

A covariant formalism is used throughout this work. Spacetime is assumed to be Minkowskian, and a Cartesian coordinate system is used, such that the metric is $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. Units are chosen such that $c = 1$. A geodetic observer $V^{\mu} = \delta^{\mu}_{0}$ is supposed to describe all quantities. Particularly the electric field is represented by $E^{\mu} = -F^{\mu\nu}V_{\nu} = (0, \vec{E})$ whose modulus is $E = (-E^{\alpha}E_{\alpha})^{1/2}$.

2. The dispersion relation

The properties of light propagation in material media are determined by the so-called dispersion relations, which can be derived, in the context of the eikonal approximation of electrodynamics, making use of the method of field discontinuities [13]. Define a surface of discontinuity Σ by $z(x^{o}, \vec{x}) = 0$. Whenever Σ is an inextendible surface, it divides the spacetime in two disjoint regions U^{-} for $z(x^{o}, \vec{x}) < 0$, and U^{+} for $z(x^{o}, \vec{x}) > 0$.

* Corresponding author.
E-mail addresses: delorenci@unifei.edu.br (V.A. De Lorenci), jsalim@cbpf.br (J.M. Salim).

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The discontinuity of an arbitrary function $f(x^o,\vec{x})$ on Σ is given by

$$[f(x^o,\vec{x})]_\Sigma \doteq \lim_{\{P^\pm\}\to P}[f(P^+)-f(P^-)] \tag{1}$$

with P^+, P^- and P belonging to U^+, U^- and Σ, respectively. The electric and magnetic fields are continuous when crossing the surface Σ. However, their derivatives behave as

$$[E^\mu{}_{,\nu}]_\Sigma = e^\mu K_\nu; \qquad [B^\mu{}_{,\nu}]_\Sigma = b^\mu K_\nu, \tag{2}$$

where e^μ and b^μ represent the discontinuities of the fields on the surface Σ and

$$K_\lambda = \frac{\partial\Sigma}{\partial x^\lambda} \tag{3}$$

is the wave vector.

When these conditions are applied to the electrodynamic field equations in local anisotropic dielectric media, the following dispersion relation for light rays [5] are obtained:

$$g_\pm^{\lambda\tau}K_\lambda K_\tau = \left\{\mu\alpha V^\lambda V^\tau + \frac{1}{2}\left[C^\nu{}_\nu - \frac{1}{\mu(v_\varphi^\pm)^2}\right]C^{(\lambda\tau)} - \frac{1}{2}C^{(\lambda}{}_\nu C^{\nu\tau)}\right\}K_\lambda K_\tau = 0, \tag{4}$$

where

$$C^\alpha{}_\tau \doteq \varepsilon^\alpha{}_\tau + \frac{\partial\varepsilon^\alpha{}_\beta}{\partial E^\tau}E^\beta + \frac{1}{\omega}\frac{\partial\varepsilon^\alpha{}_\beta}{\partial B^\rho}\eta^{\rho\lambda\gamma}{}_\tau E^\beta K_\lambda V_\gamma \tag{5}$$

and the phase velocities $v_\varphi^\pm$ are

$$v_\varphi^\pm = \sqrt{\frac{\beta}{2\alpha}\left(1\pm\sqrt{1+\frac{4\alpha\gamma}{\beta^2}}\right)}, \tag{6}$$

with $\omega \doteq K^\alpha V_\alpha$ the frequency of the electromagnetic wave and the coefficients α, β and γ given by

$$\alpha \doteq \frac{1}{6}\left[(C^\mu{}_\mu)^3 - 3C^\mu{}_\mu C^\alpha{}_\beta C^\beta{}_\alpha + 2C^\alpha{}_\beta C^\beta{}_\gamma C^\gamma{}_\alpha\right], \tag{7}$$

$$\beta \doteq \mu^{-1}(C^\lambda{}_\alpha C^{\alpha\nu} - C^\alpha{}_\alpha C^{\lambda\nu})\hat{q}_\lambda\hat{q}_\nu, \tag{8}$$

$$\gamma \doteq \mu^{-2}C^{\lambda\nu}\hat{q}_\lambda\hat{q}_\nu. \tag{9}$$

In the last two Eqs. (8)–(9) we introduced the 3-dimensional projection of the wave vector K^α as $q^\alpha = h^\alpha_\mu K^\mu = K^\alpha - \omega V^\alpha$, and $\hat{q}^\mu = q^\mu/q$. We defined the projector $h^\alpha_\mu = \delta^\alpha_\mu - V^\alpha V_\mu$.

The symmetric tensors $g_\pm^{\mu\nu}$ are the optical coefficients[1] associated with the wave propagation, and the symbol $\pm$ indicates the possibility of two distinct coefficients, one for each polarization mode—birefringence phenomena. Correspondingly to it Eq. (6) expresses the fact that, in general, the phase velocity of the electromagnetic waves inside a material medium may get two possible values $(v_\varphi^+, v_\varphi^-)$ which are associated with the two possible polarization modes. For the particular case of Maxwell linear theory in vacuum, both $g_+^{\mu\nu}$ and $g_-^{\mu\nu}$ reduce to the diagonal matrix $(+1,-1,-1,-1)$, which is identified with the Minkowski metric $\eta^{\mu\nu}$, as expected. Other particular cases are obtained by considering isotropic media [2,3,7] and some applications was recently proposed in the context of dielectric analogues of black hole spacetime [9].

[1] Such coefficients are called in the literature as the components of the optical metric, meanly in the case where they does not depend on the direction of light propagation.

3. Naturally anisotropic uniaxial media

Now, let us consider naturally anisotropic uniaxial media reacting nonlinearly when subjected to an external electric field as $\varepsilon^\alpha{}_\beta = \mathrm{diag}[0,\varepsilon_\parallel(E),\varepsilon_\perp(E),\varepsilon_\perp(E)]$. In this case $\varepsilon^\alpha{}_\beta = \varepsilon^\alpha{}_\beta(E)$ and by setting $\vec{E}$ in the x-direction (optical axis) we obtain $C^\alpha{}_\beta = \mathrm{diag}(0,\varepsilon_\parallel + E\varepsilon'_\parallel,\varepsilon_\perp,\varepsilon_\perp)$, where $\varepsilon'_\parallel = d\varepsilon_\parallel/dE$. For this particular case $C^{\alpha\beta}$ is a symmetric tensor. The phase velocities reduce to

$$(v_\varphi^+)^2 = \frac{1}{\mu\varepsilon_\perp}, \tag{10}$$

$$(v_\varphi^-)^2 = \frac{1}{\mu\varepsilon_\perp C^1{}_1}\left[\varepsilon_\perp(1-\hat{q}_1{}^2) + C^1{}_1\hat{q}_1{}^2\right]. \tag{11}$$

Note that v_φ^- depends on the direction of propagation, as it should be expected for the extraordinary ray. The two velocities coincide when either the propagation occurs along the direction of the electric field ($\hat{q}_1{}^2 = 1$), or when the no-birefringence condition $\varepsilon_\parallel + E\varepsilon'_\parallel = 0$ holds [5].

Let us also particularize to the model where

$$\varepsilon_\perp = \epsilon_\perp - 3pE^2, \tag{12}$$

$$\varepsilon_\parallel = \epsilon_\parallel - sE^2. \tag{13}$$

Thus, $C^\alpha{}_\beta = \mathrm{diag}(0,\epsilon_\parallel - 3sE^2,\epsilon_\perp - 3pE^2,\epsilon_\perp - 3pE^2)$.

For the ordinary ray the optical coefficients are

$$g_+^{00} = \mu\alpha, \tag{14}$$

$$g_+^{ii} = -\epsilon_\parallel\epsilon_\perp + 3(s\epsilon_\perp + p\epsilon_\parallel)E^2 - 9spE^4, \tag{15}$$

where

$$\alpha = -27sp^2E^6 + 9p(p\epsilon_\parallel + 2s\epsilon_\perp)E^4 - 3\epsilon_\perp(s\epsilon_\perp + 2p\epsilon_\parallel)E^2 + \epsilon_\parallel\epsilon_\perp^2. \tag{16}$$

Eqs. (14)–(15) show that for the ordinary ray there will be no anisotropy in the space section.

For the extraordinary ray the optical coefficients are

$$g_-^{00} = \mu\alpha, \tag{17}$$

$$g_-^{11} = -(\chi - \epsilon_\parallel + 3sE^2)(\epsilon_\parallel - 3sE^2), \tag{18}$$

$$g_-^{22} = g_-^{33} = -(\chi - \epsilon_\perp + 3pE^2)(\epsilon_\perp - 3pE^2), \tag{19}$$

where χ depends on the direction on wave propagation as

$$\chi = -\frac{(\epsilon_\parallel - 3sE^2)(\epsilon_\perp - 3pE^2)}{(\epsilon_\perp - 3pE^2)(1-\hat{q}_1{}^2) + (\epsilon_\parallel - 3sE^2)\hat{q}_1{}^2} + (\epsilon_\parallel - 3sE^2) + 2(\epsilon_\perp - 3pE^2). \tag{20}$$

Eqs. (17)–(19) show that for the extraordinary ray there will be anisotropy in the section ($g_-^{11} \neq g_-^{22} = g_-^{33}$).

Note that when the propagation occurs in the direction of the optical axes it follows $\chi|_{\hat{q}_1{}^2=1} = (\epsilon_\parallel - 3sE^2) + (\epsilon_\perp - 3pE^2)$ and $g_-^{11} = g_-^{22} = g_-^{33} = -(\epsilon_\parallel - 3sE^2)(\epsilon_\perp - 3pE^2)$. By the other hand, when the propagation occurs perpendicularly to the optical axes it follows $\chi|_{\hat{q}_1{}^2=0} = 2(\epsilon_\perp - 3pE^2)$ and the anisotropy remains.

4. Artificially anisotropic uniaxial media

In this section, artificially induced anisotropy is considered. Let us set the permittivity tensor as

$$\varepsilon^{\mu\nu} = \varepsilon h^{\mu\nu} - \lambda E^\mu E^\nu \tag{21}$$

with ε and λ constants (just for simplicity). Eq. (21) indicates that the anisotropy in the dielectric coefficients is operated by the action of an external electric field, which will also be responsible for the anisotropy in the resulting optical coefficient for the extraordinary ray. Indeed, from Eq. (4) it follows that

$$g_-^{\mu\nu} = \eta^{\mu\nu} + \left[\mu\left(\varepsilon + 3\lambda E^2\right) - 1\right]V^\mu V^\nu - \frac{2\lambda}{(\varepsilon + \lambda E^2)} E^\mu E^\nu. \tag{22}$$

Note that, in the absence of the external electric field, the metric gets an isotropic form [7], as expected.

5. Analogue gravity

It can be shown that the integral curves of the vectors $K_\mu^\pm$ are geodesics in an effective geometry whose components are the optical coefficients $g_\pm^{\mu\nu}$ [3]. By using this formulation, this section concerns the construction of models for the extraordinary rays[2] in such way that the paths of light can be described by means of the following line element

$$ds^2 = C\,dt^2 - A\,dx^2 - B\left(dy^2 + dz^2\right). \tag{23}$$

Since the components of $g_-^{\mu\nu}$ constitute a diagonal matrix it follows that $g_{ii}^- = 1/g_-^{ii}$. Thus $A = -1/g_-^{11}$, $B = -1/g_-^{22}$ and $C = 1/g_-^{00}$. Now, using the dispersion relation $g_-^{\mu\nu} K_\mu K_\nu = 0$ for light rays, which enables us to redefine the metric up to a conformal factor, Eq. (23) yields

$$ds^2 = dt^2 + \frac{g_-^{00}}{g_-^{11}} dx^2 + \frac{g_-^{00}}{g_-^{22}}\left(dy^2 + dz^2\right). \tag{24}$$

In what follows some analogies will be constructed using the effective geometry formalism for the cases presented in Sections 3 and 4.

5.1. Analogue model for FRW cosmology

By setting $s = p$ and $\epsilon_\perp = \epsilon_\parallel \doteq \epsilon$ in the results presented in the Section 3, an analogue model for Friedmann–Robertson–Walker (FRW) cosmology can be obtained. In this way, we are dealing with isotropic media. From Eqs. (16) and (20) we obtain $\alpha = (\epsilon - 3pE^2)^3$ and $\chi = 2(\epsilon - 3pE^2)$, respectively. Thus, the effective metric for light yields

$$ds^2 = dt^2 - \mu\left(\epsilon - 3pE^2\right)\left(dx^2 + dy^2 + dz^2\right). \tag{25}$$

Since the external electric field can be adjusted to be a function of time, the above effective metric can be used to simulate FRW cosmology.

5.2. Analogue model for Bianchi-I cosmology

A simple toy model for Bianchi type I cosmology can be produced from the results presented in Section 4 by setting the external electric field in the X-direction (optical axes), resulting in the following components for the effective metric

$$g_-^{00} = \mu\left(\varepsilon + 3\lambda E^2\right), \tag{26}$$

$$g_-^{11} = -\frac{\varepsilon + 3\lambda E^2}{\varepsilon + \lambda E^2}, \tag{27}$$

$$g_-^{22} = g_-^{33} = -1. \tag{28}$$

Now, by inserting the above equations in Eq. (24), it yields

$$ds^2 = dt^2 - \mu\left(\varepsilon + \lambda E^2\right) dx^2 - \mu\left(\varepsilon + 3\lambda E^2\right)\left(dy^2 + dz^2\right), \tag{29}$$

which, once $E = E(t)$, appears to be a Bianchi-I metric from the classification of homogeneous geometries. It should be pointed out that the redshift induced by this effective metric in the X-direction is different from the redshift induced in any direction on the plane YZ.

More elaborated models can be produced by choosing ε and λ as general functions of the external field $\vec{E}$. Yet, it is possible to produce models by interchanging natural anisotropy with artificially induced anisotropy in the dielectric coefficients. Therefore, when natural anisotropy is present, the optical coefficients appear to be dependent on the direction of light propagation, and the construction of analogies with gravity requires a more detailed analysis.

6. Conclusion

Working with propagation of monochromatic electromagnetic waves inside naturally anisotropic material media with nonlinear dielectric properties, analogue models for general relativity presenting isotropy and anisotropy in the space sections were constructed. These models are based on the idea that the trajectory of the photons can be described as a null geodesic in an effective metric. Indeed, light propagation in local anisotropic media can be used as a tool for testing kinematic aspects of cosmological models in laboratory. It must be noted, however, that the analogue model is only valid as light propagation is considered. Material particle propagates inside dielectric media without any relationship with the effective optical metric.

The optical coefficients in Eq. (4) present an explicit dependence on the wave vector K^μ, i.e., on the direction of light propagation. In this way it is also possible to look for models where the properties of the 'effective spacetime' depend on the

[2] It can be shown that the propagation associated with the ordinary ray always occurs isotropically. Indeed, it can also be used to construct an analogue model for Friedman cosmology.

direction of light propagation. In these cases there is no direct analogy with gravitational phenomena, in despite of the fact that the effective geometry approach still holds in the description of light propagation. However it should be stressed that the effective metrics presented in Section 5 for isotropic and anisotropic cosmologies [Eqs. (25) and (29), respectively] were constructed from the general optical coefficients in Eq. (4), for physical situations where the dependency on the direction of light propagation disappears. Then, such effective metrics can be used to simulate the corresponding fundamental metrics in general relativity.

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