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The effect of an intense electromagnetic wave on the dynamics of stimulated bremsstrahlung

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Abstract. Charged particle potential scattering in both arbitrary scattering and intense monochromatic electromagnetic wavefield is investigated within the framework of relativistic classical theory, treating the wavefield exactly and the scattering potential by perturbation to first order. The effect of strong wavefields on the coherent energy change of the particle in this process is revealed for large (exceed the initial momentum of the particle) momentum transfers of the scattering potential. The rate of the coherent energy exchange of an electron beam with an intense electromagnetic wave due to the stimulated bremsstrahlung process is also obtained, particularly for the high-intensity limit.

1. Introduction

Many papers have been devoted to the theoretical investigation of the electron-atom or -ion scattering processes in the presence of a laser field using quantal as well as classical considerations. These investigations have been carried out mainly within the framework of non-relativistic quantum theory, which is valid for laser fields that are not very strong. Only in a few papers (Denisov and Fedorov 1968, Kaminski 1985) have fields of arbitrary intensities been considered and the relativistic cross sections of stimulated bremsstrahlung (SB) been calculated to the first Born (Denisov and Fedorov 1968) and low-frequency (Kaminski 1985) approximations. In addition, relativistic corrections have been taken into account in single photon SB absorption-emission cross sections in the weak laser field (Fedorov 1966) and in the dynamics of the SB process for wavefields that are not very strong (Avetissian and Jivanian 1980, Avetissian *et al* 1986).

The purpose of this paper is to reveal the effect of strong monochromatic electromagnetic (EM) fields on the dynamics of the SB process, which will also enable one to clarify the possibility of electron acceleration in practice during free-free transitions related to the renormalization of electron energy in the strong wavefield in a vacuum. This renormalization can be written in the following form (Brown and Goble 1968):

$$\tilde{\mathcal{E}} = \mathcal{E}_i(1 + \alpha\xi^2)$$

where $\tilde{\mathcal{E}}$ is the mean energy of the electron in the wave, \mathcal{E}_i is the initial energy of the electron, ξ is the relativistic intensity parameter and α is a constant. This renormalization has a virtual nature because, in the absence of a third body (in our case without the scattering centre) a free electron cannot really absorb or emit a photon. From the classical viewpoint the electron energy oscillates in the wavefield and, after the wave is switched off, when $\xi \rightarrow 0$, again $\mathcal{E} = \mathcal{E}_i$. In the presence of the scattering centre the

corresponding part of the energy conditioned by the intensity of the wavefield can become real and, hence, the magnitude of acceleration will be conditioned by the full dynamics of the interaction process (the part of the energy that has been transferred from wave to electron and remains with it depends on the momentum that the scattering centre can compensate during the scattering process). In our previous papers (Avetissian and Jivanian 1980, Avetissian *et al* 1986) this problem was investigated without taking into account the intensity effect of the wavefield, i.e. the dependence $\mathcal{E}(\xi^2)$. Such an approximation is valid up to a certain wave strength. In these works the coherent (over the wavefield) energy exchange depending on the initial wave phase was derived. It has a periodic nature and, therefore, the effective energy exchange of the beam vanishes when it is averaged over the initial phases. Thus the results obtained strictly concern the coherent SB process. One can expect that accounting for the field intensity will violate the strictly periodic dependence on the initial phase of the wave, and will change the behaviour of the effective energy exchange of the electron beam in the SB process. Thus, in order to clarify the possibility of a single electron acceleration by a strong wavefield in the SB process or to study the wave absorption by an electron beam one needs an exact investigation of SB dynamics, taking into account the intensity effect of a strong wavefield on the bremsstrahlung scattering process.

Within the framework of classical theory the SB process has also been studied by Brown and Goble (1968), Pert (1972), Bunkin *et al* (1972) and Kroll and Watson (1973); however, the scattering is taken into account in the instantaneous interaction, or low-frequency approximations, which are applicable when $\tau \ll T$, where τ is the scattering duration and T the wave period. We shall exceed the limits of this approximation to take into account the dynamics of SB scattering. In this case one is able to account for the wavefield exactly if the scattering potential is treated perturbatively. However, we note that this approximation does not actually mean weak scattering potentials, because the interaction with the scattering centre is treated perturbatively not with respect to the electron initial energy (in that case one cannot expect the essential acceleration of electrons because the energy change would be less than the electron initial energy) but with respect to its energy in a strong wavefield, which can exceed many times the electron initial energy. In this case electrons can be initially non-relativistic, or even at rest. For the strong laser fields under consideration the approximation when applied in practice does not confine even the potentials of existing very strong scattering centres and, hence, one can consider that the results obtained do not practically contain the approximation.

This paper is organized as follows. In section 2 we outline the classical theory of SB and derive the energy change of a single electron in the SB process, treating the scattering potential to the first order of perturbation theory. This general result is analysed in detail in section 3 for the screened Coulomb potential and for the geometry when an electron falls in the wave propagation direction. We also consider, in section 4, the effective energy exchange of an electron beam with an intense wave arising from SB. A summary and conclusions are presented in section 5.

2. Induced energy exchange of electrons with a strong electromagnetic wavefield in the SB process

In the classical consideration of SB our concern is with the electron energy change. To derive an expression for it we shall start by presenting the relativistic Hamilton-

Jacobi equation for electron scattering by a static potential $\Phi(\mathbf{r})$ in a plane transverse monochromatic EM wavefield $\mathbf{A}(t - \nu\mathbf{r}/c)$ (Landau and Lifshitz 1973),

$$\left[\nabla S - \frac{e}{c} \mathbf{A} \left(t - \frac{\nu\mathbf{r}}{c} \right) \right]^2 - \frac{1}{c^2} \left(\frac{\partial}{\partial t} S - e\Phi(\mathbf{r}) \right)^2 + m^2 c^2 = 0 \tag{2.1}$$

where e and m are the electron charge and mass, respectively, and ν is a unit vector in the wave propagation direction.

Introducing the new coordinates $\mathbf{r} = \mathbf{r}$ and $\eta = t - \nu\mathbf{r}/c$ we write (2.1) in the following form:

$$\left(\nabla S(\mathbf{r}, \eta) - \frac{e}{c} \mathbf{A}(\eta) \right)^2 - 2 \left(\nu \nabla S(\mathbf{r}, \eta) + \frac{e}{c} \Phi(\mathbf{r}) \right) \frac{\partial}{\partial \eta} S(\mathbf{r}, \eta) - \frac{e^2}{c^2} \Phi(\mathbf{r}) + m^2 c^2 = 0 \tag{2.2}$$

where $\nabla = \partial/\partial\mathbf{r}$. We seek the solution of (2.2) in the form

$$S(\mathbf{r}, \eta) = S_0(\mathbf{r}, \eta) + S_1(\mathbf{r}, \eta) \tag{2.3}$$

$$S_0(\mathbf{r}, \eta) = -\mathcal{E}_i \eta + \mathbf{p}_i \cdot \mathbf{r} - \frac{e}{\nu \mathbf{p}_i} \int \left(\mathbf{p}_i \cdot \mathbf{A}(\eta) - \frac{e}{2c} A^2(\eta) \right) d\eta$$

where $S_0(\mathbf{r}, \eta)$ is the electron relativistic action in the EM field, $\mathbf{p}_i = \mathbf{p}_i - \nu \mathcal{E}_i/c$, and \mathbf{p}_i and \mathcal{E}_i are the electron initial momentum and energy, respectively. Inserting (2.3) into (2.2) we obtain for $S_1(\mathbf{r}, \eta)$ the following equation:

$$\left(\nabla S_1 \right)^2 + 2 \left(\nabla S_1 - \frac{\nu}{c} \frac{\partial}{\partial \eta} S_1 \right) \mathbf{p}_0(\eta) + \frac{2}{c} \left(\mathcal{E}_0(\eta) - \frac{\partial}{\partial \eta} S_1 \right) \left(\nu \nabla S_1 + \frac{e}{c} \Phi(\mathbf{r}) \right) - \frac{e^2}{c^2} \Phi(\mathbf{r}) = 0 \tag{2.4}$$

where

$$\mathbf{p}_0(\eta) = \mathbf{p}_i - \frac{e}{c} \mathbf{A}(\eta) \quad \mathcal{E}_0(\eta) = \mathcal{E}_i + \frac{e}{\nu \mathbf{p}_i} \left(\mathbf{p}_i \cdot \mathbf{A}(\eta) - \frac{e}{2c} A^2(\eta) \right) \tag{2.5}$$

are the electron momentum and energy in the EM wavefield, respectively.

We shall solve (2.4) approximately, considering the strong EM wavefield exactly, applying perturbation theory to account for the scattering potential. In order to treat the scattering potential perturbatively we require that

$$\chi \equiv U_{\max} c^2 / (\mathcal{E}_0(\eta) v_0^2(\eta)) \ll 1 \tag{2.6}$$

for any η , i.e. during motion in both scattering and EM fields the momentum of the electron is always close to the momentum of an electron which interacts only with the EM wave. In (2.6) U_{\max} is the maximum potential energy which an electron can have when scattered in the presence of an EM wave, and $v_0(\eta)$ is the electron velocity in the wave.

Confining ourselves to the first order of perturbation theory, we leave in (2.4) only terms of order χ , thus obtaining

$$\nabla S_1 \left(\mathbf{p}_0(\eta) + \frac{\nu}{c} \mathcal{E}_0(\eta) \right) - \frac{1}{c} \frac{\partial}{\partial \eta} S_1 \nu \mathbf{p}_0(\eta) = -\frac{e}{c^2} \mathcal{E}_0(\eta) \Phi(\mathbf{r}). \tag{2.7}$$

One can easily solve (2.7), making a three-dimensional Fourier transform over the space coordinates \mathbf{r} , using the initial and boundary conditions described below. At large distances from the scattering centre, where the scattering field vanishes, the particle moves only in the EM wavefield, and therefore $S_1 = 0$ when $\mathbf{r} \rightarrow \infty$. Also, as the wave is switched on adiabatically at $\eta \rightarrow -\infty$, then $S_1(\eta \rightarrow -\infty) = 0$ too. Thus, after a straightforward calculation we write the solution of (2.7) in the form

$$S_1(\mathbf{r}, \eta) = \frac{e}{c} \frac{(2\pi)^{-3}}{\nu \mathbf{p}_i} \int \left(\tilde{\Phi}(\mathbf{q}) \exp[i\mathbf{q}(\mathbf{r} - \mathbf{r}_0(\eta))] \right) \times \int_{-\infty}^{\eta} \mathcal{E}_0(\tau) \exp(i\mathbf{q} \mathbf{r}_0(\tau)) d\tau \, d^3\mathbf{q}. \quad (2.8)$$

Here $\tilde{\Phi}(\mathbf{q})$ denotes the Fourier transform of the potential and $\mathbf{r}_0(\eta)$ is the electron trajectory in the EM wave (integration constants equal to zero),

$$\mathbf{r}_0(\eta) = -\frac{c}{\nu \mathbf{p}_i} \int \mathbf{p}_0(\eta) d\eta \quad \mathbf{p}_0(\eta) = \mathbf{p}_0(\eta) + \frac{\nu}{c} \mathcal{E}_0(\eta). \quad (2.9)$$

Now let us deal with the electron energy change. Following well known procedures (Landau and Lifshitz 1973), one can find the electron energy change in the SB process, up to the first order of perturbation theory over the scattering potential as the expression $\mathcal{E}_i = -\partial S_1 / \partial \eta$ at the moment $\eta \rightarrow +\infty$, by substituting the electron trajectory in the EM wave $\mathbf{r}(\eta) = \mathbf{r}_0 + \mathbf{r}_0(\eta)$ for \mathbf{r} (\mathbf{r}_0 is the radius vector of the electron at $\eta = 0$). After straightforward calculations one has

$$\Delta \mathcal{E} = \mathcal{E}_i(\eta = +\infty) = -\frac{ie(2\pi)^{-3}}{(\nu \mathbf{p}_i)^2} \int \left(\mathbf{q} \mathbf{p}_i \tilde{\Phi}(\mathbf{q}) \exp(i\mathbf{q} \mathbf{r}_0) \int_{-\infty}^{+\infty} \mathcal{E}_0(\tau) \exp(i\mathbf{q} \mathbf{r}_0(\tau)) d\tau \right) d^3\mathbf{q}. \quad (2.10)$$

Thus, (2.10) presents the induced energy change of an electron with a plane EM wave of arbitrary polarization and field strength in an SB process. Moreover, (2.10) holds for any type of static potential satisfying criterion (2.6). However, to proceed in the evaluation of (2.10) one has to concretize the polarization or potential type. Leaving the scattering potential in a general form let us carry out integration over η for a monochromatic EM wave of frequency ω . As we shall see below, contrary to the weak field case, SB in the strong field of an EM wave is sensitive to the type of wave polarization. The reason for this is that the electron impact parameter ρ is changed essentially during the scattering process, depending upon the type of EM wave polarization caused by large amplitudes of electron oscillations in the strong wavefield. Therefore, we shall consider separately two types of polarizations, circularly and linearly polarized wavefields. These are described by, respectively,

$$\mathbf{A} = A_0[\boldsymbol{\varepsilon}_1 \cos(\omega\eta + \phi) - \boldsymbol{\varepsilon}_2 \sin(\omega\eta + \phi)] \quad (2.11a)$$

$$\mathbf{A} = A_0 \boldsymbol{\varepsilon} \cos(\omega\eta + \phi) \quad (2.11b)$$

where A_0 is the vector potential and ϕ is the initial wave phase.

Inserting the expressions for electron energy and trajectory in the wave respectively from (2.5) and (2.9) into (2.10) and using (2.11a) and (2.11b), one easily integrates over η with the help of the following well known expansion in Bessel functions of the first kind, J_n :

$$\exp(iz \sin \phi) = \sum_{n=-\infty}^{+\infty} J_n(z) \exp(in\phi).$$

As a result of calculations we obtain the following expression for the electron energy change:

$$\Delta \mathcal{E} = \sum_{n=-\infty}^{+\infty} \Delta \mathcal{E}^{(n)} \tag{2.12}$$

$$\Delta \mathcal{E}^{(n)} = -\frac{ie(2\pi)^{-2}}{(\nu p_i)^2} \int \left(\mathbf{q} p_i \tilde{\Phi}(\mathbf{q}) f_n(\mathbf{q}) \exp(i\mathbf{q} \mathbf{r}_0 - in\phi) \delta \left(n + \frac{c}{\omega} \frac{\mathbf{q} \tilde{p}_i}{\nu p_i} \right) \right) d^3 \mathbf{q}.$$

Here, for circularly and linearly polarized EM waves respectively, the functions $f_n(\mathbf{q})$ are defined as

$$f_n^{(c)}(\mathbf{q}) = \frac{\tilde{\mathcal{E}}_i}{\omega} J_n(aQ) e^{-in\phi}$$

$$-\frac{a}{2} p_i [(\epsilon_1 + i\epsilon_2) J_{n+1}(aQ) e^{-i(n+1)\phi} + (\epsilon_1 - i\epsilon_2) J_{n-1}(aQ) e^{-i(n-1)\phi}] \tag{2.13a}$$

$$f_n^{(l)}(\mathbf{q}) = \sum_{s=-\infty}^{+\infty} \left(\frac{\tilde{\mathcal{E}}_i}{\omega} - n \frac{\epsilon p_i}{\epsilon Q} - 2s \frac{\epsilon q}{\epsilon Q} \frac{\nu p_i}{\nu q} \right) J_{2s+n}(a\epsilon Q) J_s \left(\frac{a^2}{8} \frac{\omega}{c} \nu q \right) \tag{2.13b}$$

$$(\epsilon_1 + i\epsilon_2) Q = Q e^{i\phi} \quad Q = \mathbf{q} - p_i \frac{\nu \mathbf{q}}{\nu p_i}$$

where

$$\tilde{\mathcal{E}}_i = \mathcal{E}_i + \Delta \quad \tilde{p}_i = p_i + \nu \frac{\Delta}{c} \quad \Delta = -\frac{e^2 A_0^2}{c \nu p_i} \begin{cases} \frac{1}{2} & \text{for CP} \\ \frac{1}{4} & \text{for LP} \end{cases}$$

are the average energy and momentum of the electron in the EM wave and $a = -eA_0/\omega\nu p_i$ is the amplitude of electron oscillations in the wave. Vector \mathbf{r}_0 in (2.12) is the radius vector of the electron in the coordinate system introduced above at the moment $\eta = 0$. If one chooses the start of the time count so that at the moment $t = 0$ the electron appears at the point closest to the origin of the coordinate system, then one can relate \mathbf{r}_0 to the ‘impact parameter vector’ $\boldsymbol{\rho}$ as follows:

$$\mathbf{r}_0 = \boldsymbol{\rho} - \tilde{p}_i \frac{\nu \boldsymbol{\rho}}{\nu p_i} \quad \mathbf{p} \tilde{\boldsymbol{\rho}} = 0. \tag{2.14}$$

Thus, (2.12) together with (2.14) determine the induced energy exchange of the electron with an intense EM wavefield in the SB process, when the strong EM field is considered exactly, whereas the scattering potential is determined by perturbation theory to first order. As (2.12) shows, in the strong EM wavefield the electron energy change is presented as a sum of partial energy changes $\Delta \mathcal{E}^{(n)}$ on frequencies $n\omega$. We note that those energy changes of the electron in the EM wave mainly contribute to $\Delta \mathcal{E}^{(n)}$, the averages of which are equal to the quantal changes of energy during the absorption or emission of n quanta in the SB process. In reality, the electron energy in the wave reads

$$\mathcal{E}_0 = -c \frac{m^2 c^2 + p_0^2}{2\nu p_0}. \tag{2.15}$$

Let us assume that in the SB process an electron has emitted or absorbed n quanta of an external EM wave and as a result its momentum is changed by $\hbar \mathbf{q}$, becoming $\mathbf{p}_0 + \hbar \mathbf{q}$. Then, with the help of (2.15) for the electron energy change, we obtain to the lowest

order of $q: \Delta \mathcal{E} = -\hbar q \mathbf{p}_0 / \nu \mathbf{p}_0$. If its average is equal to the energy of n quanta of the EM wave

$$\langle \Delta \mathcal{E} \rangle_{av} = -\hbar q \bar{p}_i / \nu p_i = \hbar \omega n$$

then, after cancelling \hbar out, we obtain the conservation law of the δ -function entering in (2.12), which provides the main contribution to $\Delta \mathcal{E}^{(n)}$.

Unfortunately, to date no quantal consideration of SB exists which directly corresponds to the classical result (2.12) for electron energy change, derived by treating the scattering potential perturbatively to first order. Such a quantal investigation is now in progress and the comparison of this classical result with quantal calculations will be considered elsewhere. Here we shall only elucidate the connection between the classical energy change (2.12) and the other classical results obtained by applying perturbation theory to first order over both scattering and EM fields (Avetissian and Jivanian 1980) or within the approximation of instantaneous interaction (Brown and Goble 1968, Kroll and Watson 1973). With this aim for low EM intensities, when the maximum of the classical oscillatory velocity of an electron in the wave is too small compared with the incident electron velocity, namely $v_{osc} \equiv \omega a \ll v_i$ (or writing otherwise $eA_0 \omega / cp_i \Omega \ll 1$, $\Omega = \omega(1 - \nu v_i/c)$ being the Doppler-shifted frequency of the wave), we keep in (2.12) only the terms linear in the wavefield and transform it as

$$\Delta \mathcal{E} = -a \frac{e}{c} \frac{(2\pi)^{-2}}{\nu p_i} \text{Im} \left\{ e^{i\phi} \left[\omega \hat{e} p_i - i \mathcal{E}_i \left(\hat{e} - \nu \frac{\hat{e} p_i}{\nu p_i} \right) \nabla \right] I(\mathbf{r}_0) \right\} \quad (2.16)$$

$$I(\mathbf{r}_0) = \int \tilde{\Phi}(\mathbf{q}) \exp(i\mathbf{q} \mathbf{r}_0) \delta \left(1 - \frac{c}{\omega} \frac{\mathbf{q} p_i}{\nu p_i} \right) d^3 q \quad (2.17)$$

where the scattering potential is assumed to be central symmetric, and \hat{e} denotes either $\boldsymbol{\varepsilon}$ or $\boldsymbol{\varepsilon}_1 + i\boldsymbol{\varepsilon}_2$ for linearly and circularly polarized EM fields, respectively. Parenthetically, we remark that, as (2.17) shows, the electron energy change during the SB process in the EM field linear approximation does not actually depend upon the type of wavefield polarization.

Assuming a screened Coulomb potential being the potential charge Ze placed at the origin of the coordinate system and R the screening radius,

$$\Phi(\mathbf{r}) = \frac{Ze}{r} \exp\left(-\frac{r}{R}\right) \quad \tilde{\Phi}(\mathbf{q}) = \frac{4\pi Ze}{q^2 + 1/R^2} \quad (2.18)$$

integration in (2.17) yields, for the function $I(\mathbf{r}_0)$,

$$I(\mathbf{r}_0) = 8\pi^2 Ze \frac{\Omega}{v_i} K_0\left(\frac{r_{0\perp}}{\alpha}\right) \exp\left(-ir_0 \mathbf{n} \frac{\Omega}{v_i}\right) \quad (2.19)$$

where $K_0(x)$ is the McDonald function of zero order, \mathbf{n} is a unit vector in the incident electron velocity direction and $\alpha^{-2} = 2R^{-2} + (\Omega/v_i)^2$, $r_{0\perp}^2 = r_0^2 - (\mathbf{r}_0 \mathbf{n})^2$. Inserting (2.19) into (2.16) and using (2.14) we finally write the electron energy change owing to electron scattering in the screened Coulomb potential in the presence of the low-intensity EM field as

$$\Delta \mathcal{E} = \Delta \mathcal{E}_0(\boldsymbol{\rho}) \sin\left(\frac{\omega}{c} \nu \boldsymbol{\rho} - \phi + \psi(\boldsymbol{\rho})\right) \quad (2.20)$$

where

$$\Delta \mathcal{E}_0(\boldsymbol{\rho}) e^{i\psi(\boldsymbol{\rho})} = 2 \frac{Ze^3 A_0}{\beta^2 \mathcal{E}_i} \frac{\omega}{c} \left\{ (\boldsymbol{\varepsilon}_1 - i\boldsymbol{\varepsilon}_2) \mathbf{B}_0(\boldsymbol{\rho}) + i(\boldsymbol{\varepsilon}_1 - i\boldsymbol{\varepsilon}_2) \mathbf{B}_1(\boldsymbol{\rho}) \right\} \quad (2.21a)$$

$$\left\{ \boldsymbol{\varepsilon} (\mathbf{B}_0(\boldsymbol{\rho}) + i\mathbf{B}_1(\boldsymbol{\rho})) \right\} \quad (2.21b)$$

and

$$B_0(\rho) = \frac{n}{\gamma^2} \frac{\omega}{\Omega} K_0\left(\frac{\rho}{\alpha}\right) \quad B_1(\rho) = \frac{v_i}{\Omega \alpha \rho} \left(\rho + n\beta \frac{\omega}{\Omega} (\nu \rho) \right) K_1\left(\frac{\rho}{\alpha}\right).$$

Thus, for a linearly polarized wave this coincides with the classical energy change of an electron in the SB process derived by perturbation treatment of the interaction of the electron with both scattering and EM fields to first order (Avetissian and Jivanian 1980).

Now let us clarify the connection between (2.20) and the corresponding classical result obtained within the instantaneous interaction approximation (Kroll and Watson 1973). Note that in the low-intensity limit the validity condition for energy change (2.20), namely criterion (2.6), implies electron scattering at small angles without, however, restricting scattering duration. However, in the instantaneous scattering approximation, while the instantaneousness of the electron scattering is assumed, the scattering angles are arbitrary. Hence, the general domain where the energy change expressions derived by these two approximations are valid is the instantaneous scattering of electrons at small angles.

3. Energy change of an incident electron parallel to the wave propagation direction

In the preceding section a general expression was derived for the electron energy change owing to SB in an arbitrary scattering potential considering an intense wave exactly. Here we shall elucidate how the wavefield intensity and the type of wave polarization affects the classical energy change of an electron in the SB process. To make the further evaluation of (2.12) possible we restrict our study to the case of a screened Coulomb potential and to an incident electron momentum parallel to the wave propagation direction, $p_i \parallel \nu$. Then, as follows from (2.14), $r_0 = \rho$ and integration over q in (2.12) yields

$$\Delta \mathcal{E} = \sum_{n=1}^{+\infty} \Delta \mathcal{E}^{(n)} \quad \Delta \mathcal{E}^{(n)} = -4 \frac{Ze^2 p_i}{\tilde{v}_i \tilde{p}_i} n \omega f_n(\rho, \phi) \quad (3.1)$$

where $v_i = c^2 |\tilde{p}_i| / \tilde{\mathcal{E}}_i$ and $f_n(\rho, \phi)$ denotes one of the following functions depending on the circular (index c) or linear (index l) polarization of the wave:

$$f_n^{(c)}(\rho, \phi) = I_n(g_n L_{\min}) K_n(g_n L_{\max}) \sin\left(n\left(\phi + \varphi_0 - \frac{\pi}{2}\right)\right) \quad (3.2a)$$

$$f_n^{(l)}(\rho, \phi) = \sum_{s,k=-\infty}^{+\infty} \left(1 + 2 \frac{s}{n} \frac{c\tilde{p}_i}{\tilde{\mathcal{E}}_i}\right) J_s\left(\frac{n}{2} \frac{\Delta}{c\tilde{p}_i}\right) K_{2k+n}(g_n L_{\max}) I_{n+s+k}\left(\frac{1}{2} g_n L_{\min}\right) I_{k-s}\left(\frac{1}{2} g_n L_{\min}\right) \sin(n(\phi - \varphi_0 + \pi) - 2k\varphi_0). \quad (3.2b)$$

Here $I_n(z)$ is the modified Bessel function of n th order, $L_{\max(\min)}$ denotes the largest (smallest) of parameters a or ρ , and $g_n = [(n\omega\nu p_i / c\tilde{p}_i)^2 + R^{-2}]^{1/2}$. Formulae (3.1), (3.2a) and (3.2b) present the electron energy change due to SB in the presence of a strong wavefield as the sum of partial changes $\Delta \mathcal{E}^{(n)}$ of energy on the wave frequency ω and its harmonics $n\omega$. These partial changes depend on the impact parameter ρ , on the angle φ_0 between the impact parameter and the EM wavefield, and also on the initial wave phase ϕ ; moreover, their contribution to the final acceleration or deceleration of the electron in the SB process is determined by φ_0 and ϕ . Parenthetically, note that

the series in (3.1) converge; therefore, the energy change on large harmonics decreases to zero starting from a certain value of $n = n_0$. However, our primary concern is with $\Delta \mathcal{E}(\rho, \phi)$, the change in energy for the initial wave phase ϕ and a certain impact parameter ρ . In the general case it seems impossible to sum the series in (3.1) and (3.2b); therefore, temporarily, we shall restrict ourselves to the high-frequency limit. Assuming $g_n L_{\min} \gg 1$ and $g_n L_{\min} \cos \varphi_0 \gg 1$, respectively, for circularly and linearly polarized EM waves, one can substitute $K_n(x)$ and $I_n(x)$ entering in (3.2a) and (3.2b) by their asymptotes (Yahnke *et al* 1968):

$$K_n(x) \approx \sqrt{\frac{2}{\pi x}} e^{-x} \quad I_n(x) \approx \frac{e^x}{\sqrt{2\pi x}} \quad \text{when } x \gg n. \quad (3.3)$$

Then (3.2a) and (3.2b) are transformed into

$$f_b^{(c)}(\rho, \phi) = \frac{\exp(-g_n(L_{\max} - L_{\min}))}{2g_n \sqrt{L_{\min} L_{\max}}} \sin\left(n\left(\phi + \varphi_0 - \frac{\pi}{2}\right)\right) \quad (3.4a)$$

$$f_n^{(l)}(\rho, \phi) = \frac{\exp[-g_n(L_{\max} - L_{\min} \cos \varphi_0)]}{2g_n \sqrt{L_{\min} L_{\max} \cos \varphi_0}} \left(1 + \frac{\Delta}{\tilde{\mathcal{E}}_i}\right) \sin(n(\phi + \pi)). \quad (3.4b)$$

From (3.4a), (3.4b) and (3.1) we see that in the high-frequency limit the partial change of the electron energy falls exponentially with the increase in n . We carry out summation over n in (3.1) for the pure Coulomb potential, using (3.4a) and (3.4b), providing that $g_n = n\omega v p_i / c \tilde{p}_i$, and also formula 1.461 from Gradshtein and Rizik (1971). As a result one has the following expressions for the electron energy change:

$$\Delta \mathcal{E}^{(c)} = \frac{Ze^2}{\sqrt{a\rho}} \frac{\omega}{\Omega} \frac{p_i}{\tilde{p}_i} \frac{\cos(\phi + \varphi_0)}{\cosh[\tilde{\Omega}(L_{\max} - L_{\min})/\tilde{v}_i] - \sin(\phi + \varphi_0)} \quad (3.5a)$$

$$\Delta \mathcal{E}^{(l)} = \frac{Ze^2}{\sqrt{a\rho}} \frac{\omega}{\cos \varphi_0} \frac{p_i}{\Omega} \frac{\tilde{p}_i}{\tilde{p}_i} \frac{(1 + \Delta/\tilde{\mathcal{E}}_i) \sin \phi}{\cosh[\tilde{\Omega}(L_{\max} - L_{\min} \cos \varphi_0)/\tilde{v}_i] + \cos \varphi_0} \quad (3.5b)$$

where $\tilde{\Omega} = \omega(1 - v\tilde{v}_i/c)$.

From here the calculation of both the initial wave phase at which the electron energy change accepts its maximal value and the energy change at this wave phase is straightforward, and yields, respectively,

$$\begin{aligned} \phi &= \sin^{-1}\{h^{-1}[\tilde{\Omega}(L_{\max} - L_{\min})/\tilde{v}_i]\} - \varphi_0 \\ \Delta \mathcal{E}_{\max}^{(c)} &= \frac{Ze^2}{\sqrt{a\rho}} \frac{\omega}{\tilde{\Omega}} \frac{p_i}{\tilde{p}_i} \sinh^{-1}[\tilde{\Omega}(L_{\max} - L_{\min})/\tilde{v}_i] \end{aligned} \quad (3.6a)$$

for circularly polarized waves, and

$$\begin{aligned} \phi &= \pi - \cos^{-1}\{\cosh^{-1}[\tilde{\Omega}(L_{\max} - L_{\min} \cos \varphi_0)/\tilde{v}_i]\} \\ \Delta \mathcal{E}_{\max}^{(l)} &= \frac{Ze^2}{\sqrt{a\rho}} \frac{\omega}{\cos \varphi_0} \frac{p_i}{\tilde{\Omega}} \frac{\tilde{p}_i}{\tilde{p}_i} \frac{1 + \Delta/\tilde{\mathcal{E}}_i}{\sinh[\tilde{\Omega}(L_{\max} - L_{\min} \cos \varphi_0)/\tilde{v}_i]} \end{aligned} \quad (3.6b)$$

for linearly polarized waves.

As is clear from (3.6a) and (3.6b), the classical energy strongly depends on the minimum distance to which an electron can approach the scattering centre during its motion in both scattering and EM fields. In contrast to the weak field case, where this minimum distance is determined only by the impact parameter, for high EM intensities it also becomes dependent on wavefield strength and type of wave polarization.

Comparison of (3.6a) and (3.6b) shows that because of the cylindrical symmetry of the electron trajectory in the circularly polarized EM wave, in this case the minimum distance is $L_{\max} - L_{\min}$, whereas in the linearly polarized wave it depends also on the angle between the impact parameter vector and the EM field: $L_{\max} - L_{\min} \cos \varphi_0$.

To make some numerical estimates of (3.6a) we return to the validity criterion of (3.6a). For the Coulomb scattering potential and the circularly polarized EM wave, substitution of asymptotes (3.3) for $K_n(x)$ and $I_n(x)$ in (3.2a) is legitimate when

$$\tilde{\Omega}a/\tilde{v}_i \gg 1 \quad \text{and} \quad \tilde{\Omega}\rho/\tilde{v}_i \gg 1. \tag{3.7}$$

Accounting for the field dependence through $\tilde{\Omega} = \Omega \mathcal{E}_i/\tilde{\mathcal{E}}_i$ and \tilde{v}_i shows that (3.7) holds for initially non-relativistic electrons and relatively low EM intensities, when $\xi < 1$, taking the form

$$\frac{\xi}{v_i/c + 0.5 \xi^2} \gg 1 \quad \text{and} \quad \frac{\omega\rho/c}{v_i/c + 0.5 \xi^2} \gg 1. \tag{3.8}$$

Another approximation applied during derivation of (3.6a) is initial treatment of the scattering potential by perturbative theory to first order. For the Coulomb scattering potential and the circularly polarized wave in the domain in which (3.8) is valid criterion (2.6) of this approximation can be written as

$$r_{\min} = |\rho - a| \gg \frac{Ze^2}{mc^2} \frac{1}{(v_i/c)^2 + \xi^2}. \tag{3.9}$$

Let an electron with incident energy $\mathcal{E}_i = 10$ eV scatter on an α -particle in a laser field with wavelength $\lambda = 10^{-4}$ cm and intensity parameter $\xi \sim 0.1$ ($|\rho - a| \sim 10^{-10}$ cm). Then for the maximal energy change one obtains, from (3.6a), $\Delta \mathcal{E}_{\max} \approx 100$ eV. Thus, the electron energy change in the SB process exceeds its incident energy many times, as a result of the intensity effect. As follows from (3.6), the electron energy change along with the dependence on $|\rho - a|$ depends also on ρ and a . Moreover, $\Delta \mathcal{E}_{\max}$ increases with the decrease in ρ and a . However, condition (3.7) obviously breaks down for small ρ and a and one also needs to evaluate the electron energy change (3.1) for small ρ and a . In the general case there are difficulties with summation in (3.1) and we shall overcome them by firstly performing integration over q in the general expression (2.10). Then, for the screened Coulomb potential, and the above-adopted geometry $p_i \parallel v$, one can present (2.10) in the following form:

$$\Delta \mathcal{E} = -\frac{Ze^2 p_i}{c \tilde{p}_i v p_i} \int_{-\infty}^{+\infty} \frac{\partial}{\partial \phi} \left(\frac{\exp(-r(\tau, \phi)/R)}{r(\tau, \phi)} \right) \mathcal{E}_0(\tau) d\tau \tag{3.10}$$

where τ denotes the wave total phase $\omega\eta + \phi$, with $r(\tau, \phi) = |r_0 + r_0(\tau)|$ and $\mathcal{E}_0(\tau)$ being, respectively, the distance between the scattering centre and the electron and the electron energy in the wave,

$$r_c^2(\tau, \phi) = \rho^2 + a^2 - 2a\rho \sin(\tau + \varphi_0) + (\tau - \phi)^2 (\tilde{v}_i/\tilde{\Omega})^2 \quad \mathcal{E}_0(\tau) = \tilde{\mathcal{E}}_i \tag{3.11a}$$

and

$$r_l^2(\tau, \phi) = \rho^2 - 2a\epsilon\rho \sin \tau + a^2 \sin^2 \tau + \left(\tau - \phi + \frac{1}{2} \frac{\Delta}{c\tilde{p}_i} \sin 2\tau \right)^2 (\tilde{v}_i/\tilde{\Omega})^2$$

$$\mathcal{E}_0(\tau) = \tilde{\mathcal{E}}_i + \Delta \cos 2\tau \tag{3.11b}$$

respectively, for circularly and linearly polarized EM waves.

For short-range potentials, when $R \ll r(\tau, \phi)$, we can obtain an analytical estimate of the integral appearing in (3.10) using the fact that the area around the minimum of $r(\tau, \phi)$ mainly contributes to this integral. Then the use of the Laplace method (e.g. see Lavrentev and Shabat 1973) after some calculation yields for the electron energy change

$$\Delta \mathcal{E} = -\frac{Ze^2 p_i}{c \tilde{p}_i \nu p_i} \frac{\partial}{\partial \phi} \left[\mathcal{E}_0(\tau_0) \sqrt{\frac{2\pi R}{g(\tau_0, \phi) r_{\min}(\phi)}} \exp\left(-\frac{r_{\min}(\phi)}{R}\right) \right] \quad (3.12)$$

where $r_{\min}(\phi) = r(\tau_0, \phi)$, $g(\tau_0, \phi)$ (particularly for the circularly polarized wave) has the form

$$g_c(\tau_0, \phi) = a\rho \sin(\tau_0 + \phi) + (\tilde{v}_i/\tilde{\Omega})^2 \quad (3.13)$$

and τ_0 is the solution of the transcendental equation $\partial r(\tau, \phi)/\partial \tau = 0$.

In the case of small impact parameters and oscillation amplitudes when $a\rho(\tilde{\Omega}/\tilde{v}_i)^2 \ll 1$, this equation has the solution $\tau_0 = \phi$. Then, inserting (3.11a) and (3.13) into (3.12) one obtains

$$\Delta \mathcal{E} = -\frac{Ze^2}{r_{\min}(\phi)} \frac{\omega a \rho p_i}{\tilde{p}_i \tilde{v}_i} \sqrt{\frac{2\pi R}{r_{\min}(\phi)}} \times \left(\frac{1}{R} + \frac{1}{2r_{\min}(\phi)} \right) \exp\left(-\frac{r_{\min}(\phi)}{R}\right) \cos(\phi + \varphi_0). \quad (3.14)$$

Thus, as (3.14) shows, the dependence of the electron energy change on the initial phase ϕ is crucial for short-range potentials when the electron impact parameter ρ and oscillation amplitude a are of the same order. For the linearly polarized wave one is able to obtain an expression for the energy change similar to (3.14), requiring $\tilde{\Omega}a/\tilde{v}_i \ll 1$, $\epsilon\rho\tilde{\Omega}/\tilde{v}_i \ll 1$ and $\Delta/\tilde{p}_i c \ll 1$. However, in this case the energy change depends essentially on the initial wave phase ϕ for short-range potentials and for high EM intensities and small impact parameters, when $a \gg \rho$.

For long-range potentials, particularly for the Coulomb field, and the circularly polarized wave with the help of (3.11a), (3.10) can be transformed to

$$\Delta \mathcal{E} = -\frac{Ze^2 \omega p_i}{\tilde{p}_i \tilde{v}_i} \int_{-\infty}^{+\infty} \frac{\partial}{\partial \phi} \left[1 + \frac{L_{\min}^2}{L_{\max}^2} - 2 \frac{L_{\min}}{L_{\max}} \sin\left(x \frac{\tilde{\Omega}}{\tilde{v}_i} L_{\max} + \varphi_0 + \phi\right) + x^2 \right]^{-1/2} dx. \quad (3.15)$$

Expanding (3.15) up to terms of first order in the small parameter $L_{\min}/L_{\max} \ll 1$, we have

$$\Delta \mathcal{E} = -2 \frac{Ze^2 \omega p_i}{\tilde{p}_i \tilde{v}_i} \frac{L_{\min}}{L_{\max}} K_0\left(\frac{\tilde{\Omega}}{\tilde{v}_i} L_{\max}\right) \cos(\phi + \varphi_0). \quad (3.16)$$

Note that such an expression is valid for two limits, when $a \gg \rho$ (high intensities) and $\rho \gg a$ (large impact parameters).

In contrast to the weak-field limit, where the classical energy change is proportional to the field strength, we find from (3.16) that in the high-intensity limit, when $\xi \gg \gamma$, the energy change varies with the field strength as $\ln \xi/\xi^3$. This result relates to SB in the presence of the circularly polarized wave. For the linearly polarized wave one is unable to derive a simple expression similar to (3.16) in the high EM intensity limit when $a \gg \rho$. Such an approximation for the linearly polarized wave is inconsistent with criterion (2.6) of the perturbative treatment of the scattering potential because for certain initial wave phases the electron can approach the scattering centre very closely.

For low intensities of the linearly polarized wave, when $\Delta/\tilde{p}_i c \ll 1$ and $\rho \gg a$, one can obtain, similar to (3.16), the expression

$$\Delta \mathcal{E} = -2 \frac{Ze^2 \omega p_i}{\tilde{p}_i \tilde{v}_i} \frac{a \epsilon \rho}{\rho^2} K_0 \left(\frac{\tilde{\Omega}}{\tilde{v}_i} \rho \right) \cos \phi. \tag{3.17}$$

Thus, though variation of the electron energy change in the SB process with the field strength is the same for both circularly and linearly polarized weak wavefields, in the high intensity limit its dependence on the type of wave polarization becomes crucial.

As the analytical investigation of energy change is mathematically difficult, we have done some numerical calculations using (3.15), revealing the dependence of the classical change in electron energy on the initial phase (figure 1) and on the EM intensity (figure 2) for the circularly polarized wave. As is apparent from figure 1, the energy change varies sinusoidally for weak wavefields with a peak corresponding to $\pi/2$ (see also Avetissian and Jivanian 1980). However, upon increasing the field strength deviations from sinusoidal type variation appear and the energy change peak moves to π . However, as follows from (3.16) for very high intensities, the energy change peak again corresponds to the phases $\phi + \varphi_0 = \pi/2$ (as in the case of large impact parameters or weak wavefields). Note that as $\Delta \mathcal{E}[2\pi - (\phi + \varphi_0)] = -\Delta \mathcal{E}[\phi + \varphi_0]$, then one has a symmetrical picture of electron deceleration ($\Delta \mathcal{E} < 0$) in the phase interval $[\pi, 2\pi]$.

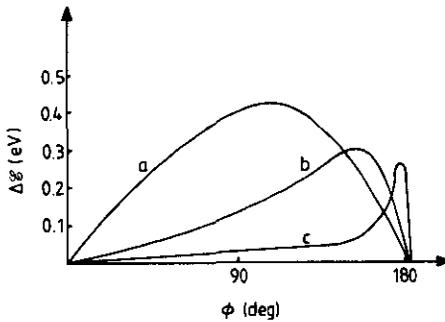


Figure 1. Graph of $\Delta \mathcal{E}(\phi)$ for electrons with incident energy $\mathcal{E}_i = 1$ keV, when $\hbar\omega = 1$ eV and $r_{\min} = 2 \times 10^{-9}$ cm for different wavefield intensities: (a) $\xi = 10^{-5}$, (b) $\xi = 10^{-4}$ and (c) $\xi = 10^{-3}$. For curve (a) the ordinate is increased 10 times and curve (c) it is decreased 10 times.

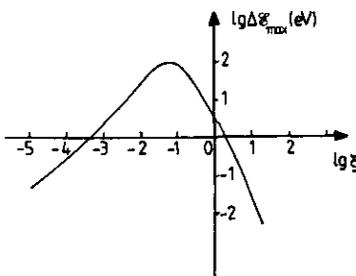


Figure 2. The maximum of the electron energy change (over the initial wave phase) as a function of the wavefield intensity parameter ξ , when $\mathcal{E}_i = 1$ keV, $\hbar\omega = 1$ eV and $r_{\min} = 2.10^{-9}$ cm.

Finally, in figure 2 we see that the maximum of the electron energy change over the phase due to SB depends on the wave intensity non-linearly, in contrast to the weak field case. In particular it has a maximum at $\xi \sim v_i/c$ (see figure 2).

4. Effective interaction of an electron beam with an intense EM wave in the SB process

Up to now we have studied the dynamics of a single electron scattering in the SB process, considering the EM wavefield exactly. Let us now analyse the effective interaction of an electron beam with an intense EM wave during the SB process.

Let an electron beam (for simplicity the beam is assumed to be monoenergetic) scatter on a static potential in the presence of a strong EM field. Then the rate of energy exchange (EE) between the electron beam and the EM wave (EE per unit time) is defined by the expression

$$dE/dt = \frac{1}{2\pi} \tilde{v}_i \int \int_0^{2\pi} \Delta \mathcal{E}(\boldsymbol{\rho}, \phi) f(\boldsymbol{\rho}, \phi) d^2 \boldsymbol{\rho} d\phi \quad (4.1)$$

where $\Delta \mathcal{E}(\boldsymbol{\rho}, \phi)$ is the energy change of one electron due to the SB process and is defined by (2.12)–(2.14), and $f(\boldsymbol{\rho}, \phi)$ is the distribution function of the electrons in the beam over the vector of the impact parameter and initial wave phases, in a unit volume. Integration over $\boldsymbol{\rho}$ is carried out in the plane perpendicular to \tilde{v}_i , and over the initial wave phase ϕ between the limits 0 and 2π .

As is seen from (2.12) and (4.1), the rate of EE between the electron beam and the EM wave in the SB process essentially depends on the distribution of electrons in the beam over $\boldsymbol{\rho}$ and ϕ , when the wavefield intensity is taken into account. Thus, starting from these expressions it is easy to show that the rate of EE due to SB for an electron beam homogeneously distributed over $\boldsymbol{\rho}$ and ϕ is zero. This result is a direct consequence of the adopted approximation of perturbation theory for the scattering potential, according to which, for the first order of this approximation, the contribution of the scattering to the electron trajectory is not taken into account. If the beam with an electron density N_e is homogeneous over the transverse section and is modulated and enters the interaction region with a certain wave phase, then, with the help of (2.12), (2.13a), (2.13b) and by means of (2.14) from (4.1) for the EE rate, one obtains

$$\frac{dE(\phi)}{dt} = 2N_e \frac{ec^2}{\mathcal{E}_i \mathcal{E}_i} \frac{\omega^2}{\Omega} \text{Im} \left(\sum_{n=1}^{+\infty} \mathbf{q}_n p_i \tilde{\Phi}(\mathbf{q}_n) f_n(\mathbf{q}_n) e^{-in\phi} \right) \quad (4.2)$$

where

$$\mathbf{q}_n = n \frac{\omega}{\tilde{v}_i} \left(\tilde{\mathbf{n}} - \nu \frac{\tilde{v}_i}{c} \right)$$

and $\tilde{\mathbf{n}} = \tilde{\mathbf{p}}_i / \tilde{p}_i$ is a unit vector in the normalized electron momentum direction in the wave, $\tilde{\Phi}(\mathbf{q}_n)$ is the Fourier transform of the scattering potential, and the function $f_n(\mathbf{q}_n)$ is defined by (2.13a) or (2.13b) for the circularly or linearly polarized wave, respectively.

In the weak field limit, when $eE_0/p_i\Omega \ll 1$, one can keep in (2.13a) and (2.13b) only terms up to the first order of the field strength. As a result, for the EE rate of electrons with the weak wavefield during stimulated scattering in the screened Coulomb potential (2.18), from (4.2) we obtain

$$\frac{dE(\phi)}{dt} = -4\pi N_e \frac{Ze^3}{\beta^2 \mathcal{E}_i} \frac{E_0 v_i}{\gamma^2} \frac{\omega}{\Omega} \frac{(\omega/\Omega - \beta^2 \gamma^2) \text{Im}(e^{i\Phi} \hat{\mathbf{e}} \mathbf{n})}{(\omega/v_i)^2 (1 - 2\beta \nu n + \beta^2) + 1/R^2} \quad (4.3)$$

where \hat{e} is defined as in section 2. We remark that for the Coulomb potential and the linearly polarized wave (4.3) in the non-relativistic limit coincides with the weak Coulomb field limit of the EE rate derived by Avetissian *et al* (1986) for the SB process treating the Coulomb field exactly and the wave as a perturbation. In contrast with the weak field limit, in the case of the strong EM field to (4.2) contribute all terms of the sum and as a result the EE rate becomes sensitive to the type of wave polarization. For the circularly polarized wave one can sum the series in (4.2) when

$$z \equiv eE_0 \frac{\omega}{\Omega^2} \frac{p_{\perp i}}{\tilde{p}_i^2} \left(1 - \frac{\tilde{v}_i^2}{c^2} \right) < 1. \tag{4.4}$$

Then, using the following formulae (Proodnikov *et al* 1983),

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n} \sin(na) I_n(nz) &= \frac{1}{2}(t-a) & t - z \sin t &= a \\ \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \begin{matrix} \sin(na) \\ \cos(na) \end{matrix} \right\} \left\{ I_{n-1}(nz) \pm I_{n+1}(nz) \right\} &= \begin{cases} \sin t \\ \cos t \end{cases} \end{aligned} \tag{4.5}$$

for the EE rate of the electron beam with the circularly polarized intense wave we obtain the expression

$$\frac{dE(\psi)}{dt} = 4\pi N_e Z e^3 E_0 \frac{v_i v_{i\perp}}{\tilde{p}_i \Omega^2} \frac{\mathbf{n}(\tilde{\mathbf{n}} - \nu \tilde{\boldsymbol{\beta}})}{1 - 2\tilde{\boldsymbol{\beta}} \nu \tilde{\mathbf{n}} + \tilde{\beta}^2} \left(\tilde{\beta}^2 - \frac{\omega}{\Omega} \frac{\tilde{\mathcal{E}}_i}{\mathcal{E}_i} (1 - \tilde{\beta}^2) \right) \sin \psi \tag{4.6}$$

where $\tilde{\boldsymbol{\beta}} = \tilde{v}_i/c$, and the phase ψ is the solution of the transcendental equation

$$\psi + z \sin \psi = \phi + \phi_0. \tag{4.7}$$

From (4.6) it follows that an incident beam parallel to the propagation direction of the wave does not change as a whole its energy due to SB. The absence of EE in this case is because of the axial symmetry of the electron trajectories in the circularly polarized EM wave.

For an incident beam perpendicular to the wave propagation direction, from (4.6) for the maximum of the EE rate one has

$$\left(\frac{dE}{dt} \right)_{\max} = -4\pi N_e \frac{Ze^2 \beta^3 c^2}{1 + \beta^2} \frac{\xi'}{\omega} \frac{1 - 2\beta^2 + 0.5(3 - \beta^2)\xi'^2 - 0.25\xi'^4}{\beta^2 + 0.25\xi'^4} \tag{4.8}$$

where $\xi' = \xi/\gamma$. As is seen from (4.8), the dependence of the maximum of the EE rate upon the field of the circularly polarized EM wave for high intensities, when $\xi' \gg 1$, becomes linear, as in the case of the weak field. However, for intermediate wave intensities, when $\xi' \ll 1$, but $\xi'^2 \geq \beta$, this dependence is non-linear.

In contrast to the circularly polarized wave case, for the linearly polarized wave, the EE of an incident beam parallel to the propagation direction of the wave is not zero. In this case one can sum the series in (4.2) using the first formula from (4.5), if

$$z_i \equiv \frac{\xi^2(1 + \beta)}{4\beta + \xi^2(1 - \beta^2)} < 1. \tag{4.9}$$

For the rate of EE we obtain

$$\frac{dE^{(1)}}{dt} = 2\pi N_e Z e^2 \frac{c}{\omega} \frac{(1 + \beta)}{(1 - \beta)} \frac{\xi^2 \sin \psi_1}{4 + \xi^2(1 - \beta^2)} \tag{4.10}$$

where the phase ψ_1 is the solution of the transcendental equation

$$\psi_1 + z_i \sin \psi_1 = 2\phi. \tag{4.11}$$

When the EM field is relatively weak, $\xi \ll \gamma$, then the maximum of the EE rate over the wave phase depends upon the wavefield quadratically,

$$\left(\frac{dE^{(1)}}{dt}\right)_{\max} = \frac{\pi}{2} N_e Z e^2 v_i \frac{c}{\omega} \xi^2 \frac{(1+\beta)}{(1-\beta)} \quad (4.12)$$

and increases as the beam velocity increases. Note that the energy change of the incident electron beam parallel to the wavevector direction is a relativistic intensity effect, vanishing in the weak field limit (see (4.3)).

Now let us consider the situation when a beam with a certain impact parameter ρ scatters in a Coulomb field and the electrons of the beam are homogeneously distributed over the wave phases. For the circularly polarized wave (2.11a), using (2.12), (2.13a) and (4.1), and performing integration over ϕ in (4.1), for the effective EE of the beam with an EM wave per unit time and per unit transverse section of the beam we obtain

$$\Delta E(\rho) = N_e \frac{Z e^2 c}{\xi_i} \operatorname{Re} \left[\frac{(\tilde{n}\rho, \mathbf{b})}{\sqrt{(\tilde{n}\rho\mathbf{b})^2 - a^2 d}} \left(\frac{\tilde{\mathcal{E}}_i}{\nu p_i} + i \frac{\omega a^2 (\nu \mathbf{b}) (\tilde{n}\epsilon\epsilon^*)}{2 \sqrt{(\tilde{n}\rho\mathbf{b})^2 - a^2 d} - (\tilde{n}\rho\mathbf{b})} \right) \right] \quad (4.13)$$

where

$$d = \epsilon^2 (\tilde{n}\epsilon\epsilon^*)^2 (\epsilon\epsilon^* + i(\tilde{n}\epsilon\epsilon^*))$$

$$\mathbf{b} = [\epsilon[\epsilon\epsilon^*]] + i\epsilon(\tilde{n}\epsilon\epsilon^*)$$

$$\epsilon = (\epsilon_1 + i\epsilon_2) - \nu \frac{(\epsilon_1 + i\epsilon_2)\rho_i}{\nu p_i}$$

Equation (4.13) is valid when $a < R(\rho) = |2i(\tilde{n}\rho\epsilon)/(\tilde{n}\epsilon\epsilon^*)|$. In the inverse case, $a > R(\rho)$, the effective EE does not depend on ρ :

$$\Delta E(\rho) = -2N_e \frac{Z e^2 c}{\xi_i} \omega \frac{\operatorname{Re}[(\tilde{n}\rho, \mathbf{b})(\nu\epsilon^*)]}{i(\tilde{n}\epsilon\epsilon^*)[\epsilon\epsilon^* + i(\tilde{n}\epsilon\epsilon^*)]} \quad (4.14)$$

From (4.14) it follows that, for very large intensities of the wavefield, when $\xi \gg \gamma$, $\Delta E(\rho)$ decreases to zero as $1/\xi^2$.

Thus, in contrast to the weak field case, when the energy change of the beam averaged over the initial phases is zero (Avetissian and Jivanian 1980, Avetissian *et al* 1986), the effective EE of the beam in the strong field of the wave differs from zero, which directly reveals the intensity effect in the SB process.

Finally, we note that the analysis of the obtained results shows that one can reach an appreciable amplification due to coherent SB for millimetric waves, when $\xi \ll v_i/c \sim 1$. For example, when a modulated electron beam with a density $N_e \sim 10^8 \text{ cm}^{-3}$ and incident energy 625 keV is scattered on an ionized gas stream with ion density $N_i \sim 10^{18} \text{ cm}^{-3}$ and diameter $30 \mu\text{m}$, an EM wave with a wavelength of 1 mm and intensity of 2.7 W cm^{-2} can be amplified three times.

5. Summary and conclusions

The intensity effect of a strong EM field on both the dynamics of the SB process and the coherent effective energy exchange between an electron beam and an intense wave due to the SB process has been investigated. Within relativistic theory we derived the electron energy change during electron scattering in an arbitrary static scattering

potential and an intense EM wavefield. Unlike previous investigations we have treated the electron interaction with an EM wave exactly, whereas the scattering potential is dealt with as a perturbation. This enabled us to reveal the non-linear variation of energy change with the field strength, and also its essential dependence on the polarization of the wave. In addition, as an apparent effect of intensity on the dynamics of SB one finds that in a strong EM field the average of the electron energy change over the initial wave phase differs from zero, in contrast to the weak field case.

Despite the generality of the results, the energy change is discussed for a screened Coulomb potential and for an incident electron parallel to the wave propagation direction. This shows that the electron energy change in an intense field due to SB can exceed its incident energy. Also, the crucial role of the initial wave phase in the electron energy exchange with an intense wave for a short-range potential is elucidated. Numerical calculations for a circularly polarized wave help to form a notion of the dependence of the energy change on the initial wave phase and EM field strength.

The calculations carried out for the rate of coherent energy exchange of the electron beam with an intense field due to SB also show its non-linear dependence on the field strength. In addition, it becomes very sensitive to the type of wave polarization for high intensities. Thus, there is no coherent energy exchange between an incident electron beam parallel to the wave propagation direction when the wave has circular polarization, whereas such an exchange does occur for a linearly polarized wave, as a result of the intensity effect. It is also shown that the non-linear dependence of the rate of coherent energy exchange on the field strength does not always follow for high intensities. Thus, the rate of coherent EE of an incident electron beam perpendicular to the circularly polarized wave propagation direction varies with the field strength linearly in the 'relativistic intensity' limit, as is the case in the low field limit.

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