Upper Limits on Gravitational-Wave Emission in Association with the 27 Dec 2004 Giant Flare of SGR1806-20

L. Baggio,[1] M. Bignotto,[2] M. Bonaldi,[3] M. Cerdonio,[2,*] L. Conti,[2] M. De Rosa,[4] P. Falferi,[3] P. Fortini,[5] M. Inguscio,[6] N. Liguori,[2] F. Marin,[6] R. Mezzena,[7] A. Mion,[7] A. Ortolan,[8] G. A. Prodi,[7] S. Poggi,[7] F. Salemi,[6] G. Soranzo,[9] L. Taffarello,[9] G. Vedovato,[9] A. Vinante,[7] S. Vitale,[7] and J. P. Zendri[9]

(AURIGA Collaboration)[†]

[1]*Institute for Cosmic Ray Research, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba, 277-8582, Japan*
[2]*INFN Padova Section and Department of Physics, University of Padova, I-35131 Padova, Italy*
[3]*Istituto di Fotonica e Nanotecnologie CNR-ITC and INFN Gruppo Collegato di Trento, Padova Section, I-38050 Povo (Trento), Italy*
[4]*INOA I-80078 Pozzuoli (Napoli), Italy and INFN Firenze Section, I-50121 Firenze, Italy*
[5]*Physics Department, University of Ferrara and INFN Ferrara Section, I-44100 Ferrara, Italy*
[6]*LENS and Physics Department, University of Firenze and INFN Firenze Section, I-50121 Firenze, Italy*
[7]*Physics Department, University of Trento and INFN Gruppo Collegato di Trento, Padova Section, I-38050 Povo (Trento), Italy*
[8]*INFN, Laboratori Nazionali di Legnaro, I-35020 Legnaro (Padova) Italy*
[9]*INFN Padova Section, I-35100 Padova, Italy*
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At the time when the giant flare of SGR1806-20 occurred, the AURIGA "bar" gravitational-wave (GW) detector was on the air with a noise performance close to stationary Gaussian. This allows us to set relevant upper limits, at a number of frequencies in the vicinities of 900 Hz, on the amplitude of the damped GW wave trains, which, according to current models, could have been emitted, due to the excitation of normal modes of the star associated with the peak in x-ray luminosity.

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On 27 December 2004 the Soft Gamma-ray Repeater SGR1806-20 gave a giant flare, which was observed by a number of instruments [1].

The fluence, if the emission is assumed isotropic, at the distance of $d \sim 15$ kpc would imply an energy some hundred times larger than any other known giant flare [2,3]. Soft gamma-ray repeaters are thought to be magnetars (see [2] and references therein). It has been suggested [2,4] that the extreme energy event of 27 December 2004 is due to a catastrophic instability involving global crustal failure and magnetic reconnection [5]. Observations by CLUSTER and TC-2, in combination with data from GEOTAIL, gave evidence that the steep initial rise contains two exponential phases, of e-folding times 4.9 and 67 ms, respectively, which covered the 24 ms before the time of the peak intensity t_p; all the time scales support the notion of a sudden reconfiguration of the star magnetic field, producing large fractures in the crust [4]. In particular, these authors note that the intermediate ≈ 5 ms time is naturally explained if the rising time is limited by the propagation of a triggering fracture of size ≈ 5 km, as it would be predicted by the theory of Ref. [6].

According to a few somewhat different models, as a consequence of crustal cracking [7] or reconfiguration of the moment of inertia tensor [8], nonradial kHz oscillation modes of the neutron star would be excited, giving emission of gravitational waves (GW), possibly at frequencies where the GW bar detector AURIGA [9] is sensitive (see inset of Fig. 1). Both the above quoted models predict GW emission, starting very close to t_p, which involves kHz nonradial modes of oscillation of a neutron star with few hundred ms damping time. The expected waveforms can be approximately parametrized as $h(t) = h_0 \exp(-t/\tau_s) \times \sin(2\pi f_s t)$, where h_0 is the maximum GW amplitude, f_s

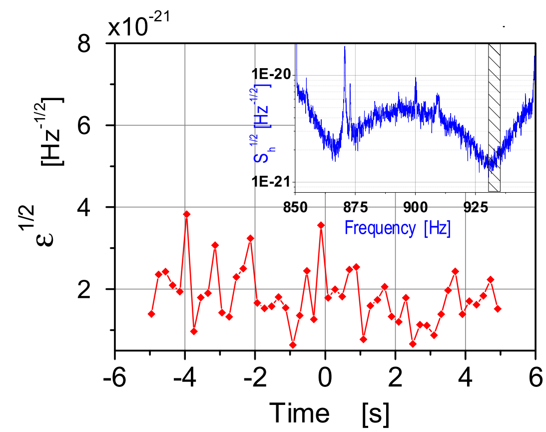

FIG. 1 (color online). Plot of $\mathcal{E}^{1/2}$ in the frequency band 930–935 Hz as a function of time; the origin in the x axis corresponds to the arrival time of the flare X of SGR1806-20 at the AURIGA site. The inset shows the AURIGA one-sided noise spectral density, as expressed in equivalent GW amplitude $h_r(t)$ at input. The vertical dashed area shows the position of the frequency bin 930 Hz (see text).

and τ_s are the frequencies and damping times of normal modes; the polarization of the wave is not known. The frequencies of the various modes are still under study and depend on a variety of factors as equation of state (EOS), temperature, density, age, rotational state of the star, etc. [10], so that we are unable to anticipate with any confidence what specific set of GW emission frequencies could be the one expected for a magnetar ready to undergo a supergiant flare. Still, the lowest lying modes, g, f, and, marginally, *p modes* could well be in the frequency range 500–1500 Hz, depending on the status of the star.

Within a factor of 10 in GW amplitude, AURIGA is sensitive to GWs from ~800 to 1050 Hz. Here we limit the analysis to the most sensitive part of the band, namely, between 850 and 950 Hz (see the inset of Fig. 1), where the detector sensitivity varies no more than a factor of 4 in amplitude. Since the upgrading of the suspensions on 2 December 2004 the detector is well behaved in the sense that performs stationary Gaussian, after epochs of environmental disturbances are vetoed by means of auxiliary channels (i.e., signals at frequencies where the detector is GW insensitive). During nights and weekends the vetoed epochs become less frequent and shorter, so that the detector achieves close to 90% stationary Gaussian operation (see also *Note added*). in particular, on time spans of minutes we can use the data, without even applying vetoes. This is the case for the time span of about ± 100 s around the epoch of the 27 December 2004 giant flare of SGR1806-20, which we use in this analysis. We show in the following that the noise is driven by a zero mean stochastic Gaussian process with a stationary correlation function. At the time of the giant flare, the antenna pattern of AURIGA, after averaging over the GW polarization angles, was optimally oriented in the direction of SGR1806-20 was such that, averaged over polarizations, gave maximal sensitivity. Then we have a unique opportunity to search in our data for gravitational waves emitted at the peak time of the giant flare. We take the peak time t_p to be 21:30:26.68 UT of 27 December 2004 after taking into account the time difference between the arrival time at INTEGRAL [11] and at AURIGA sites of 133.427 ms [12]. This time corresponds also to the peak position of the CLUSTER data which show, after the last exponential rise, the evident start of a phase in which damping occurs until the signal gets below 1/10 of the peak value, ~300 ms after the peak [4]. Following the models quoted above, in both cases we can assume the peak time t_p as the start of the GW excitation and $\tau_s = 100$ ms, that is 1/3 of 300 ms, as the corresponding damping time. In order to extract the signal power first we reconstruct the GW amplitude $h_r(t)$ at input through the detector transfer function. Then we slice the GW sensitive frequency band of AURIGA in contiguous and nonoverlapping subbands f_j of constant width Δf, and centered in $f_j^c = f_j + \Delta f/2$, by means of digital top-hat filters in the frequency domain $T_j(f) = \vartheta(|f| - f_j) - \vartheta(|f| - f_j - \Delta f)$. Within each subband, we compute the equivalent input signal power over a time span Δt

$$\mathcal{E}_j \equiv \int_{\Delta t} T_j * h_r^2(t + k\Delta t)dt, \qquad (1)$$

where $*$ stands for time convolution. The $\mathcal{E}_j(t)$ is sampled every Δt to construct the time series $\mathcal{E}_j(k)$ with k integer. We decided *a priori* a fixed partition of the time frequency plane: $\Delta f = 5$ Hz $\simeq 1/(2\tau_s)$ and $\Delta t = 201.5$ ms $\simeq 2\tau_s$. For each subband f_j, we analyzed the resulting time series of $\mathcal{E}_j(k)$ over a time span of ± 100 s around the peak time t_p to check the "off source" noise statistics. The $\mathcal{E}_j(k)$ sample including the peak time t_p is then compared to the measured noise statistics, looking for any evidence of excess power. To be more precise, the *a priori* choice of our sampling time made t_p to fall 120 ms after the beginning of the integration time Δt of the "on source" sample. Figure 1 shows how $\mathcal{E}_j$ fluctuates on the time spans of ± 5 s around the time of the flare t_p for the subband $f_j =$ 930 Hz. A GW emission at frequency f_s would give an excess power in the band Δf centered at the f_j^c such that $|f_s - f_j^c| < \Delta f/2$. The released energy would be maximum in the "on source" sample. The excess signal power in each subband Δf can easily be calculated from the expected waveform and reads

$$\mathcal{E}_s \simeq (h_0^2\tau_s/4)\left\{\left[\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{2x}{\delta^2 - x^2}\right)\right] + O\left(\frac{1}{f_s\tau_s}\right)^2\right\}, \qquad (2)$$

where $x \equiv (2\pi\tau_s)^{-1}/\Delta f$ and $\delta \equiv |f_s - f_j^c|/\Delta f$; δ is the ratio between the detuning of the signal frequency and the bandwidth Δf. With our choice of parameters for the analysis, the excess signal power is approximately (within a few percent error)

$$\mathcal{E}_s \approx \frac{h_0^2\tau_s}{6}\left[1 - \left(\frac{f_s - f_j^c}{\Delta f_{\rm eff}}\right)^2\right], \qquad (3)$$

where $\Delta f_{\rm eff} = 4$ Hz. To check the statistics of the "off source" samples, we histogram each time series $\mathcal{E}_j(k)$ and compare them with the predicted probability density functions assuming Gaussian noise, by fitting for the variance separately in each subband. The fitting probability density function is a χ^2 distribution with α effective degrees of freedom $p(\mathcal{E}; \sigma^2) = 2^{-\alpha/2}(\mathcal{E}/\sigma^2)^{\alpha/2-1}\exp(-\mathcal{E}/2\sigma^2)/\Gamma(\alpha/2)/\sigma^2$, where σ^2 is the variance of the underlying Gaussian stochastic process. We show in Fig. 2 the close agreement with the prediction of the data for the frequency bin $f_j = 930$ Hz, over the extended time span of ± 100 s. The results for all other frequency bins are similar. The goodness of the fit has been checked by a χ^2 test, and the resulting p values for all the subbands are consistent with a uniform distribution in the unit interval, as expected (see the inset of Fig. 2). In Table I we report the parameter σ^2 of

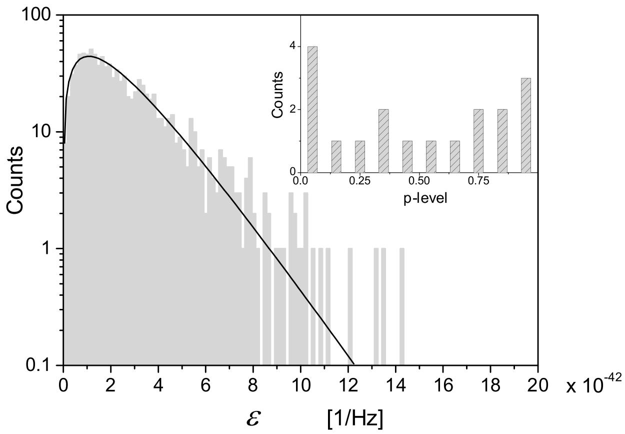

FIG. 2. Histogram of $\mathcal{E}$ in the frequency band 930–935 Hz; solid line represents the best fit curve of a χ^2 distribution with $\alpha = 3.6$ effective degrees of freedom. The inset shows the p-level distribution of the same fit for the histograms of the 18 frequency bins.

the fit of $p(\mathcal{E})$ to the experimental data. We find that the dependence on the effective degrees of freedom α of the p levels is very weak and, within the statistical errors, we can fix $\alpha = 3.6$ for all the frequency bins. The p-level distribution is uniform in the unit interval (see the inset of Fig. 2). The stationary behavior, at least for time scales of a few minutes, is shown by the constancy in time of the parameters needed to fit the noise model. We take advantage of the classical theory of hypothesis testing to establish whether the samples $\mathcal{E}_{t_p}$ corresponding to the arrival time t_p are affected by the presence of a GW signal. To test the null hypothesis $\mathcal{H}_0$, i.e., that the sample is drawn from the estimated noise probability distribution in the absence of signals, we set a threshold $\mathcal{E}_{\rm cr}$ corresponding to a confidence level (C.L.) $p(\mathcal{E} < \mathcal{E}_{\rm cr}) \geq 1 - p_{\rm cr}$. The threshold for $1 - p_{\rm cr} = 95\%$ C.L. corresponds to $\mathcal{E}_{\rm cr} = 8.8 \times \sigma_j^2$. Thus one sees from Table I that no excess of GW power is found at t_p, and therefore we have to set upper limits. We set conservative confidence intervals for $\mathcal{E}_s$ using a confidence belt construction [13] that ensures nonuniform coverage greater than or equal to 90%. The confidence belt construction proceeds as follows. Assume that the signal magnitude is $\mathcal{E}_s$. The measured $\mathcal{E}$ in each subband [Eq. (1)] obeys a noncentral χ^2 distribution with central parameter equal to $\mathcal{E}_s/\sigma^2$ (here we drop for simplicity the index of the subband). Its corresponding probability density function can be written as

$$p(\mathcal{E}; \mathcal{E}_s, \sigma) = \frac{1}{2\sigma^2} \exp\left(-\frac{\mathcal{E} + \mathcal{E}_s}{2\sigma^2}\right) \times \left(\frac{\mathcal{E}}{\mathcal{E}_s}\right)^{(\alpha-2)/4} I_{\alpha/2-1}\left(\sqrt{\mathcal{E}\mathcal{E}_s/\sigma^2}\right), \quad (4)$$

where $I_k(x)$ are the modified Bessel functions of the first kind of order k. The q quantile of this distribution, $\mathcal{E}_q(\mathcal{E}_s, \sigma)$, is implicitly defined by $q = \int_0^{\mathcal{E}_q} p(\mathcal{E}; \mathcal{E}_s, \sigma) d\mathcal{E}$. For each value of the unknown $\mathcal{E}_s$ we define the 95% confidence belt boundaries $\mathcal{E}_{\rm hi}$ and $\mathcal{E}_{\rm low}$ as

$$\mathcal{E}_{\rm hi}(\mathcal{E}_s, \sigma) = \begin{cases} 0 & \text{if } \mathcal{E}_s < \mathcal{E}_s^{\rm cr}(\sigma), \\ \mathcal{E}_{5\%}(\mathcal{E}_s, \sigma) & \text{otherwise,} \end{cases}$$
$$\mathcal{E}_{\rm low}(\mathcal{E}_s, \sigma) = \mathcal{E}_{95\%}(\mathcal{E}_s, \sigma), \quad (5)$$

where $\mathcal{E}_s^{\rm cr}$ is implicitly defined by $\mathcal{E}_{5\%}(\mathcal{E}_s^{\rm cr}, \sigma) = \mathcal{E}_{95\%}(0, \sigma)$. This confidence belt defines a set of confidence intervals on $\mathcal{E}_s$, whose frequentist coverage is—by construction—90% for $\mathcal{E}_s > \mathcal{E}_s^{\rm cr}$, and 95% for $\mathcal{E}_s \leq \mathcal{E}_s^{\rm cr}$. In other words, for every value of $\mathcal{E}_{t_p}$ from each subband, if $\mathcal{E}_{t_p} < \mathcal{E}_{95\%}(0, \sigma_j)$ we set an upper limit equal to $\mathcal{E}_s^{\rm cr}$; otherwise, our procedure gives a two-sided confidence interval. In all subbands we obtain upper limits, which can be written as $\mathcal{E}_s^{\rm cr} \simeq 18 \times \sigma_j^2$. These limits range from $\mathcal{E}^{1/2} = 3.5 \times 10^{-21}$ Hz$^{-1/2}$ to $\mathcal{E}^{1/2} = 1.4 \times 10^{-21}$ Hz$^{-1/2}$, according to AURIGA sensitivity.

The initial amplitude of the neutron star normal modes h_0 is related to $\mathcal{E}$ by Eq. (3) that gives, for the best upper limit, $h_0 \leq 2.7 \times 10^{-20}$. We discuss now the upper limit in terms of the total GW energy $\epsilon_{\rm gw} = E_{\rm gw}/M_\odot c^2$ emitted by the normal mode excitation during the peak of the giant flare of SGR1806-20. The well known formula of the quadrupolar radiation, for the expected GW signal [7],

TABLE I. List of fit parameter σ^2 of the histogrammed $\mathcal{E}$ data samples ±100 s around t_p tabulated as increasing subband frequencies; the data for the 870 Hz subband have been discarded *a priori* as this band is contaminated by environmental noise. The "on source" value of $\mathcal{E}$ including the trigger time t_p is also reported as well as the computed upper limit with confidence ≥ 95%.

f_j (Hz)	$\sigma_j^2 \times 10^{42}$ (Hz^{-1})	$\mathcal{E}_{t_p} \times 10^{42}$ (Hz^{-1})	$\mathcal{E}_{95} \times 10^{40}$ (Hz^{-1})
855	4.78	4.90	0.86
860	1.89	4.15	0.34
865	1.96	9.83	0.35
875	2.94	2.93	0.53
880	4.30	10.3	0.77
885	5.11	4.52	0.92
890	6.15	6.51	1.11
895	5.89	8.57	1.06
900	6.93	6.60	1.25
905	6.18	6.78	1.11
910	3.69	19.7	0.66
915	2.60	7.06	0.47
920	1.61	6.57	0.29
925	0.87	3.19	0.16
930	0.71	5.25	0.13
935	1.24	2.57	0.22
940	3.56	19.3	0.64
945	11.2	30.2	2.01

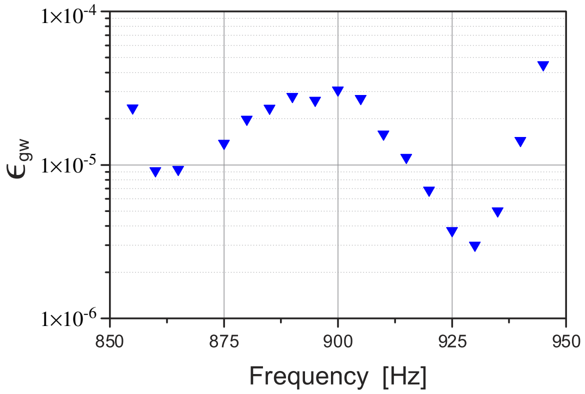

FIG. 3 (color online). Upper limits on the GW energy released at the source around the flare peak time t_p expressed as a fraction of $M_\odot c^2$.

can be written as $h_0 = [\epsilon_{gw} c R_S/(4\pi^2 \tau_s)]^{1/2}/(f_s d)$, where R_S is the Swartzchild radius of one solar mass black hole. Thus the resultant upper limit on ϵ_{gw} reads

$$\epsilon_{gw} \leq 3 \times 10^{-6}\left(\frac{\mathcal{E}}{1.3 \times 10^{-41}\ \mathrm{Hz}^{-1}}\right)\left(\frac{15\ \mathrm{kpc}}{d}\right) \times \left(\frac{930\ \mathrm{Hz}}{f_s}\right)^2\left(\frac{\tau_s}{0.1 s}\right). \quad (6)$$

The ϵ_{gw} upper limits for each frequency bin f_s are shown in Fig. 3. We should notice that a GW bar detector has a polarization dependent sensitivity; hence, for an unpolarized or linearly polarized GW, the result in Eq. (6) should be multiplied by a factor of 2 or $\cos^2(2\psi)$, respectively, where ψ is the angle between the bar axis and the polarization of the wave. We conclude that, if the star ever emitted GWs from excitation of its normal modes at any of the frequencies studied here, in the time span Δt containing the flare time t_p, the GW amplitudes and energetics are limited as above. If the giant flare of SGR1806-20 on 27 December 2004 is, indeed, some 100 times more energetic (however, see Ref. [14]) and if the GW luminosity scales with the electromagnetic luminosity, then, for the frequencies considered, our upper limits come close to the predictions of the models of Refs. [7,8], which give an energetics of the order of $\epsilon_{gw} \approx 5 \times 10^{-6}$. The method used here is, of course, suboptimal, and the upper limits are somewhat weaker than the "optimal" matched filter. In any case this work shows that, as there is the specific peak time t_p to be used as an external trigger, it is worth to making searches even with a single detector if its noise is well behaved. An extension of such searches involving the GW detectors on the air in a coincidence search would also allow one to use the information of the GW travel delays between the detectors to select against spuria, and would give the most exhaustive and efficient search, in terms of frequency coverage and confidence in improving the limits, if not to get a candidate detection.

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Note added.—After a subsequent upgrade of suspensions on 19 May 2005, AURIGA shows 95% stationary Gaussian operation, irrespective of time of day or day of the week.

*Corresponding author.
Electronic address: cerdonio@pd.infn.it
†Electronic address: http://www.auriga.lnl.infn.it

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