Radiation-Induced "Zero-Resistance State" and the Photon-Assisted Transport

Junren Shi[1] and X. C. Xie[2,3]

[1]*Condensed Matter Sciences Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[2]*Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA*
[3]*International Center for Quantum Structures, Chinese Academy of Sciences, Beijing 100080, China*

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We demonstrate that the radiation-induced "zero-resistance state" observed in a two-dimensional electron gas is a result of the nontrivial structure of the density of states of the systems and the photon-assisted transport. A toy model of a quantum tunneling junction with oscillatory density of states in leads catches most of the important features of the experiments. We present a generalized Kubo-Greenwood conductivity formula for the photon-assisted transport in a general system and show essentially the same nature of the transport anomaly in a uniform system.

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The recent discovery of the "zero-resistance state" in a two-dimensional electron gas (2DEG) presents a surprise to the physics community [1–6]. In these experiments [1–3], the magnetoresistance of a 2DEG under the influence of a microwave radiation exhibits strong oscillations vs magnetic field. Unlike the well-known Shubnikov–de Haas oscillation, the period of such an oscillation is determined by the frequency of the microwave radiation, and the resistance shows minima near $\omega/\omega_c = n + 1/4$, where ω is the frequency of the microwave radiation and ω_c is the cyclotron frequency of electron in the magnetic field. When the microwave radiation is strong enough, the zero-resistance states are observed around the resistance minima. Durst *et al.* proposed a theory [4] that successfully explains the period and the phase of the magnetoresistance oscillation and also yields the negative resistance at the positions where the zero-resistance state was observed in the experiments. The existence of the negative resistance was also predicted by Ryzhii [7]. Andreev *et al.* [5] pointed out that such a negative resistance state is essential to understanding the zero-resistance state, because the negative resistance is unstable in nature and could be interpreted as the "zero resistance" by the measurement techniques employed in those experiments. A similar conclusion is also reached in Ref. [6]. In essence, the existence of the negative resistance state is crucial in the current stage of theoretical understanding of the phenomenon.

In this Letter, we show that such a negative resistance state is the result of the nontrivial structure of the density of states of the 2DEG system and the photon-assisted transport. The similar effect of photon-assisted transport could be observed in other systems. A generalized Kubo-Greenwood formula is presented to provide a formal theory for such phenomena.

To demonstrate our point in a clear and simple way, first we consider the transport through a quantum tunneling junction. We show that such a toy model catches most of the qualitative feature of the 2DEG experiments [1–3]. At the same time, the simplicity of the model provides us a clear view to the origin of the transport anomaly. Then we present a generalized Kubo-Greenwood formula to calculate the conductivity of a general system under the influence of radiation and provide a natural explanation of the success of the simple toy model.

The structure of the toy model is shown in Fig. 1. An ac voltage $V_{\rm ac} = \Delta\cos\omega t$ is applied across the junction to model the microwave radiation. The current through the junction is written as [8]

$$I = eD\int d\epsilon \sum_n J_n^2\left(\frac{\Delta}{\hbar\omega}\right)[f(\epsilon) - f(\epsilon + n\hbar\omega + eV)] \times \rho_{\rm L}(\epsilon)\rho_{\rm R}(\epsilon + n\hbar\omega + eV), \quad (1)$$

where $\rho_{\rm L(R)}$ is the density of states of the left (right) lead, $f(\epsilon)$ is the Fermi distribution function, D is the transmission constant of the junction, and $J_n(x)$ is the Bessel function of nth order.

For simplicity, we consider a symmetric system, $\rho_{\rm L}(\epsilon) = \rho_{\rm R}(\epsilon) = \rho(\epsilon)$. In this case, the zero-bias conductance $\sigma = dI/dV|_{V=0}$ can be written as

$$\sigma = e^2D\int d\epsilon \sum_n J_n^2\left(\frac{\Delta}{\hbar\omega}\right) \times \{[-f'(\epsilon)]\rho(\epsilon)\rho(\epsilon + n\hbar\omega) + [f(\epsilon) - f(\epsilon + n\hbar\omega)]\rho(\epsilon)\rho'(\epsilon + n\hbar\omega)\}. \quad (2)$$

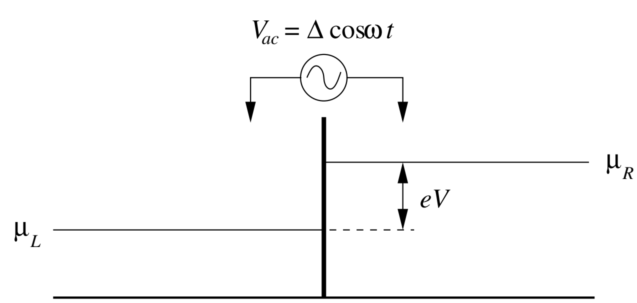

FIG. 1. Toy model of a quantum tunneling junction. A dc voltage V and an ac field $V_{\rm ac} = \Delta\cos\omega t$ are applied on a quantum tunneling junction.

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The first term of the equation is always positive, while the second term depends on the derivative of the density of states and can be negative. The contribution from the second term is purely due to the photon-assisted tunneling process and vanishes when there is no ac field.

Based on Eq. (2), it is not difficult to design a system with the necessary form of the density of states to realize a negative conductance. This is especially feasible for artificial quantum systems [9]. However, to make our following discussion more focused, we assume the density of states in the leads is a periodic function of energy near the Fermi surface with a period $\hbar\omega_c$. By assuming that, we show that such a simple toy model catches most of the important features of the experiments [1–3].

Without invoking a special form for the density of states, we can show that the conductance at the ac frequency $\omega = n\omega_c$ is identical to its dark field value, as observed in the experiments [1–3]. In this case, $\rho(\epsilon + n\hbar\omega) = \rho(\epsilon)$, the second term of Eq. (2) vanishes, leading to

$$\sigma = e^2 D \int d\epsilon \sum_n J_n^2\left(\frac{\Delta}{\hbar\omega}\right)[-f'(\epsilon)]\rho^2(\epsilon)$$
$$= e^2 D \int d\epsilon [-f'(\epsilon)]\rho^2(\epsilon) \equiv \sigma_{\text{dark}}, \quad (3)$$

where we have used the identity $\sum_n J_n^2(x) = 1$.

We now assume the density of states has the following form:

$$\rho(\epsilon) = \left(1 + \lambda \cos\frac{2\pi\epsilon}{\hbar\omega_c}\right)\rho_0, \quad (4)$$

with λ being a dimensionless constant. A straightforward calculation yields the conductance of the system,

$$\sigma(T)/\sigma_0 = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{\Delta}{\hbar\omega}\right)\left[1 + \frac{\lambda^2}{2}\cos\left(2\pi n\frac{\omega}{\omega_c}\right) - n\pi\lambda^2\frac{\omega}{\omega_c}\sin\left(2\pi n\frac{\omega}{\omega_c}\right)\right] + g\left(\frac{\mu}{\hbar\omega_c}, T\right), \quad (5)$$

where $\sigma_0 = e^2 D\rho_0^2$, and $g(\mu/\hbar\omega_c, T)$ is the contribution from the Shubnikov–de Haas oscillation which diminishes rapidly at finite temperatures. The conductance oscillation minima can be easily determined from Eq. (5): for the kth harmonics of the oscillation, the positions of the conductance minima are given by the equation $\tan x = -x/2$, where $x = 2\pi k\omega/\omega_c$. For $k = 1$, it yields the conductance minimum positions very close to $\omega/\omega_c = n + 1/4$, although not exactly. When the higher orders of harmonics become important, we expect that the conductance minima deviate from the $n + 1/4$ rule. The amplitude of oscillation is independent of the temperature, indicating any temperature dependence observed in the experiments should come from the temperature dependence of the density of states, i.e., λ.

Next we use a more realistic density of states for the leads: when the leads are the 2DEGs under a perpendicular weak magnetic field, λ is a function of ω_c [10],

$$\lambda = 2\exp\left(-\frac{\pi}{\omega_c\tau_f}\right), \quad (6)$$

where ω_c is the cyclotron frequency of the electron and τ_f is the relaxation time of the electron which depends on the scattering mechanisms of the system and the temperature. The conductance for such a system is shown in Fig. 2. It is evident that our model system, although very different and much simpler, shows striking resemblances to the experimental observations [1–3] and the more realistic calculation [4]. The conductance minima are found at the positions near $\omega/\omega_c = n + 1/4$ for the low and intermediate intensities of the ac field. When the intensity becomes even higher, the multiphoton process sets in, presenting the high order harmonic components to the conductance oscillation. As in the experiments [1–3], one can see two sets of crossing points at $\omega/\omega_c = n$ and $\omega/\omega_c = n + 1/2$. As we have shown above, the former is a general property of the periodic density of states, whereas the latter will be destroyed by high intensities of radiation, as shown in Fig. 2.

The system becomes unstable when entering into the negative conductance regime. The consequence of such instability can be easily foreseen in our toy model. In the case of constant voltage measurement, a negative conductance means the current across the junction is in the reversed direction to the electric field applied, as shown in Fig. 3(a). As a result, the tunneling electrons cannot be removed from the junction by the electric field in the leads. Instead, they accumulate near the junction and increase the effective voltage difference. The process

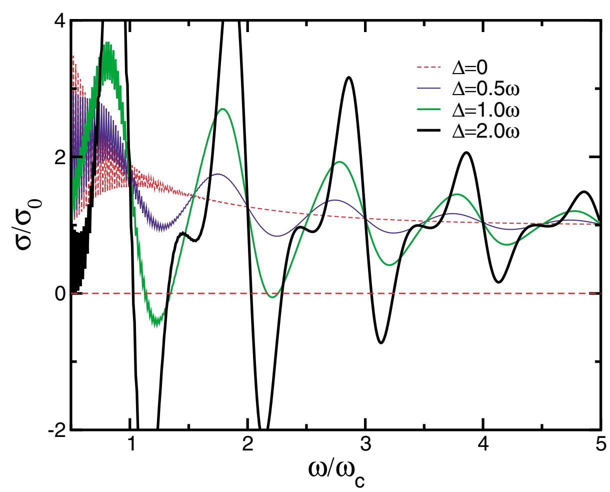

FIG. 2 (color online). Conductance dependence on $1/\omega_c$ for different radiation intensities. The parameters are the same as those used in Ref. [4]: $\mu = 50\hbar\omega$, $kT = 0.25\hbar\omega$, $\omega\tau_f = 6.25$.

continues on until the effective voltage difference between the junction reaches such a point that the current becomes zero, as shown in Fig. 3(b). Consequently, the measurement will yield a zero conductance. On the other hand, in a constant current measurement, the system will be in a bistable state with either positive or negative junction voltage, depending on the history of the applied current, as shown in Fig. 3(c). We stress that the analysis is very sensitive to the detailed setup of the system. In the case of this toy model, many parameters such as barrier thickness, lead configurations, and dielectric constants may affect the resulting phase. However, the instability itself is totally determined by the radiation power and the density of states.

The photon-assisted transport process, which is responsible for the transport anomaly in the tunneling junction, also exists in a uniform system like the 2DEG. This becomes clear when we look at the Kubo-Greenwood conductivity formula [11], where the total conductivity is a summation over all possible "tunneling" between single electron states. A generalization of the Kubo-Greenwood formula [see Eqs. (12) and (15)] shows the similar contribution of the photon-assisted tunneling. The effective ac voltage $\Delta_{\rm eff}$ in such tunneling is determined by the spatial separation between the involved single electron states and on average is of the order of $E_\omega l$, where l is the mean free path of the electron and E_ω is the radiation field strength. Based on the parameters given in the experiment [1], we deduce that $l \sim 10^{-4}$ m, $E_\omega \sim 10$ V/m; thus $\Delta_{\rm eff} \sim 1$ meV, which is the same order of the radiation frequency $\hbar\omega \sim 0.4$ meV. The estimation indicates the photon-assisted process is sufficient to understand the observed transport anomaly.

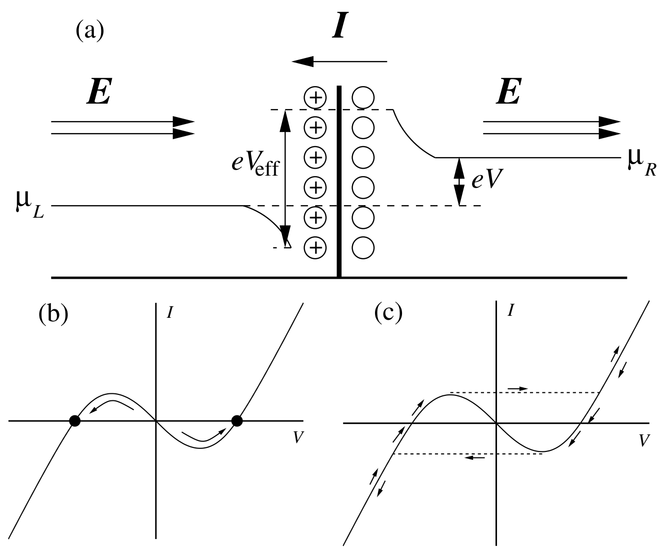

FIG. 3. (a) Charge buildup in the negative conductance regime at a constant voltage measurement. (b) Instability in a constant voltage measurement. (c) Bistability in a constant current measurement.

Now we derive a generalized Kubo-Greenwood formula for the system under the influence of radiation. For a general system considered, the Hamiltonian can be written as

$$H = H_0 + H_{\rm ac}(\omega) + H_{\rm dc}. \quad (7)$$

Here H_0 is the original Hamiltonian of a general system. $H_{\rm ac}(\omega)$ is the coupling to the external radiation of frequency ω. $H_{\rm dc}$ is the potential induced by a small dc field and is treated as a small perturbation to the system defined by $H_1 = H_0 + H_{\rm ac}$. The standard linear response theory yields the current density as [12]

$$\mathbf{J}(t) = \lim_{\omega_0\to 0} \frac{2\mathbf{E}_0}{\hbar\omega_0} \int_{-\infty}^{t} dt' \langle[\hat{\mathbf{j}}(t), \hat{\mathbf{j}}(t')]\rangle e^{-i\omega_0 t' + \eta t'}, \quad (8)$$

where the dc field is simulated by an electric field $\mathbf{E}_0 e^{-i\omega_0 t + \eta t}$ with the infinitesimal frequency ω_0, and $\hat{\mathbf{j}} = e/2\sum_i[\mathbf{v}_i\delta(\mathbf{x}-\mathbf{x}_i) + \delta(\mathbf{x}-\mathbf{x}_i)\mathbf{v}_i]$ is the current operator. We have omitted the gauge term which has no contribution to the dc current. For brevity, we drop $\lim_{\omega_0\to 0}$ in the following derivations.

According to the Floquet theorem, the wave function of the system defined by H_1 can be written as

$$|\alpha(t)\rangle = e^{-i\tilde{E}_\alpha t/\hbar} \sum_{n=-\infty}^{\infty} e^{-in\omega t}|\alpha, n\rangle, \quad (9)$$

where $\tilde{E}_\alpha$ is quasienergy of the Floquet state. We assume the external radiation is applied onto the system adiabatically, and the system keeps the adiabaticity during the process [13], so each Floquet state can be uniquely mapped to an eigenstate $|\alpha\rangle$ for the system without the radiation.

Turning to the Heisenberg representation and using the eigenstates of H_0 as the basis, we can expand the current operator as

$$\hat{\mathbf{j}}(t) = e^{i\tilde{H}_0 t/\hbar}\left[\sum_{n=-\infty}^{\infty} \hat{\mathbf{j}}_n e^{-in\omega t}\right] e^{-i\tilde{H}_0 t/\hbar},$$
$$\hat{\mathbf{j}}_n = \sum_{m,\alpha,\beta} |\alpha\rangle\langle\alpha, m|\hat{\mathbf{j}}|\beta, m+n\rangle\langle\beta|, \quad (10)$$

where $\tilde{H}_0$ is the quasienergy operator defined by $\tilde{H}_0|\alpha\rangle = \tilde{E}_\alpha|\alpha\rangle$.

Substituting Eq. (10) into Eq. (8) and following the usual process of the derivation of the Kubo formula [12], we have

$$\sigma_{\rm dc} = \frac{2\pi}{V}\frac{\partial}{\partial\omega_0}\sum_{f,i}\sum_n (P_i - P_f)|\langle f|\hat{\mathbf{j}}_n|i\rangle|^2 \times \delta(\hbar\omega_0 + n\hbar\omega - \tilde{E}_f + \tilde{E}_i), \quad (11)$$

where $P_{i(f)} \approx e^{-\beta\tilde{E}_{i(f)}}/Z$ is the probability of the system at the state $|i\rangle$ ($|f\rangle$) in the limit that the radiation power is not extremely strong and $\omega\tau \gg 1$, where τ is the population relaxation time of the system.

For the noninteracting system, the generalized Kubo-Greenwood formula reads

$$\sigma_{\rm dc} = 2\pi \frac{\partial}{\partial \omega_0} \sum_n \int d\epsilon_\alpha [f(\tilde{\epsilon}_\alpha) - f(\tilde{\epsilon}_\alpha + \hbar\omega_0 + n\hbar\omega)] \times \overline{|\langle\beta|\hat{\mathbf{j}}_n|\alpha\rangle|^2} \rho(\tilde{\epsilon}_\alpha)\rho(\tilde{\epsilon}_\alpha + \hbar\omega_0 + n\hbar\omega), \qquad (12)$$

where $\overline{|\langle\beta|\hat{\mathbf{j}}_n|\alpha\rangle|^2}$ is the average over all possible initial and final states for the transitions $\tilde{\epsilon}_\alpha \rightarrow \tilde{\epsilon}_\alpha + n\hbar\omega + \hbar\omega_0$. The same contribution of the photon-assisted process is evident by comparing it with Eq. (2).

To gain more insight to Eq. (12), we study a limiting case where the wavelength of the radiation is much longer than the spatial extent of the electron wave function. As a result, the ac electric field felt by the individual electron state can be approximately considered as spatially independent. So for state $|\alpha\rangle$, $H_{\rm ac}^\alpha \approx \Delta\cos(\omega t - \mathbf{k}\cdot\mathbf{r}_\alpha)$, where $\Delta = eE_\omega c/\omega$, and $\mathbf{r}_\alpha$ is the average center of the wave function. Now the Floquet state can be obtained analytically,

$$|\alpha(t)\rangle \approx e^{-i\epsilon_\alpha t/\hbar} \sum_{n=-\infty}^{\infty} J_n\left(\frac{\Delta}{\hbar\omega}\right) e^{-in(\omega t - \mathbf{k}\cdot\mathbf{r}_\alpha)}|\alpha\rangle. \qquad (13)$$

Comparing Eq. (13) with Eq. (9), we conclude $|\alpha, n\rangle = J_n(\Delta/\hbar\omega)\exp(in\mathbf{k}\cdot\mathbf{r}_\alpha)|\alpha\rangle$ and $\epsilon_\alpha = \tilde{\epsilon}_\alpha$. With Eq. (10), it is straightforward to get

$$\langle\alpha|\hat{\mathbf{j}}_n|\beta\rangle = e^{in\mathbf{k}\cdot(\mathbf{r}_\alpha+\mathbf{r}_\beta)/2} J_n\left(\frac{\Delta_{\alpha\beta}}{\hbar\omega}\right)\hat{\mathbf{j}}_{\alpha\beta}, \qquad (14)$$

where $\Delta_{\alpha\beta} = \Delta|\exp(i\mathbf{k}\cdot\mathbf{r}_\alpha) - \exp(i\mathbf{k}\cdot\mathbf{r}_\beta)| \approx eE_\omega|\mathbf{r}_{\alpha\beta}\cdot\mathbf{k}/k|$ is the effective ac potential between two states, and $\hat{\mathbf{j}}_{\alpha\beta} = \langle\alpha|\hat{\mathbf{j}}|\beta\rangle$.

We obtain the total conductivity:

$$\sigma_{\rm dc} = \frac{\partial}{\partial\epsilon_0}\sum_n \int d\epsilon D_n(\epsilon, \epsilon + n\hbar\omega)[f(\epsilon) - f(\epsilon + \epsilon_0 + n\hbar\omega)] \times \rho(\epsilon)\rho(\epsilon + \epsilon_0 + n\hbar\omega), \qquad (15)$$

where $D_n(\epsilon, \epsilon + n\hbar\omega) = 2\pi\hbar\overline{J_n^2(\Delta_{\alpha\beta}/\hbar\omega)|\hat{\mathbf{j}}_{\alpha\beta}|^2}$. The total conductivity can be considered as the summation of photon-assisted tunneling between the electron states. This proves our previous qualitative argument.

Finally, we discuss the implication of negative conductance. In general, negative conductance signifies the instability of the driven system by an external microwave radiation. Such instability may drive the system to a far-from-equilibrium regime where nonlinear and self-organizing effects dominate, resulting in intriguing and rich phenomena [14]. One possible phase of such a kind has been proposed to understand the zero-resistance state [5,6]. On the other hand, we stress that these are two separate issues: (a) origin of the instability and (b) a new phase induced by the instability. As demonstrated in the toy model, while the instability is totally determined by the radiation power and the density of states, the determination of a new phase requires detailed knowledge of the system and deserves more studies.

In summary, we demonstrate the existence of the negative conductance in a quantum tunneling junction under the influence of a radiation field. We trace the origin of such a transport anomaly to the nontrivial structure of the density of states of the system and the photon-assisted transport. A generalized Kubo-Greenwood conductivity formula is presented to show essentially the same nature of the anomalies observed in tunneling junctions and in 2DEG systems. We expect similar transport anomalies could be observed in other systems.

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