

# The ground-state lifetime of bound polaron in a parabolic quantum dot<sup>☆</sup>

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## ABSTRACT

The ground-state energy of bound polaron was obtained with strong electron–LO–phonon coupling using a variational method of the Pekar type in a parabolic quantum dot (QD). Quantum transition occurred in the quantum system due to the electron–phonon interaction and the influence of temperature. That is the polaron transit from the ground state to the first-excited state after absorbing a LO-phonon and it changes the polaron lifetime. Numerical calculations are performed and the results illustrate the relations of the ground-state lifetime of the bound polaron on the ground-state energy of polaron, the electron–LO–phonon coupling strength, the confinement length of the quantum dot, the temperature and the Coulomb binding parameter.

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## 1. Introduction

With the recent progress in microfabrication technology, such as molecular-beam epitaxy and nanolithography, it became possible to fabricate synthetic polar semiconductor structures with low dimensionality such as dielectrics, heterojunctions, quantum wells, quantum dots and quantum wires. Consequently, there has been a large amount of work, both experimental [1–3] and theoretical [4–6], especially with the development of quantum information and quantum computing. In recent years, the most feasible approach is quantum dots for realizing the quantum computer because of the advantage of being integrated. The two quantum states of electron are employed as a single qubit in a quantum dot [7–9]. In practice, the electron–phonon interaction is essential to understand the optical absorption spectra in semiconductors. Research on the polaron effect has become a main subject in the physics of low-dimensional quantum systems. Especially in a quantum dot system, electron–phonon interactions are enhanced by the geometric confinement. Therefore, a number of studies have focused on the influence of electron–phonon interactions on impurity bound polaron in parabolic quantum dots. Chen et al. [10] studied the thickness dependence of the binding energy of an impurity bound polaron in a parabolic QC in magnetic fields using the second-

order perturbation theory. Au-Leang et al. [11] investigated the combined effects of a parabolic potential and a Coulomb impurity on the cyclotron resonance of a three-dimensional bound magnetopolaron using Larsen's perturbation method, under the condition of strong parabolic potential. Wang et al. [12] recently studied the binding energy of hydrogenic impurities in a GaAs cylindrical quantum dot (QD) using a two-parameter variational wave function. By introducing a trial wave function constructed as a direct form of an electronic part and a part of coherent phonons, Kandemir et al. [13] investigated the polaronic effect on the low-lying energy levels of an electron bound to a hydrogenic impurity in a three-dimensional anisotropic harmonic potential subjected to a uniform magnetic field. Boucaud et al. [14] studied polaron decay in InAs/GaAs self-assembled quantum dots by pump-probe spectroscopy, and Zibik et al. [15] investigated polaron decay in n-type InAs quantum dots using energy-dependent, mid-infrared pump-probe spectroscopy. Verzelen et al. [16] calculated the polaron relaxation time to thermo-dynamical equilibrium when its lifetime is limited by the decay of its phonon component due to crystal anharmonicity. In these works, however, the ground-state lifetime of bound polaron in a parabolic QD has not been investigated so far.

The purpose of the present paper is to explore the ground-state lifetime of bound polaron in a parabolic QD with strong electron–LO–phonon coupling using a variational method of Pekar type. The relations between the ground-state lifetime of polaron and the ground-state energy of polaron, the electron–LO–phonon coupling strength, the confinement length of the quantum dot, the Coulomb binding parameter and the temperature are discussed.

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## 2. Theoretical model

We consider the system in which the electrons are bounded by the confinement potential in a single QD as parabolic:

$$V = \frac{1}{2}m^*\omega_0^2r^2, \quad (1)$$

where  $m^*$  is the band mass of the electron,  $r$  the coordinate of the electron and  $\omega_0$  the confinement strength. The Hamiltonian of the electron–phonon system is given by

$$H = -\frac{\hbar^2}{2m^*}\nabla_r^2 + \frac{1}{2}m^*\omega_0^2r^2 + \sum_q \hbar\omega_{LO}b_q^+b_q + \sum_q (V_q e^{iqr}b_q + hc) - \frac{e^2}{\epsilon_\infty r}, \quad (2)$$

where  $-e^2/\epsilon_\infty r$  is the Coulomb bound potential and  $\beta = e^2/\epsilon_\infty$  is the Coulomb binding parameter,  $b_q^+$  ( $b_q$ ) is the creation (annihilation) operator of a bulk LO-phonon with wave vector  $q$  ( $q = q_{||}, q_z$ ). And

$$V_q = i(\hbar\omega_{LO}/q)(\hbar/2m^*\omega_{LO})^{1/4}(4\pi\alpha/V)^{1/2}, \quad (3)$$

$$\alpha = (e^2/2\hbar\omega_{LO})(2m^*\omega_{LO}/\hbar)^{1/2}(1/\epsilon_\infty - 1/\epsilon_0). \quad (4)$$

Using the Fourier expansion to the Coulomb bound potential, we get

$$-\frac{\beta}{r} = -\sum_q \frac{4\pi\beta}{Vq^2} \exp(-iqr). \quad (5)$$

We carry out the Lee-Low-Pines transformation to Eq. (2):

$$U = \exp \left[ \sum_q (f_q b_q^+ - f_q^* b_q) \right], \quad (6)$$

where  $f_q$  will be treated as a variational function, we have

$$H' = U^{-1}HU. \quad (7)$$

Suppose the Gaussian function approximation is still valid in the ground state of the electron–phonon system according to the variational method of Pekar type. The ground-state wave function of the system is chosen as

$$|\varphi_{e-p}\rangle = \left(\frac{\lambda}{\sqrt{\pi}}\right)^{3/2} \exp\left(-\frac{\lambda^2 r^2}{2}\right) |0_{ph}\rangle, \quad (8)$$

where  $\lambda$  is the variational parameter and  $|0\rangle$  is zero phonon state that satisfies  $b_q|0\rangle = 0$ . Then the obtained ground-state energy of polaron can be written as follows:

$$E_0(\lambda) = \langle \varphi_{e-p} | H' | \varphi_{e-p} \rangle = \frac{3\hbar^2\lambda^2}{4m^*} + \frac{3\hbar^2}{4m^*\lambda^2 l_0^4} - \sqrt{\frac{2}{\pi}}\alpha\hbar\omega_{LO}r_0\lambda - \frac{2}{\sqrt{\pi}}\beta\lambda, \quad (9)$$

where the confinement length of a QD is  $l_0 = (\hbar/m \times \omega_0)^{1/2}$  and the polaron radius is  $l_0 = (\hbar/m^*\omega_0)^{1/2}$ . Then the ground-state energy of polaron in a parabolic quantum dot can be written as

$$E_0 = \frac{3\hbar^2\lambda_0^2}{4m^*} + \frac{3\hbar^2}{4m^*\lambda_0^2 l_0^4} - \sqrt{\frac{2}{\pi}}\alpha\hbar\omega_{LO}r_0\lambda_0 - \frac{2}{\sqrt{\pi}}\beta\lambda_0. \quad (10)$$

Under the influence of the electron–phonon interaction and the temperature based on the Fermi Golden Rule, the transit rate that the polaron transits from the ground state to the first-excited state after absorbing a LO-phonon is given by

$$T^{-1} = \frac{\alpha\omega_{LO}}{2\lambda_0} \sqrt{\frac{2m^*\omega_{LO}}{\hbar}} n_q \ln \frac{\left(\sqrt{\lambda_0^2 + 2m^*\omega_{LO}/\hbar} + \lambda_0\right)^2}{\left(\sqrt{\lambda_0^2 - 2m^*\omega_{LO}/\hbar} - \lambda_0\right)^2}. \quad (11)$$

According to quantum statistics, we have

$$n_q = \left[ \exp\left(\frac{\hbar\omega_{LO}}{K_B T}\right) - 1 \right]^{-1}, \quad (12)$$

where the Boltzmann constant is  $K_B$ ,  $T$  is the lifetime of polaron and  $\lambda_0$  the ground state wave vector of polaron; Eq. (12) should be self-consistent with the  $n_q$  of Eq. (11). Meanwhile, we assumed that  $\hbar\omega_{LO}/K_B T = \gamma$  is the temperature parameter and then Eq. (12) is converted into  $\bar{n}_q = [\exp \gamma - 1]^{-1}$ .

## 3. Results and discussion

The numerical results of the dependence of the ground-state lifetime of bound polaron on the electron–LO–phonon coupling strength, the confinement length of QD, the ground-state energy of polaron, the Coulomb binding parameter and the temperature in a parabolic QD are presented in Figs. 1–4, chosen in  $\hbar = 2m^* = \omega_{LO} = 1$  polaron units.

Fig. 1 presents the ground-state lifetime of bound polaron as a function of the ground state of polaron for different Coulomb binding parameters at temperature parameter  $\gamma = 1$ . It can be seen that the lifetime increases on increasing the ground-state energy. This is because the higher the ground-state energy of polaron, the lower the transition probability per second, which is a measure of how quickly the polaron is scattered from its ground state into the first-excited state under absorption of a LO-phonon.

Fig. 2 shows the lifetime of bound polaron as a function of the confinement length for different Coulomb binding parameters at the electron–LO–phonon coupling strength  $\alpha = 5$  and the temperature parameter  $\gamma = 1$ . This shows that the lifetime decreases on increasing the confinement length when the Coulomb binding parameter  $\beta = 0$ . The reason is that the movement of the electron is confined to the existence of the confine potential (parabolic potential) in a QD. On decreasing the confinement length ( $\omega_0$  increasing), that is, on increasing  $r_0$ , the thermal motion energy of electrons and the interaction between electron and phonons, which take phonon as medium, are reduced because the range of particles motion becomes large. So the ground-state energy in a parabolic QD decreases. As a result, the transition probability per second becomes fast and it leads to the reduction in the lifetime. However, the ground-state energy reduces on

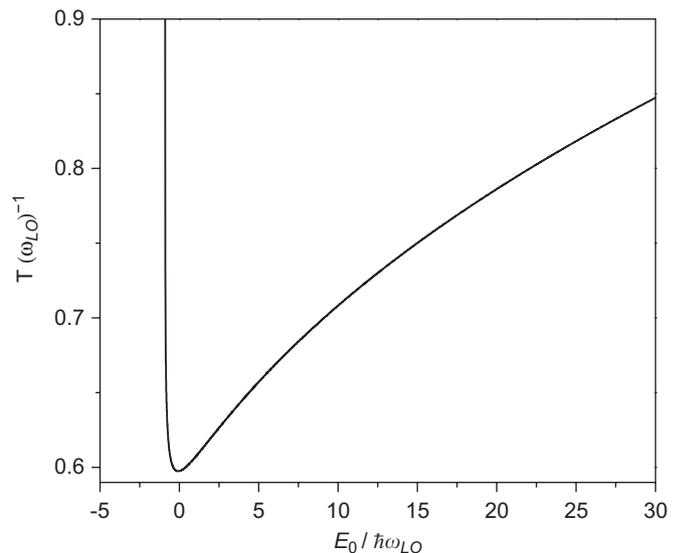


Fig. 1. The ground-state lifetime of polaron as a function of the ground-state energy in a parabolic QD.

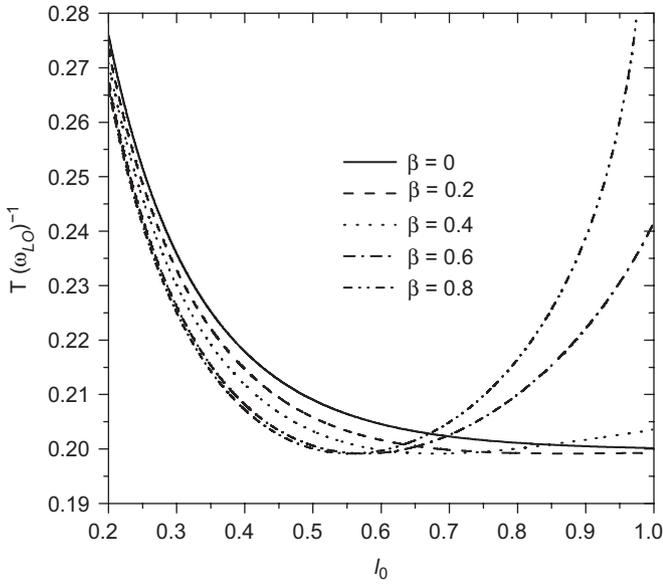


Fig. 2. The ground-state lifetime of bound polaron as a function of the confinement length in a parabolic QD for different Coulomb binding parameters.

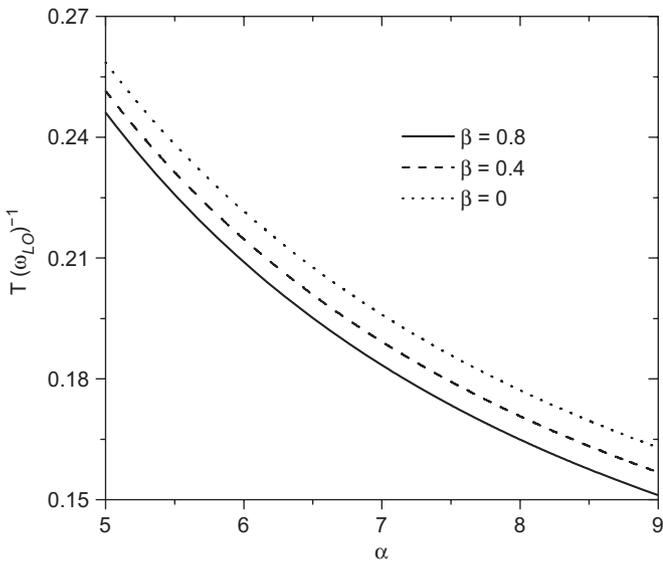


Fig. 3. The ground-state lifetime of bound polaron as a function of the electron-LO-phonon coupling strength in a parabolic QD for different Coulomb binding parameters.

increasing the confinement length when the Coulomb binding parameter is not equal to zero. And the ground-state energy reduces gradually to zero at a certain confinement. Then on increasing the confinement length, the value of the ground-state energy turns negative and the absolute value of the ground-state energy increases with decreasing confinement length. After this, the lifetime increases with increasing confinement length. At the same time, we found that when the ground-state energy  $E > 0$ , with increasing the Coulomb binding parameter the lifetime decreases and when the ground state energy  $E < 0$  the lifetime increases.

Fig. 3 illustrates the ground-state lifetime of polaron as a function of the electron-LO-phonon coupling strength for different Coulomb binding parameters  $\beta = 0, 0.4$  and  $0.8$  at confinement length  $l_0 = 0.4$  and temperature parameter  $\gamma = 1$ . It

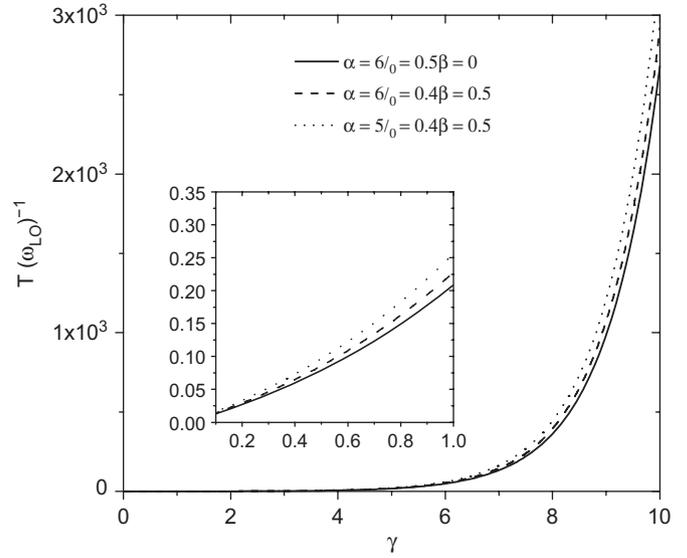


Fig. 4. The ground-state lifetime of bound polaron as a function of the temperature parameter at different coupling strengths and different Coulomb binding parameters.

shows that the ground-state lifetime decreases with increasing the electron-LO-phonon coupling strength. This is derived from the fact that the electron-phonon interaction to the ground-state energy is decreased. So on increasing the electron-phonon coupling strength, the ground-state energy will decrease. And then the transition probability per second becomes quick. Consequently, the ground-state lifetime is reduced with the result. This also shows that the ground-state lifetime of polaron decreases on increasing the Coulomb binding parameter.

Fig. 4 plots the ground-state lifetime of polaron as a function of the temperature parameter for different electron-LO-phonon coupling strengths, the Coulomb binding parameter and the confinement length, respectively. From them, we can see that on increasing the parameter (the temperature falling), the lifetime increases rapidly. This is because the lattice thermal vibrations become slow on decreasing the temperature. And it decreases sharply the numbers of phonon around the electron. For this reason, the probability that the electron absorbs a phonon becomes small. Meanwhile, we also see that the influence of temperature on the lifetime is dominant in a quantum dot. Only in certain temperatures, the influence of coupling strength and the confinement length to the lifetime is mainly considered.

#### 4. Conclusion

With strong electron-LO-phonon coupling by using a variational method of Pekar type in a QD, the ground-state energy lifetime of bound polaron was studied. The relations between the ground-state lifetime of bound polaron and the ground-state energy of polaron, the electron-LO-phonon coupling strength, the confinement length of a quantum dot, the temperature and the Coulomb binding parameter are discussed. The results indicate that the ground-state lifetime of bound polaron will increase on increasing the ground-state energy of polaron and decrease on increasing the electron-LO-phonon coupling strength, the confinement length of a quantum dot and the temperature. The influence of confinement length and the Coulomb binding parameter on the lifetime of bound polaron varies with the influence of them on the ground-state energy.

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