Noiseless Transfer of Nonclassical Light through Bistable Systems

Yuri M. Golubev[1] and Mikhail I. Kolobov[1,2]
[1]Physics Institute, St. Petersburg University, 198904 Petrodvorets, St. Petersburg, Russia
[2]Fachbereich Physik, Universität-GH Essen, D-45117 Essen, Deutschland
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We consider dispersive optical bistability with nonclassical driving field from a sub-Poissonian laser. The bistable system consists of an optical cavity filled with a resonant atomic medium. We calculate analytically the photocurrent noise spectrum at the output of the bistable system and find complete noise reduction close to the upper turning point of the bistability curve. This proves a possibility of using nonclassical light in optical bistability without destructing its quantum statistics. [S0031-9007(97)03565-5]

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In recent years the phenomenon of optical bistability has been in the scope of the quantum optics community primarily as a tool for generating the squeezed states of light [1]. Nevertheless, the traditional area for applications of optical bistability is in optical logic, optical computing, etc. At present, there are not many papers investigating such phenomena from the point of view of quantum noise which puts fundamental limits to the performance of devices based on optical bistability.

In Ref. [2] it was shown that dispersive optical bistability in a regime of an optical transistor can be used for noiseless amplification of a small input signal in coherent state. In this Letter we go one step further and consider driving of an optical bistable system with nonclassical, namely, sub-Poissonian, light. We take the sub-Poissonian laser as an external driving source mostly because its theory is well developed and the properties are well known [3]. When the light from such a laser is photodetected, the photocurrent noise at low frequencies can be reduced below the shot-noise level and in an ideal case even completely. The natural question arises: what happens with quantum noise of such nonclassical input signal in the bistable optical system? In other words, can nonclassical light be used in optical logic, optical computing, and other applications of optical bistability without destruction of its regular quantum statistics? Such a possibility would improve the signal-to-noise ratio of a prototype device beyond the standard quantum limit.

Below we show that driving a bistable optical system with sub-Poissonian laser light we can obtain the same degree of the shot-noise reduction at the output of the system as at its input very close to the upper turning point of the bistability curve. Since this point is very sensitive to a small amplitude modulation of the input signal, this gives a possibility of noiseless control (amplification, for example) of a small sub-Poissonian input signal.

We consider the scheme of quantum driving of a bistable optical system shown in Fig. 1. Two running-wave optical cavities are coupled in a unidirectional cascade. The first cavity 1(a) is a driving source, in our case a single-mode sub-Poissonian laser. The second (driven) cavity 1(b) contains a resonant optical medium. It is known that under certain conditions such a system interacting with an external field can have more than one steady state, manifesting optical bistability. The light on the output of the second cavity is photodetected. There are two possible photodetection schemes shown in Fig. 1: (1) When only transmitted light is detected, and (2) when a superposition of reflected and transmitted light is detected. For simplicity we consider the first scheme.

The main difference (and complication) of our problem from the standard description of the optical bistability is that we do not assume the driving field to be in coherent state. Thus, we cannot describe it as a complex amplitude in the interaction Hamiltonian of the atoms and the external field. Moreover, it is well known that such description is not applicable for the nonclassical driving field. Therefore, we choose to write the master equation for the density matrix $\rho(t)$ of the two cavity modes: the mode a of the source and the mode b of the driven cavity, excited by the external signal. This master equation has a structure,

$$\dot{\rho} = (L_a + L_b + L_{ab})\rho , \tag{1}$$

where the generators L_a and L_b describe the independent development of the field modes inside the (a) and (b)

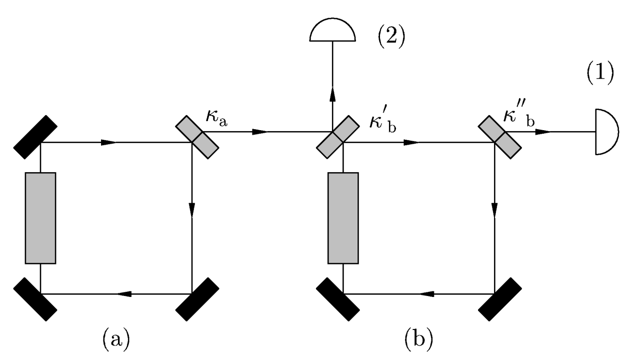

FIG. 1. The scheme of driving bistable optical resonator (b) by a sub-Poissonian laser (a) with unidirectional coupling.

resonators, and the generator L_{ab} gives the coupling between them. For the unidirectional cascaded scheme shown in Fig. 1 this coupling generator was obtained by Kolobov and Sokolov, Carmichael, and Gardiner [4], and reads

$$L_{ab}\rho = \kappa([\rho a^\dagger, b] + [b^\dagger, a\rho]). \qquad (2)$$

Here $a, a^\dagger (b, b^\dagger)$ are the annihilation and creation operators in the first (second) cavity obeying the standard boson commutation relations, $[a, a^\dagger] = [b, b^\dagger] = 1$; κ is the coupling constant equal to

$$\kappa = (\kappa_a \kappa_b')^{1/2}, \qquad (3)$$

where κ_a is the decay rate of the first cavity, $\kappa_a = c|\tau_a|^2 l_a^{-1}$, τ_a being the amplitude transmission coefficient of the outcoupling mirror, and l_a the perimeter of the cavity; analogously, κ_b' and κ_b'' are the decay rates associated with transmission of the input and the output mirrors of the second cavity; the total decay rate of the second cavity is $\kappa_b = \kappa_b' + \kappa_b''$.

It is worth noting that the generator L_{ab} is not symmetric with respect to the operators a and b. Physically speaking, it describes the process of annihilation of a photon in the source cavity and creation of a photon in the driven cavity (term $b^\dagger a$), but not the inverse process (no term $ab^\dagger$). Such an asymmetry would contradict the reciprocity principle for a closed system. However, we are dealing with the open system constituted by two damped optical cavities coupled via nonideal mirrors. The asymmetry in the generator L_{ab} is due to the unidirectional coupling of these cavities.

We choose the Glauber P representation for the density matrix ρ. This choice needs a word of comment. When the state of the field inside the first or the second cavity has no classical analog (like in the case of a sub-Poissonian laser in the first cavity), not all the diffusion coefficients in the equation for the distribution function $P(\alpha, \beta; t)$ are positive. Therefore, strictly speaking, we have to keep the α and β derivatives of P of all orders. However, for evaluation of the photocurrent noise spectrum we need the correlation functions of a and b modes not higher than second order. The derivatives of the order higher than two do not contribute to these correlation functions. Thus, for our purpose we can use the Fokker-Plank equation,

$$\dot{P}(\alpha, \beta; t) = (\mathcal{L}_a + \mathcal{L}_b + \mathcal{L}_{ab})P(\alpha, \beta; t), \qquad (4)$$

for the distribution function $P(\alpha, \beta; t)$. For calculations of the correlation functions and fluctuation spectra we shall write a set of equivalent Langevin equations.

We introduce the polar coordinates on the complex planes α and β as

$$\alpha = u_a^{1/2}\exp[i\varphi_a], \qquad \beta = u_b^{1/2}\exp[i\varphi_b], \qquad (5)$$

and linearize the radial components as $u_q = n_q + \epsilon_q, q = a, b,$ where $n_a(n_b)$ gives the mean value of the radial component $u_a(u_b)$, equal to the mean number of photons in the first (second) cavity, and $\epsilon_q \ll n_q$ is a small fluctuation. After such a linearization the generator $\mathcal{L}_a$ reads

$$\mathcal{L}_a = \Gamma_a\left\{\frac{\partial}{\partial\epsilon_a}\epsilon_a + n_a\xi_a\frac{\partial^2}{\partial\epsilon_a^2}\right\} + D_a\frac{\partial^2}{\partial\varphi_a^2}. \qquad (6)$$

Here Γ_a is the width of the amplitude fluctuations, D_a is the phase diffusion coefficient, and ξ_a is a statistical parameter,

$$\Gamma_a = \frac{\kappa_a I}{1 + I}, \qquad D_a = \frac{\kappa_a}{4n_a}, \qquad \xi_a = \frac{1}{I} - \frac{1}{2}, \qquad (7)$$

where I is the dimensionless laser intensity; the statistical parameter ξ_a determines the photon fluctuations inside the cavity as follows:

$$\langle(\Delta n_a)^2\rangle = (1 + \xi_a)n_a. \qquad (8)$$

We observe from (7) that ξ_a is bounded from below as $\xi_a \geq -1/2$. For $-1/2 \leq \xi_a < 0$ the photon statistics inside the cavity is sub-Poissonian, for $\xi_a = 0$—Poissonian, and for $\xi_a \geq 0$—super-Poissonian. When the light is photodetected at the output of the laser resonator, the photocurrent noise spectrum $(\delta i)^2_\omega$ has the following form:

$$(\delta i)^2_\omega = 1 + 2\xi_a\frac{\kappa_a\Gamma_a}{\Gamma_a^2 + \omega^2}. \qquad (9)$$

Here we have normalized the photocurrent noise spectrum to the shot-noise level and put the quantum efficiency of the photodetector to unity. From (9) we observe that in the case of $\xi_a = -1/2$ and far above threshold operation, $I \gg 1$ (when $\Gamma_a \approx \kappa_a$), there is a complete reduction of the shot noise at zero frequency $\omega = 0$ in the noise spectrum.

We assume that the second cavity contains N two-level atoms with atomic transition frequency ω_0, longitudinal decay rate $\gamma_\parallel$, and transverse decay rate $\gamma_\perp$. The lower atomic level is considered to be a ground state to ensure the conservation of the number of atoms in the scheme. We also assume that the decay of atomic polarization is due only to spontaneous emission, so that $\gamma_\parallel = 2\gamma_\perp$. The case of an additional nonradiative decay will be considered elsewhere.

We remind one as well of a set of parameters appearing in the theory of the optical bistability [1]. These are $\theta = (\omega_b - \omega_a)/(\kappa_b/2)$—the dimensionless detuning of two resonators, where $\omega_a(\omega_b)$ is the frequency of the source (driven) cavity; $\Delta = (\omega_0 - \omega_b)/\gamma_\perp$—the dimensionless atomic detuning; $n_0 = \gamma_\perp\gamma_\parallel/(4g^2)$—the saturation photon number, where g is the atom-field coupling constant; $C = g^2N/(\kappa_b\gamma_\perp)$—the cooperativity parameter.

When the rates of atomic decay are much larger than that of the field, $\gamma_\parallel, \gamma_\perp \gg \kappa_b$, the atoms can be eliminated adiabatically. In this case the generator $\mathcal{L}_b$

was obtained by Drummond and Walls in [5]. Adding to it the coupling generator $\mathcal{L}_{ab}$ we arrive at

$$\mathcal{L}_{ab} + \mathcal{L}_b = \frac{\partial}{\partial \beta}(A\beta - \kappa\alpha) + \frac{\partial^2}{\partial \beta^2}\beta^2 D_{\beta\beta} + \frac{\partial}{\partial \beta}\frac{\partial}{\partial \beta^\star}|\beta|^2 D_{\beta\beta^\star} + \text{c.c.}, \quad (10)$$

with the following drift and diffusion coefficients $A, D_{\beta\beta}$, and $D_{\beta\beta^\star}$,

$$A = \frac{\kappa_b}{2}\left(1 + i\theta + 2C\frac{1 - i\Delta}{1 + X + \Delta^2}\right), \quad (11)$$

$$D_{\beta\beta} = -\kappa_b \frac{C}{2n_0}\frac{(1 - i\Delta)^3 + X^2/2}{(1 + X + \Delta^2)^3}, \quad (12)$$

$$D_{\beta\beta^\star} = \kappa_b \frac{C}{2n_0}\frac{X(2 + X/2)}{(1 + X + \Delta^2)^3}. \quad (13)$$

Here we have used a shorthand, $X = |\beta|^2/n_0$.

Following the literature on optical bistability, we introduce two dimensionless intensities,

$$I_a = \frac{4\kappa^2}{\kappa_b^2}\frac{n_a}{n_0}, \qquad I_b = \frac{n_b}{n_0}, \quad (14)$$

related to the source and the driven cavity, respectively.

The stationary semiclassical solution of Eq. (4) is found from $A\beta - \kappa\alpha = 0$, with A given by (11). The latter gives two equations for the dimensionless intensity I_b and stationary phase difference $\bar{\varphi}$ defined as $\varphi_a - \varphi_b = \bar{\varphi} + \varphi$, where $\bar{\varphi}$ is the stationary mean value, φ is the fluctuation,

$$\sqrt{\frac{I_a}{I_b}}\cos\bar{\varphi} = 1 + \frac{2C}{1 + I_b + \Delta^2},$$
$$\sqrt{\frac{I_a}{I_b}}\sin\bar{\varphi} = \theta - \frac{2C\Delta}{1 + I_b + \Delta^2}. \quad (15)$$

Below we consider the limit of dispersive bistability, when $\Delta \gg 1$ [6]. The general case will be investigated elsewhere. For dispersive bistability it is convenient to employ the following scaled variables,

$$x = \frac{2C}{\Delta^3}I_b, \quad y = \frac{2C}{\Delta^3}I_a, \quad \delta = -\theta + \frac{2C}{\Delta}. \quad (16)$$

Without losing generality we can assume that $\Delta > 0$ and consider x and y as scaled intensities of the driven and the source cavities. From (15) we obtain the following relation between these scaled intensities [2],

$$y = x(1 + z^2), \qquad z = \delta - x. \quad (17)$$

The steady-state curve $x(y)$ is bistable for $\delta \geq \sqrt{3}$. It is shown in Fig. 2(a) for different values of δ. The turning points $x_\pm$ are found from $dy/dx = 0$ as

$$x_\pm = \frac{1}{3}[2\delta \pm \sqrt{\delta^2 - 3}]. \quad (18)$$

The system (15) gives also the stationary mean value of the phase $\bar{\varphi}$. Linearizing the Fokker-Plank equation (4) around the stationary solution (17) and $\bar{\varphi}$ we obtain

$$\frac{\partial}{\partial t}P = \left\{\frac{\kappa_b}{2}\frac{\partial}{\partial \epsilon_b}\left(\epsilon_b - \frac{n_b}{n_a}\epsilon_a + 2n_b z\varphi\right) - \frac{\kappa_b}{2}\frac{\partial}{\partial \varphi}\left(\varphi + \frac{z}{2n_b}\epsilon_a + z(2x - z)\epsilon_b\right) + \Gamma_a\frac{\partial}{\partial \epsilon_a}\epsilon_a + \Gamma_a n_a \xi_a \frac{\partial^2}{\partial \epsilon_a^2} + D_a\frac{\partial^2}{\partial \varphi_a^2} + \frac{\kappa_b}{2}x\frac{\partial^2}{\partial \varphi \partial \epsilon_b}\right\}P. \quad (19)$$

The rest is the standard technique used for calculation of the fluctuation spectra (see, for example, [7]). We write the Langevin equations for the fluctuations $\epsilon_a(t), \epsilon_b(t), \varphi_a(t)$, and $\varphi_b(t)$, perform their Fourier transform, and obtain a linear algebraic system of equations for the Fourier amplitudes $\epsilon_a(\omega)$, etc. Then we use the Wiener-Khinchine theorem which says that the noise power spectrum of $\epsilon_b(t)$, for example, is equal to $|\epsilon_b(\omega)|^2$. Finally, we employ the relation between the photocurrent noise spectrum $(\delta i)^2_\omega$ at the output of the second cavity and $|\epsilon_b(\omega)|^2$,

$$(\delta i)^2_\omega = 1 + \frac{\kappa_b''}{n_b}|\epsilon_b(\omega)|^2. \quad (20)$$

Surprisingly, the final result for the noise spectrum $(\delta i)^2_\omega$ looks relatively simple. We give it for the case of a symmetric b cavity, $\kappa_b' = \kappa_b''$, when there is no reflected signal in Fig. 1. This is an optimal situation for noise reduction in our observation scheme. The photocurrent noise spectrum reads

$$(\delta i)^2_\omega = 1 + \frac{(\kappa_b/2)^4}{|\kappa_+ - i\omega|^2|\kappa_- - i\omega|^2} \times \left[2\xi_a\left(\frac{\Gamma_a\kappa_a}{\Gamma_a^2 + \omega^2}\right)\frac{(1 + z^2)^2 + \omega^2/(\kappa_b/2)^2}{1 + z^2} + 4xz + 8\left(\frac{\kappa_a}{\kappa_b}\right)^2\frac{z^2}{1 + z^2}\right], \quad (21)$$

with

$$\kappa_\pm = \frac{\kappa_b}{2}[1 \pm \sqrt{z(2x - z)}]. \quad (22)$$

Choosing $\xi_a = -1/2$ and $\Gamma_a = \kappa_a$, which gives the complete noise reduction at $\omega = 0$ for the input signal [see (9)], we write the photocurrent noise spectrum at zero frequency for the output signal,

$$(\delta i)^2_{\omega=0} = \frac{z^2}{1 + z^2}\left[1 + 4\frac{x^2 + 4\kappa_a^2/\kappa_b^2}{(1 + z^2 - 2xz)^2}\right]. \quad (23)$$

From (23) it is easy to see that $(\delta i)^2_{\omega=0} = 0$ for $z = 0$, i.e., $x = \delta$. This point lies on the upper branch of the

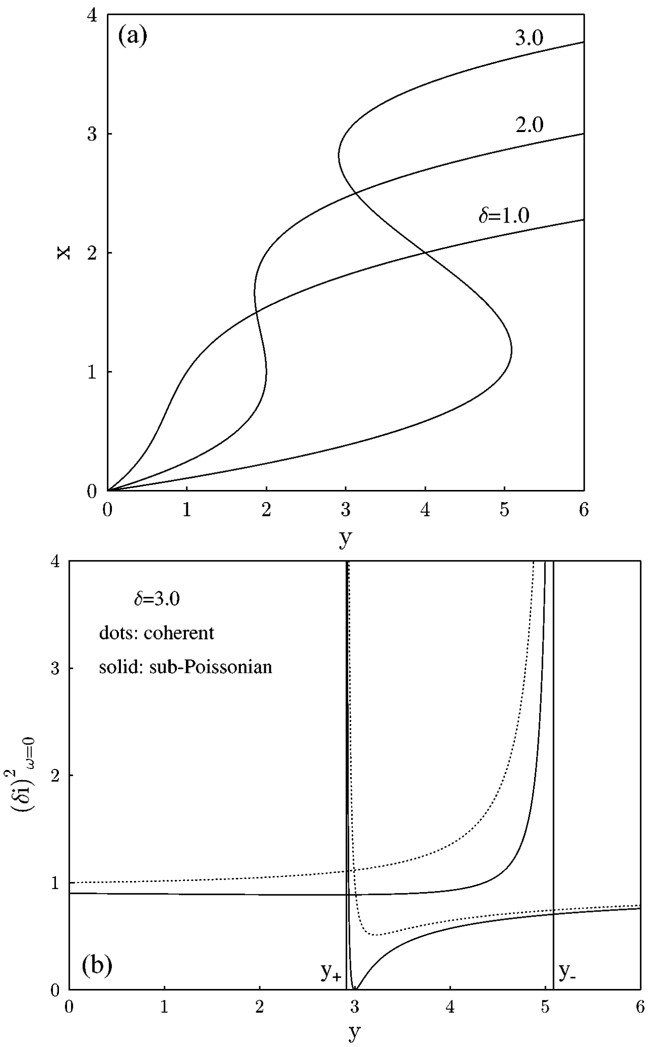

FIG. 2. (a) Bistability curve: intracavity scaled intensity x vs input intensity y for different parameters δ, $\delta = 1.0, 2.0, 3.0$; (b) photocurrent noise spectrum at zero frequency, $(\delta i)^2_{\omega=0}$, for coherent (dots) and sub-Poissonian (solid) input signal, $\delta = 3.0$.

bistability curve and for $\delta \gg 1$ is very close to the upper turning point $x_+ = (2\delta + \sqrt{\delta^2 - 3})/3$. The noise spectrum at zero frequency, $(\delta i)^2_{\omega=0}$, is shown in Fig. 2(b) as a function of the scaled input intensity y.

For comparison in Fig. 2(b) we have drawn also the noise spectrum $(\delta i)^2_{\omega=0}$ for a coherent input signal. In this case there is about 50% of shot-noise reduction at the output of the second cavity at the same point of the bistability curve. This ability of a bistable optical cavity to produce squeezing on its output is well known. It is, however, not what we are looking for. In our case we already have complete noise reduction in the input signal and want to process this signal *without adding noise*. This would allow us to improve the signal-to-noise ratio of a prototype device based on the phenomenon of optical bistability beyond the standard quantum limit.

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