Natural Quintessence with Gauge Coupling Unification

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We show that a positive accelerating universe can be obtained simply by the dynamics of a non-Abelian gauge group. The condensates of the chiral fields obtain a negative power potential below the condensation scale Λ_c and allow for a quintessence interpretation of these fields. The only free parameters are N_c, N_f, and the number of dynamically gauge singlet bilinear fields ϕ generated below Λ_c. We show that it is possible to have unification of all coupling constants, while having an acceptable phenomenology of ϕ as quintessance, without any fine-tuning of the initial conditions. The coincidence problem is not solved but it is put at the same level as that of the particle content of the standard model.

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In the past few years, different observations have led one to conclude that the universe is flat and filled with an energy density with negative pressure, a cosmological constant [1,2]. The cosmological constant is perhaps best understood, from an elementary particle point of view, as the contribution from a scalar field that interacts with all other fields only gravitationally, i.e., quintessence [3]. Recent observational results constrain the class of potentials since they require an energy density $\Omega_{\phi o} = 0.7 \pm 0.1$ with an equation of state parameter $w_{\phi o} = p_{\phi o}/\rho_{\phi o} \leq -0.6$, where the subscript o refers to present day quantities [1,2,4].

In this Letter, we show that a non-Abelian gauge group with N_c the number of colors and N_f that of chiral fields leads to an acceptable quintessence potential. We show that the only degrees of freedom are precisely the simple choice of N_c, N_f, and the number of dynamically generated bilinear fields which set the scale of condensation and the power in the potential of the scalar field responsible for present day acceleration of the universe. Of course, we are not able to determine from first principles the values of N_c, N_f but they are at the same footing as the choice of gauge groups and matter content of the standard model.

The model is quite simple: We start with a non-Abelian gauge group at a high energy scale (could be the unification scale of the standard model gauge groups) with massless matter fields, and we let it evolve to lower scales. By lowering the energy scale, the gauge coupling constant becomes large and all fields become strongly interacting at the condensation scale Λ_c. Below this scale, there are no more free elementary fields, chiral nor gauge fields, similar to what happens with QCD and we are left with gauge singlet bilinear fields $\phi^2 \equiv \langle Q\tilde{Q}\rangle$ (the square in ϕ is to give the field a mass dimension one). We use the Affleck-Seiberg superpotential [5] to determine the form of the scalar potential V in terms of ϕ (related work can be seen in [6]). Afterwards, we solve Einstein's general relativity equations in a Friedmann-Robertson-Walker flat metric and determine the cosmological evolution of ϕ. We show that a positive accelerating universe at present time with $\Omega_{\phi o} \simeq 0.7$ and $w_{\phi o} < -0.6$ is possible. We will bear in mind that the second restriction can be set in terms of an effective equation of state parameter $w_{\rm eff} \equiv \int da\, \Omega_\phi(a) w_\phi(a)/\int da\, \Omega_\phi(a) < -0.7$ [4].

Furthermore, we constrain the model to have the same unification scale and gauge coupling as the standard model gauge groups. This is by all means not a necessary condition, but it gives a very interesting model. We could think of this model as coming from string theory after compactifying the extra dimensions. The gauge coupling is unified for all gauge groups, the standard and nonstandard model gauge groups, at the compactification scale which is, in this case, also the unification scale. We then allow all fields to evolve cosmologically. Since at the beginning all fields are massless, they behave as radiation until a gauge group becomes strongly coupled and there is a phase transition. Below this scale, the particles charged under the strongly coupled gauge group condense while the other fields still evolve as radiation. Finally, we take into account the matter domination period and determine today's relevant cosmological quantities.

Let us begin by writing the scalar potential for a non-Abelian SU(N_c) gauge group with N_f (chiral + antichiral) massless matter fields. The superpotential is given by [5]

$$W = (N_c - N_f)\left(\frac{\Lambda^b}{\det\langle Q\tilde{Q}\rangle}\right)^{1/(N_c-N_f)}$$

and the scalar potential in global supersymmetry is $V = |W_\phi|^2$, with $W_\phi = \partial W/\partial\phi$, giving

$$V = (2N_f)^2 \Lambda_c^{4+n} \phi^{-n}, \tag{1}$$

where we have taken $\det\langle Q\tilde{Q}\rangle = \Pi_{j=1}^{N_f}\phi_j^2$, $n = 2(N_c + N_f)/(N_c - N_f)$, and Λ_c is the condensation scale of the gauge group SU(N_c). We have taken ϕ canonically normalized; however, the full Kahler potential K is not known, and for $\phi \simeq 1$ other terms may become relevant [6] and could spoil the runaway and quintessence behavior of ϕ. Expanding the Kahler potential as a series power $K = |\phi|^2 + \Sigma_i a_i |\phi|^{2i}/2i$, the canonically normalized field ϕ' can be approximated

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by $\phi' = (K_\phi^\phi)^{1/2}\phi$, and Eq. (1) would be given by $V = (K_\phi^\phi)^{-1}|W_\phi|^2 = (2N_f)^2\Lambda_c^{4+n}\phi^{-n}(K_\phi^\phi)^{(n/2-1)}$. [The canonically normalized field ϕ' is defined as $\phi' = g(\phi,\bar{\phi})\phi$ with $K_\phi^\phi = (g + \phi g_\phi + \bar{\phi} g_{\bar{\phi}})^2$.] For $n < 2$, the exponent term of K_ϕ^ϕ is negative so it would not spoil the runaway behavior of ϕ.

In terms of the evolution of the gauge coupling constant, we have

$$\Lambda_c = \Lambda_0 e^{-1/(2b_0 g_0^2)}, \tag{2}$$

with Λ_0, g_0 the energy scale and coupling constant at a high energy scale where the gauge group is weakly coupled and $b_0 = (3N_c - N_f)/16\pi^2$ the one-loop beta function. We would like to take Λ_0 as the unification scale $\Lambda_{\rm GUT} \simeq 10^{16}$ GeV and g_0 as the unification coupling $g_{\rm GUT} = \sqrt{4\pi/25.7}$ [7].

The presence of the field ϕ with potential Eq. (1) begins only at the condensation scale Λ_c. We can relate the scale Λ_c to the Hubble constant using $H^2 = \rho/3 \simeq \Lambda_c^4/3$ giving $\Lambda_c \simeq (3H^2)^{1/4}$, where we have set the reduced Planck mass to one (i.e., $m_p^2 = 8\pi G = 1$). By dimensional analysis, we set the initial condition for ϕ to be $\phi_i = \Lambda_c$ which is the natural choice.

The cosmological evolution of inverse power potential has been studied in [3,8]. The equations to be solved, for a spatially flat Friedmann-Robertson-Walker universe, in the presence of a barotropic fluid, which can be either radiation or matter given by an energy density ρ_γ, are given by [9,10]

$$\begin{aligned} x_N &= -3x + \sqrt{3/2}\,\lambda y^2 + \tfrac{3}{2}x[2x^2 + \gamma_\gamma(1 - x^2 - y^2)], \\ y_N &= -\sqrt{3/2}\,\lambda xy + \tfrac{3}{2}y[2x^2 + \gamma_\gamma(1 - x^2 - y^2)], \\ H_N &= -\tfrac{3}{2}H[2x^2 + \gamma_\gamma(1 - x^2 - y^2)], \end{aligned} \tag{3}$$

where N is the logarithm of the scale factor a, $N \equiv \ln(a)$, $f_N \equiv df/dN$ for $f = x, y, H$, and $\gamma_\gamma = 4/3, 1$ for radiation or matter fields, respectively, and $\lambda(N) \equiv -V'/V$. We have defined the variables $x \equiv \dot{\phi}/\sqrt{6}H$, $y \equiv \sqrt{V}/\sqrt{3}H$. In terms of x and y, one has $\Omega_\phi = x^2 + y^2$, and the equation of state for the quintessence field is given by $w_\phi = (x^2 - y^2)/(x^2 + y^2)$. Generic solutions to Eqs. (3) can be found in [10,11].

Notice that all model dependence in Eqs. (3) is through the quantity $\lambda(N)$. Using the potential given in Eq. (1) we have $\lambda = \frac{n}{\phi} = n(H_i y_i)^{-1/2}(Hy)^{2/n}/3^{1/4}$, where i stands for initial conditions. Since $\Lambda_c \ll m_p$ and $\phi_i = \Lambda_c$, the initial value of λ is very large $\lambda_i = nm_p/\phi_i \gg 1$ and this has interesting consequences.

From Eqs. (3), the evolution of Ω_ϕ is to drop rapidly, $\Omega_\phi \ll 1$, in about three e-folds; i.e., $\delta N \simeq 3$, regardless of its initial value and it remains very small for a large period of time [see Fig. 2 (below)]. These properties are due to the fact that λ_i is large. The evolution of ϕ enters a scaling regime with $\lambda =$ const and $\Omega_\phi \ll 1$ during all of this period. The scaling regime ends when $x = O(1/10)$ and Ω_ϕ becomes also of the order of 0.1.

In this Letter, we have determined $w_{\rm eff}$ for different values of n and we have concluded that, for $\Omega_{\phi i} \leq 0.25$, one needs $n < 2.7$ for $w_{\rm eff}$ to be smaller than -0.7 as required [4]. In Fig. 1 we show n as a function of $w_{\rm eff}$ assuming $\Omega_{\phi o} = 0.7$ and $h_o = 0.7$, where the Hubble constant is given by $H = 100$ h km/Mpc s. This result constrains many inverse power models. In fact, for $N_c > N_f$ one has $n > 2$. It is important to point out that we find a decreasing value of $w_{\phi o}$ with decreasing value of n in contrast with [3]. The main difference may be that in our models the value of $\Omega_\phi = 0.7$ is reached before w_ϕ joins the tracker solution.

Furthermore, to avoid any conflict with the standard big bang nucleosynthesis (NS) results, one requires $\Omega_\phi({\rm NS}) < 0.1$ at the energy scale of NS [12]; i.e., $E_{\rm NS} = 0.1-10$ MeV. The condition of not spoiling the NS results rules out the values of n between $1.2 < n < 2.1$ for models with $\Omega_{\phi i} > 0.1$. This is because for those values of n the initial value H_i lies within the value of the Hubble parameter at NS, and Ω_ϕ is not yet smaller than 0.1. Of course we could start with a small value of Ω_ϕ but we would lose our democratic choice of initial conditions. These constraints would leave a window as small as $2.1 < n < 2.7$ for $N_c > N_f$. However, if we insist deriving Λ_c from Eq. (2) with Λ_0, g_0 the unification scale and coupling, the above conditions constrain the models even more. In fact for $N_c > N_f$ $(n > 2)$ there are no models available that satisfy all constraints: $\Omega_{\phi o} = 0.7$, $w_{\rm eff} < -0.7$, and Λ_c given by Eq. (2) with $\Lambda_0 = \Lambda_{\rm GUT}$ and $g_0 = g_{\rm GUT}$. A full analysis of all cases will be presented elsewhere [13].

In order to have $\Lambda_0 = \Lambda_{\rm GUT}$ and $g_0 = g_{\rm GUT}$, we require the number of dynamically bilinear fields $Q\tilde{Q}$ to be different from N_f. Some of these fields may be fixed at their condensate constant vacuum expectation value (v.e.v.) with $\langle Q\tilde{Q}\rangle = \Lambda_c^2$, or we could have a gauge group with an unmatching number of chiral and antichiral fields.

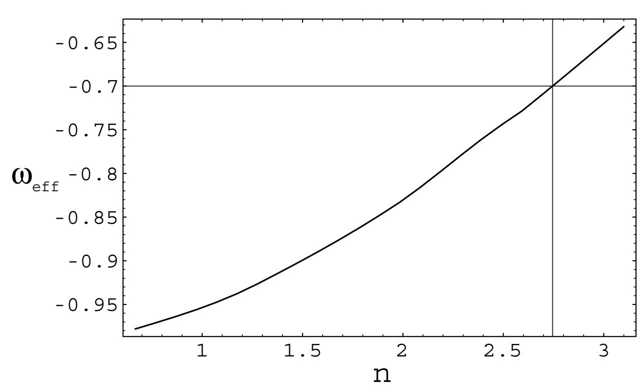

FIG. 1. Restriction on n from the upper limit $w_{\rm eff} \leq -0.7$.

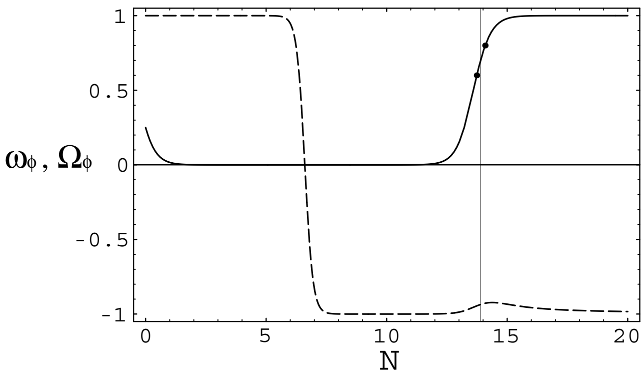

FIG. 2. Evolution of ω_ϕ and Ω_ϕ (dotted and solid lines, respectively). The vertical line represents the point for which $\Omega_\phi = \Omega_{\phi o} = 0.7$ and $h = h_0 = 0.7$. The lower dot marks $\Omega_\phi = 0.6$ while the upper one stands for $\Omega_\phi = 0.8$.

Here, we will present the case of an SU(3) gauge group with $N_f = 6$ chiral fields in the chiral and antichiral representation, and we will assume that only one bilinear field $\phi^2 = Q\tilde{Q}$ becomes dynamical with all other condensates remaining constant with a v.e.v. equal to Λ_c. Notice that this gauge group is self-dual ($\tilde{N}_c = N_f - N_c = 3$ with N_f flavors) under Seiberg's duality transformation [14]. (It is important to point out that, even though it has been argued that for $N_f > N_c$ there is no nonperturbative superpotential W generated [5], this is not always the case [15].)

The potential generated in this case is

$$V = 4\Lambda^{4+n}\phi^{-n}, \tag{4}$$

with $n = 2(1 + \frac{2}{N_c - N_f}) = 2/3$ and $\phi_i = \Lambda_c$. Using Eq. (2) with $16\pi^2 b_0 = 3N_c - N_f = 3$, one has $\Lambda_c = 4 \times 10^{-8}$ GeV, which is well below the NS energy scale. Notice that $n < 2$ so the noncanonical terms in K will not spoil the quintessence behavior of ϕ, and the mass is $m \simeq H_o$ so it is cosmologically fine [16].

Solving Eqs. (3) with the potential given in Eq. (4) and initial conditions $\Omega_i = 0.25$ and $H_i = (4\Lambda_c^4/3y_i^2)^{1/2} = 1 \times 10^{-33}$ GeV gives for $h_o = 0.7$ the values $\Omega_{\phi o} = 0.7$ and an equation of state parameter $w_{\phi o} = -0.97$ (with an effective $w_{\rm eff} = -0.98$). We see that the present day value of the parameters agrees with the analysis of recent observations [4] and there is no conflict with nucleosynthesis, since during nucleosynthesis the SU(3) gauge group was not strongly coupled and all those fields were massless and behaved as radiation at that epoch. The choice of initial conditions is not very sensitive and we took it as $\Omega_{\phi i} = 0.25$ to be democratic with the standard model gauge groups. A variation of 40% in the initial value of $\Omega_{\phi i}$ gives still a final result within the range of $h_o = 0.7 \pm 0.1$ and $\Omega_{\phi o} = 0.7 \pm 0.1$. Finally, we show in Fig. 2 the evolution of Ω_ϕ and w_ϕ as a function of N.

In conclusion, we have shown that starting with a non-Abelian gauge group with a gauge coupling constant unified with the standard model gauge couplings at the unification scale, a gauge singlet bilinear field ϕ, arising due to nonperturbative effects of the strongly interacting non-Abelian gauge group at the condensation scale, gives an acceptable phenomenology for the cosmological constant, and it is therefore a natural candidate for quintessence.

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