Optical response in a quantum dot superlattice nanoring under a lateral electric field

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The optical absorption of a GaAs/AlGaAs quantum dot superlattice nanoring (QDSLNR) under a lateral dc electric field and with magnetic flux threading the ring is investigated. This structure and configuration provides a unique opportunity to study the optical response of a superlattice under an inhomogeneous electric field, which is not easily realized for general quantum well superlattices (QWSLs) but naturally realized for QDSLNRs under a homogeneous lateral electric field. It has been shown that a lateral dc electric field gives rise to a substantial change of the optical absorption spectra. Under a low field, the excitonic optical absorption is dominated by a 1*s* exciton. And with the electric field increasing, the optical absorption undergoes a transition from 1*s* excitonic absorption to 0 excitronic WSL absorption. (The number of 0, and −1 and +1 below are WSLs index.) The −1 and the +1 WSLs corresponding to the maximum effective field can also be identified. Due to the inhomogeneity of the electric field, the peaks of the −1 and the +1 WSLs are diminished and between them there exist rich and complicated structures. This is in contrast to the general QWSLs under a homogenous electric field. The complicated structures can be understood by considering the inhomogeneity of the electric field along the ring, which results in the nearest-neighbor transition, the next-nearest-neighbor transition, etc., have a different value repectively, at different sites along the ring. This may give rise to multiple WSLs. We have also shown that the line shape of the optical absorption is not sensitive to the threading magnetic flux. The threading magnetic flux only gives rise to a slight diamagnetic shift. Thus the enhancement of the sensitivity to the flux allowing for observation of the excitonic Aharanov-Bohm effect in the plain nanoring is not expected in QDSLNRs.

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I. INTRODUCTION

Semiconductor superlattices have been investigated intensively by several generations of physicists since the first proposal by Esaki and Tsu in the early 1970s.[1] In contrast to natural crystals, the properties of the periodic potential of superlattices can be controlled and adjusted by material and growth parameters. Thus, superlattices are ideal objects to test many basic properties proposed for natural crystal but not experimentally observed due to the large bandwidth. Indeed, Bloch oscillation,[2,3] Wannier Stark ladder (WSL),[4,5] Zener tunnelling,[6,7] and miniband collapse and dynamic localization,[8,9] have been observed in superlattices, where their counterparts in natural crystals have been prohibited from being observed due to their small period. Recently, quantum dot superlattice nanowire structure consisting of a series of quantum dots interlaced along a nanowire has been grown.[10–14] These quasi-one-dimensional (1D) superlattices have unique features differing from the above-mentioned three-dimensional (3D) quantum well superlattices (QWSLs). These structures have a wide range of potential applications in nanoscale electronics, photonics, thermoelectrics, and magnetics, such as nanolasers,[10] nanobarcodes,[11] 1D waveguides, resonant tunneling diodes,[12,13] and thermoelectric systems,[15] thus attracting much attention. Very recently, Citrin suggested that there would be interesting magnetic Bloch oscillations, if a quantum dot superlattice nanowire is made to form a ring (QDSLNR) with time-dependent magnetic flux threading the ring.[16] Nanorings are natural systems to study quantum interference phenomenon and Aharanov-Bohm effect (ABE), thus have gained considerable attention in past decades. With rapid advances in fabrication of nanoscale devices, semiconductor nanorings that are disorder free and contain only a few electrons have been successfully fabricated and have been shown exhibiting pronounced optical features associated with the optical generation of excitons.[17–20] Many works have been made to investigate the excitonic ABE optically in semiconductor nanorings.[21–29]

The problem of quantum states of crystal electrons in a homogenous electric field is of considerable importance in physics and has been discussed and debated for decades. Now, it is well known that the states of an electron in a homogenous electric field are a sequence of resonances separated by equal energy intervals, the so-called WSL of resonances. Although QDSLNR structure remains beyond the present technological feasibility, it is conceivable that this QDSLNR might be realized in the not-too-distant future with the advances in material growth and fabrication technology. In this paper, we analyze numerically the optical absorption in a QDSLNR under a lateral electric field with magnetic flux threading the ring. One purpose of this work is to show that excitonic sensitivity to the threading magnetic flux shows up in this QDSLNR structure. More importantly, the envisioned structure and configuration provides us with a unique opportunity to investigate the characteristic optical spectrum of a superlattice under an inhomogeneous electric field applied along the growth direction of the superlattice.

To the best of our knowledge, thus far no theoretic effort about the inhomogeneous electric field effect on the optical absorption of superlattices has ever been made because of the difficulty in the experiments. In contrast, however, the lateral electric field applied to a QDSLNR leads naturally to an inhomogeneous electric field along the QDSLNR.

The paper is organized as follows. Section II presents a simplified theoretic model of the QDSLNRs and the equation-of-motion method used to obtain the differential optical absorption spectrum. In Sec. III we present the numerical results and related analyses. Finally in Sec. IV we summarize the main results and their implications.

II. PHYSICAL MODEL AND THEORETICAL APPROACH

A complete analysis of the excitonic absorption in the semiconductor including the band-structure degeneracy, confining potential, the *e-h* interaction, anisotropy, strain, and disorder, etc., is a very difficult task and needs tremendous efforts. To capture the main physical properties of the QDSLNRs, we consider a simplified 1D model that contains the periodic potential modulation of superlattice and annular topology in geometry. In this paper, we use the effective mass approximation and two band model. Furthermore, we only account for heavy holes and assume that the carrier confinement inside the QDSLNR is strong enough, so the extension of the QDSLNR is neglected. The physical model used here is a modified model of the plain nanorings used by Maslov and Citrin.[27]

The optical absorption can be traced back to the solution of an eigenvalue problem of an electron-hole pair subjected to Coulomb interaction, and many analytical and numerical methods have been used to solve it.[30,31] In this paper, we use the equation-of-motion approach which has been widely used to investigate the optical absorption of nanostructures,[32] such as quantum wells,[33,34] quantum wires,[35] quantum rings,[27,29] and superlattices,[36–38] as well as bulk semiconductors.[39,40] The optical properties of the QDSLNR are characterized by the linear optical susceptibility $\chi(t;t')$ which relates the optical field $E(t)$ with the macroscopic polarization $P(t)$ induced by $E(t)$

$$P(t)=\int_{-\infty}^{+\infty}dt'\,\epsilon_0\chi(t;t')E(t'), \tag{1}$$

where ϵ_0 is the permittivity of vacuum. The optical field is assumed to be incident normally on the QDSLNRs, thus, it is polarized in the QDSLNR's plane, and $E(t)=E_0\exp(-t^2/\tau^2)$ is the slowly varying envelope of the incident optical pulse. In a time-independent system, the optical susceptibility only depends on the time distance, i.e., $\chi(t,t')=\chi(t-t')$. By performing a Fourier transform to Eq.(1), we obtain the optical susceptibility,

$$\chi(\omega)=\frac{P(\omega)}{\epsilon_0E(\omega)}. \tag{2}$$

The optical susceptibility can be calculated from the exciton envelope function. In the QDSLNRs the exciton envelope function can be approximated by

$$\Psi(\mathbf{r}_e,\mathbf{r}_h)=\psi(\phi_e,\phi_h)g_e(\mathbf{r}_e)g_h(\mathbf{r}_h), \tag{3}$$

where $g_e(\mathbf{r}_e)$ and $g_h(\mathbf{r}_h)$ are the lowest subband wave functions for the electrons and the holes, respectively, ϕ_e and ϕ_h are the polar angles of the electrons and the holes, respectively. The envelope $\psi(\phi_e,\phi_h)$ obeys the inhomogeneous Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi-i\hbar\gamma\psi-edE(t)\delta(\phi_e-\phi_h), \tag{4}$$

with the Hamiltonian

$$\begin{aligned}\hat{H}=&\frac{\hbar^2}{2m_eR^2}\left(-i\frac{\partial}{\partial\phi_e}+\frac{\Phi}{\Phi_0}\right)^2+U_e(\phi_e)+eRF\cos\phi_e\\&+\frac{\hbar^2}{2m_hR^2}\left(-i\frac{\partial}{\partial\phi_h}-\frac{\Phi}{\Phi_0}\right)^2+U_h(\phi_h)-eRF\cos\phi_h\\&+\hat{V}_{\mathrm{Coul}}(\phi_e-\phi_h),\end{aligned} \tag{5}$$

where e is the elemental charge ($-e$ for electrons), m_e and m_h are the effective mass of the electron and hole, respectively, $U_e(\phi_e)$ and $U_h(\phi_h)$ represent the discontinuous superlattice potential along the ring for the electrons and the holes, respectively, R is the radius of the QDSLNR, F is the lateral electric field, Φ is the threading magnetic flux, Φ_0 is the elemental quantum flux h/e, and $\hat{V}_{\mathrm{Coul}}(\phi_e-\phi_h)$ is the Coulomb interactions between the electrons and the holes. The last term on the right-hand side of Eq.(4) describes the *e-h* excitation by an optical field $E(t)$, d is the interband dipole moment, γ is the phenomenological dephasing rate resulting from various scattering in the system.

The effective Coulomb potential in Eq.(4) is

$$\hat{V}_{\mathrm{Coul}}(\phi_e-\phi_h)=-\frac{e^2}{4\pi\epsilon_0\epsilon}\int\frac{|g_e(\mathbf{r}_e)|^2|g_h(\mathbf{r}_h)|^2}{|\mathbf{r}_e-\mathbf{r}_h|}d\mathbf{r}_ed\mathbf{r}_h, \tag{6}$$

where ϵ is the relative permittivity of the semiconductor material used to fabricate the QDSLNRs, and the difference of ϵ in the dots and in the barrier is neglected. The specific form of $V_{\mathrm{Coul}}(\phi)$ depends on the shape of QDSLNR's cross section. However, for narrow QDSLNRs one can use the cusp-type potential, which agrees very well with the actual potential in quantum wires[41]

$$\hat{V}_{\mathrm{Coul}}(\phi_e-\phi_h)=-\frac{e^2}{4\pi\epsilon_0\epsilon}\left(2R\sin\left|\frac{\phi_e-\phi_h}{2}\right|+a_0\right)^{-1}, \tag{7}$$

where parameter a_0 is proportional to the lateral size of the QDSLNR's cross section. The approximation of true Coulomb potential by a cusp-type one becomes better as the QDSLNR width decreases;a this corresponds to smaller values of the parameter a_0 in Eq. (7). Once $\psi(\phi_e,\phi_h)$ is calculated by solving Eq.(4), the optical susceptibility in the frequency domain is found after Fourier transforming of the complex optical polarization, which is determined by $\psi(\phi_e,\phi_h)$, via

$$P(t) = d^* \int_0^{2\pi} \psi(\phi,\phi) d\phi. \quad (8)$$

III. NUMERICAL RESULTS AND ANALYSES OF OPTICAL ABSORPTION IN QDSLNRS

We assume the calculated QDSLNR consists of 32 GaAs quantum dots separated by 32 AlGaAs barrier layers with the same thickness as the dots. The radius of the ring R $=50$ nm. Thus the length of the quantum dots and the spacing layers is approximately 4.9 nm, respectively. The depth of the potential of quantum dots is 237 meV for the electrons and 147 meV for the holes, respectively. We use the following parameters: $a_0=5$ nm, $m_e=0.067m_0$, $m_h=0.4m_0$ (with m_0 the free-electron mass) for GaAs and AlGaAs, $\epsilon=13.1\epsilon_0$, and $\hbar\gamma=0.2$ meV. The width of the QDSLNR is estimated to be around 20–25 nm for $a_0=5$ nm and the bulk Bohr radius is about 11 nm. To solve Eq. (4), we use the central finite difference for space with spatial step $\Delta\phi_e=\Delta\phi_h=2\pi/320$ and explicit method for time with a typical time step 0.2 fs. The total integration time $t_{\max}$ is chosen in such a way that the optical polarization becomes practically zero at the end of integration, $t_{\max}=2\pi/\gamma$.

Although the free electron-hole pair interband absorption is never realized in the practical situation, it is still of theoretical interest as it shows the optical density of states and allows one to clearly identify the role of excitonic effects. In all of the figures in this paper, the energy is measured with respect to the free-particle interband transitions and the magnetic flux is measured in units of the elemental quantum flux Φ_0. Figure 1 shows the effect of a lateral electric field on the optical density of states of the QDSLNR under different field strength. In Fig.1(a), we show the evolution of the optical density of states with the field strength varying from 0 to 10 kV/cm at a step of 1.25 kV/cm. In the absence of external electric field, the optical density of states is in a band form as expected due to the periodic potential. In Fig.1(a), only the first combined miniband is shown in the energy range. The combined miniband is about 11 meV in width and is modulated by a series of peaks corresponding to the quantized angular momentum due to the annular topology of the QDSLNR. The peaks in the middle of the miniband have Lorentzian line shape, but the shapes of the peaks near the two edges of the combined miniband distort partially due to the Hopf singularity of the 1D density of states and partially due to the overlap of the closer peaks near the edges. In the presence of a lateral electric field, the optical density of states change dramatically. The peaks due to the quantized angular momentum are smoothed out by the electric field. Because the applied external electric field distorts the eigenfunctions of the angular momentum, thus breaks the selection rule of transition of the electrons and the holes with the same angular momentum number, and gives up to transitions of the electrons and the holes with different angular momentum number. However, there arise new peaks from the smoothed combined miniband. We attribute these new peaks to WSLs. To show this, we plot three dotted arrows in Fig.1(a). The cross points of these three arrows with the base lines of each trace correspond to the position of the peak in the respective spectrum. The left and the right arrows labeled by −1 and +1 are guides to the eye to indicate the linear shift of the peaks with increasing field. The distance of the two arrows from the arrow of the main peaks is eFd in energy, just corresponding to the expected value of WSL. Thus, the −1 and the +1 are the WSL index. There are also many complex structures in the energy range between the −1 and the +1 WSL. This is in contradistinction to the WSL of the usual QWSLs, where optical absorption peaks of the WSL are well separated cleanly. These complex structures can be understood as follows. Since F is the maximum of the electric field along the ring due to the cosine factor, there are many field values between 0 and F. In contrast to a general QWSL under a homogenous growth-direction electric field, where the nearest-neighbor transition, the next-nearest-neighbor transition, etc., have the same value, respectively, at different sites along the whole growth axis, the nearest-neighbor transition, the next-nearest-neighbor transition, etc., have a different value, repectively, at different sites along the ring. Thus there may exist multiple WSLs resulting from the less effective fields along the ring at different sites. In the traces for field $F=1.25$, 2.5 kV/cm, the main peak is absent. This can be explained by the coexistence of the FK and

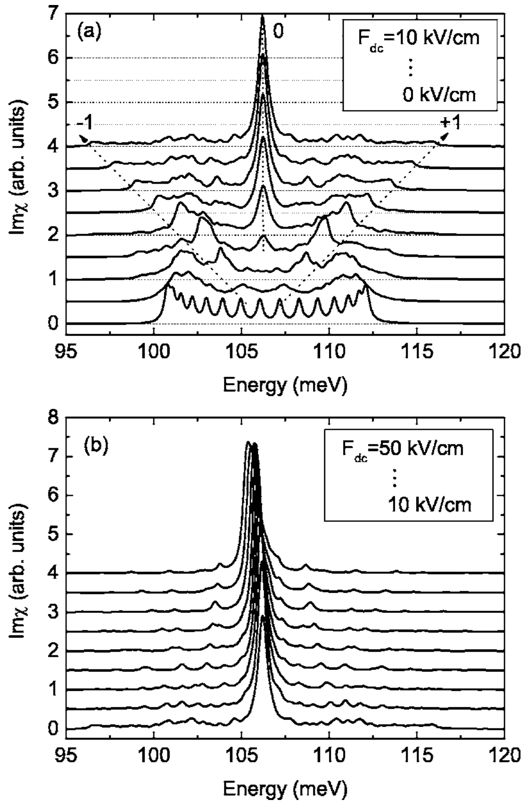

FIG. 1. Optical density of state of QDSLNR under different electric fields F. (a) Optical density of state of QDSLNR under different electric fields with strength from 0 to 10 kV/cm at a step of 1.25 kV/cm. The three dotted arrows labeled by 0, −1, and +1, are guides to the eye to indicate the linear shift of the WSLs with the applied field. The numbers 0, −1, and +1 are the WSL index. (b) Optical density of state of QDSLNR under different electric fields with strength from 10 to 50 kV/cm at a step of 5 kV/cm.

Wannier-Stark effect (WSE) in an intermediate field as in the usual QWSLs investigated by Linder *et al.*[42,43] With the field increasing, the main peak corresponding to the same well transition along the ring dominate, and the peaks of the other WSL index is suppressed. Figure 1(b) shows the influence of a strong field on the optical density of states. The field increases from 10 kV/cm to 50 kV/cm at a step of 5 kV/cm. In the presence of a strong field, there are multiple WSL-like around the main peak, which corresponds to the WSLs with different effective field.

In Fig. 2 we show the optical absorption of the QDSLNR including the Coulomb interaction between the electrons and the holes which results in formation of bound excitonic states. Figure 2(a) shows the effect of the applied lateral field on the main exciton absorption peak. It can be seen that the field results in the reduction in height, broadening, emerging of shoulders. The position of the main exciton peak redshift with the field increasing, but at field values at about $F = 8.5$ kV/cm the position is blueshift, and becomes redshift again when the field increases further. These are characteristics of the well-known Franz-Keldysh effect. Unlike quantum wires, in which application of F mostly results in the broadening of the peak due to field-induced tunneling,[35] in QDSLNR's the continuum is discrete and this allows one to observe a complicated structure in the exciton peak. The broadened peak is in fact composed of a large number of states, which become optically allowed in the presence of F. In addition, there occurs a peak shift to the main exciton peak linearly with the field increasing. Figure 2(b) shows the optical absorption under a strong field. The field increases from 10 kV/cm to 50 kV/cm at a step of 5 kV/cm. With the field increasing, the main exciton peak deceases and the new increasing peak dominates and redshifts. To identify the origin of the new peak, in Fig. 2(c) we replot optical absorption in a larger energy range and enlarge the bottom to show where the peak came from. In the absence of the electric field, the optical absorption is dominated by the strong exciton peak at about 12 meV below the first combined miniband. The 2*s* exciton peak about 2 meV below the first miniband also can be identified. As in the case without Coulomb interaction, the quasi-1D *e-h* continuum is broken into a set of discrete states by the requirement of periodicity imposed by the circular topology. The modulated combined miniband is redshifted 1.8 meV with respect to the combined miniband of optical density of states as a whole by the attactive Coulomb interaction. And the nearly symmetric combined miniband without Coulomb interaction become asymmetric combined miniband with the lower edge dominating the upper edge. The small absorption beyond the first exciton peak is related to the reduced quasi-1D Sommerfeld enhancement factor.[41] When a dc lateral field F is applied, the exciton peak reduces in height, broadens, and moves slightly to the red. Simultaneously, the peaks arising from the middle of the combined miniband increases and redshift rapidly. As in the analyses of Fig.1(a), when the field exceeds 2.5 kV/cm, we can identify this increasing peak with field in the $n=0(X)$ excitonic WSL with n being the WSL index and X being the exciton. The trace of field $F=1.25$ and 2.5 kV/cm can be considered as the intermediate field, where the WSE and FK effect can coexist. We can also identify the $-1(X)$ and $+1(X)$

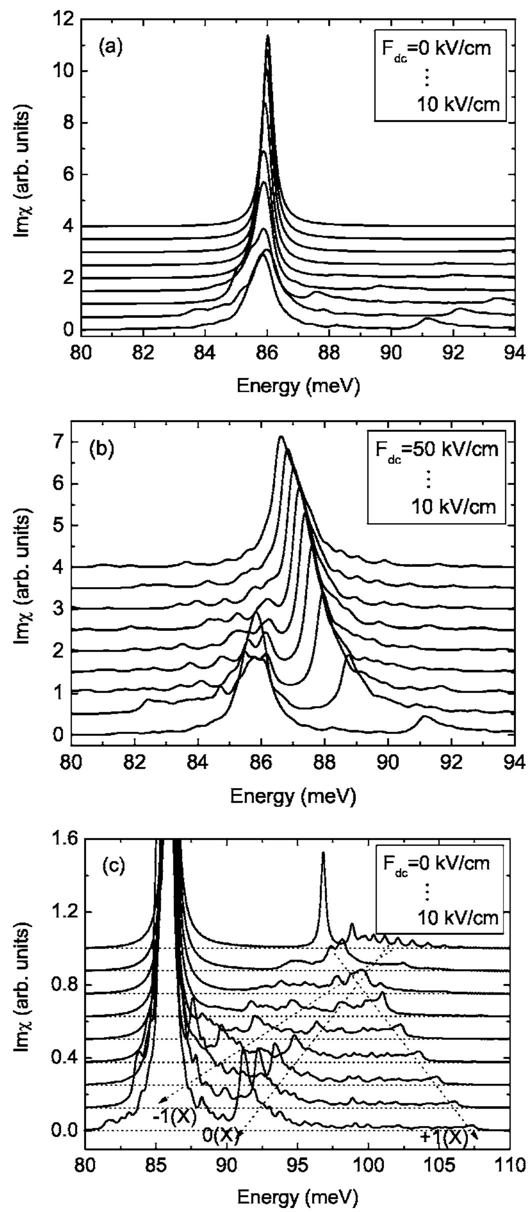

FIG. 2. Optical absorption of QDSLNR with excitonic effects under different electric fields F. (a) The reduction in height, broadening or splitting, and redshift of the main exciton peak induced by the applied lateral dc fields. The field's strength increases from 0 to 10 kV/cm at a step of 1.25 kV/cm. (b) The increasing and redshift of the main $0(X)$ WSL and the diminishing of the 1*s* main excitonic peak induced by the applied lateral dc field. The field's strength increases from 10 to 50 kV/cm at a step of 5 kV/cm. (c) The enlarged and wider energy range plot of (a) to show the details of the bottom and identification of the $0(X)$, $-1(X)$, and $+1(X)$ excitonic WSLs. The number of 0, −1, and +1 are WSL index and X indicating exciton.

excitonic WSLs transition corresponding to the field F. These WSL excitronic peaks are indicated by the three dotted arrows labeled as $0(X)$, $-1(X)$ and $+1(X)$, respectively. As in Fig. 1(a), the main WS excitronic peak $0(X)$ increasing with field while $-1(X)$ and $+1(X)$ excitronic WSL peaks suppressed with field increasing. However, there are prominent differences with and without Coulomb interaction. Without

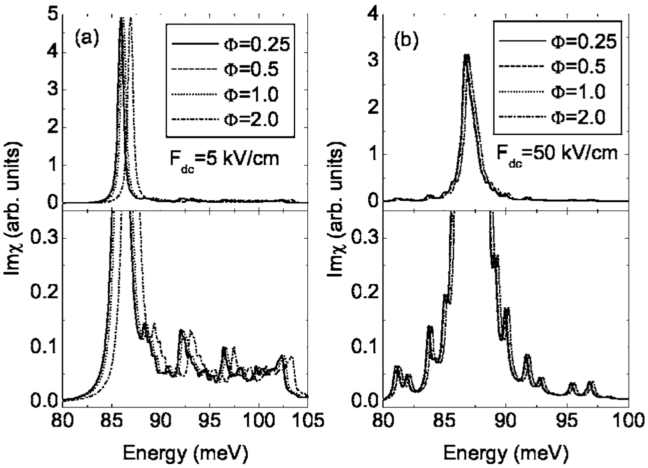

FIG. 3. Optical absorption of QDSLNR with different threading magnetic flux Φ. (a) The effect of magnetic flux on the main exciton peak in the presence of a moderate lateral dc field with field strength 5 kV/cm. (b) The effect of magnetic flux on the main exciton peak in the presence of a strong lateral dc field with field strength 50 kV/cm.

Coulomb interaction, the main WSL peak hardly any redshift in low field and only a slight redshift at very high field. In addition, the peaks of −1 and +1 WSL are nearly completely symmetric with respect to the peak of main WSL. With Coulomb interaction, as we have noted, the peaks of the −1 and the main excitronic WSLs redshift rapidly with increasing field. The peak of the +1 excitronic WSL blueshifts with field increasing. The peaks of the $-1(X)$ and the $+1(X)$ WSLs are asymmetric with respect to the peak of main WSL. The $-1(X)$ WSL is less apart than the $+1(X)$ from the main WSL peak. This excitron asymmetric has been obtained for usual QWSLs under homogenous electric field by Dignam and Sipe,[44] and by Linder.[43] Between peaks of the $-1(X)$ and the $+1(X)$ WSLs, there are also complicated structures, which can be explained as for Fig. 1(a).

Figure 3 shows the effect of the threading magnetic flux on the excitonic and the free electron-hole pair absorption. Figures 3(a) and 3(b) show the effect of piercing magnetic flux on the main exciton peak under a lateral field with different field strength. The upper panels show the overview and the lower panels are the enlarged bottom part to show the details. From Figs. 3(a) and 3(b), we see that the magnetic flux has little influence on the line shape, the shape of the strong exciton peak does not change noticeably with magnetic flux Φ, neither does the height of the exciton peak. There is just only a slight diamagnetic shift, the position of the exciton peak shifts slightly to the higher energy side. The two lower panels of (a) and (b) also show that under 5 kV/cm the optical absorption is the main $1s$ excitonic absorption where the combined miniband can be discernible, and under 50 kV/cm the main excitonic absorption peak is the 0 WSL from the miniband, where the discrete absorption due to the complicated structures is around it. This is in contrast to the plain nanoring, where a lateral electric field enhances the sensitivity of the optical absorption to the threading magnetic flux.[27] This result is not unexpected, since the QDs confine the separation of the electrons and the holes.

IV. CONCLUSIONS

In conclusion, we have calculated numerically the optical response in QDSLNRs in the presence of a lateral dc electric field and with magnetic flux threading the rings. This structure and configuration also provides a-unique opportunity to study the optical response of a superlattice under an inhomogeneous electric field, which is not easy realized for general QWSLs but naturally realized for QDSLNRs under a homogeneous lateral electric field considered in this paper. We have shown that a lateral dc electric field applied to a semiconductor QDSLNR gives rise to a substantial change of the optical absorption spectra. Under a low field, the excitonic optical absorption is dominated by $1s$ exciton. And with the electric field increasing, the optical absorption undergoes a transition from $1s$ excitonic absorption to 0 WSL absorption coming from the middle of combined miniband. The −1 and the +1 WSLs corresponding to the maximum effective field can also be identified. However, due to the inhomogeneity of the electric field, the peaks of the −1 and the +1 WSLs are diminished and between them there exist complicated structures. This is in contrast to the general QWSLs under a homogeneous electric field along the grown axis, where the different WSL peaks separate clearly and cleanly. The complicated structures can be understood by considering the inhomogeneity of the electric field, which results in the nearest-neighbor transition, the next-nearest-neighbor transition, etc., have a different value, respectively, at different sites along the ring. We suggest the existence of multiple WSLs. We have also shown that the line shape of the optical absorption is not sensitive to the threading magnetic flux. The threading magnetic flux only gives rise to a slight bit of diamagnetic shift. Thus the enhancement of the sensitivity to the flux allows the unexpected observation of the excitonic *AB* effect in QDSLNRs. This result can be anticipated as the quantum dots confine the separations of the electrons and the holes.

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