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Enhancement of Kerr nonlinearity via spontaneously generated coherence in a four-level N-type atomic system

Wen-juan Jiang [a], Xiang'an Yan [a,b], Jian-ping Song [a,*], Huai-bin Zheng [a], Chuanxing Wu [a], Bao-yin Yin [a], Yanpeng Zhang [a]

[a] Key Laboratory for Physical Electronics and Devices of Education Ministry, Xi'an Jiaotong University, Xi'an 710049, China
[b] School of Science, Xi'an Polytechnic University, Xi'an 710048, China

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ABSTRACT

A theoretical investigation is carried out into the effect of spontaneously generated coherence (SGC) on Kerr nonlinearity of a four-level N-type system. It's found that the Kerr nonlinearity can be obviously enhanced with SGC present. We attribute the enhancement of Kerr nonlinearity mainly to the generation of extra coherences induced by the superposition of the two SGC channels, and when the superposition is controlled by the interference between two SGC channels properly, the maximal Kerr nonlinearity does not only enter the electromagnetically induced transparency window as the spontaneous generated coherences intensify, but also gets enhanced about 10 times with very large SGC coefficients than that with no SGC effect.

1. Introduction

Optical properties of an atomic medium may be controlled and modified by the application of a strong electromagnetic field. Phenomena associated with electromagnetically induced transparency (EIT) have attracted considerable attention in recent years and offer a variety of interesting and potentially important applications. The essence of EIT is that an atomic coherence is induced in a multilevel system by a strong controlling laser field, which alters the response of the system to a probe laser field, and under the right circumstances, the absorption of a weak-probe beam at the resonance frequency can be substantially reduced.

It is now well known that another kind of coherence, spontaneously generated coherence (SGC), which refers to the interference of spontaneous emission channels, can also change the steady-state response of the medium. It can be created by the interference spontaneously emission of either a single excited level to two closely lying atomic levels (Λ-type atom) [1] or two closely lying atomic levels to a common atomic level (V-type atom) [2,3]. In a cascade three-level (Ξ-type) system, it can also be created in a nearly spaced atomic levels case [4–6]. The existence of such coherence requires that two closely lying levels be near-degenerate and that the two atomic dipole matrix elements be non-orthogonal when the atom is placed in free space.

The effects of SGC on absorption and dispersion [7], transparency [8], resonance fluorescence [9] and population inversion [10], spectral narrowing and elimination [11–15] have been extensively investigated recently. Cheng et al. [16] found, in the case of small dephasing in a Λ system, that instead of EIT at resonance, electromagnetically induced absorption (EIA) can occur due to the effect of SGC, besides, optical bistability can be realized with the existence of strong SGC, and the appearance or disappearance of bistability can be controlled by adjusting the relative phase between the two coherent fields [17]. Furthermore, it also makes possible to switch between bistability and multistability via SGC in a three-level ladder-type system [18]. Bai et al. found that owing to the effect of SGC, LWI can be realized without the incoherent pumping or the decay between two upper levels in V system [19]. Xu et al. [20] found it also because of SGC effect, that the probe gain can be greatly enhanced and can be modulated by changing the relative phase between the applied fields in a N-type system. Xia et al. [21] carried out the first experimental investigation of constructive and destructive interference effects in spontaneous emission interference. However, repetition of this experiment failed [22] and some controversies arose [23,24]. Recently, there is experimental evidence that SGC plays a role in charged GaAs quantum dots [25], which have been proposed as elements in quantum-information networks. In atomic systems, although it is difficult to realize SGC experimentally, there are several proposals to simulate this effect. Ficek and Swain [26] have put forward a project to obtain SGC from a V-type three-level system with perpendicular dipole moments by a dc field.

As is known, Kerr-type nonlinear coefficients play a crucial role in nonlinear optics. The large enhancement of nonlinear susceptibilities with small absorption causes the nonlinear optics to be

* Corresponding author. Tel.: +8613689288177.
E-mail addresses: jpsong@mail.xjtu.edu.cn, jwjxuat@yahoo.cn (J.-p. Song).

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studied at low light levels [27–30], and it have been found various useful applications such as optical Kerr shutters, generation of optical solitons, quantum logic operation [31] and so on. Large cross-Kerr nonlinearity with vanishing linear susceptibility can be observed in coherently prepared four-level N-type rubidium atoms with a cross-phase modulation (XPM) scheme in experimental [32]. Nevertheless, the effect of SGC on the self-Kerr nonlinearity in multilevel systems has never been reported to our best knowledge. Therefore, in this case, we intend to investigate the enhancement of self-Kerr nonlinearity via SGC effect in a four-level N-type system. Our research was based on a system in ^{87}Rb atoms [33], the results show that the Kerr nonlinearity can be obviously enhanced with SGC present. We attribute the enhancement of Kerr nonlinearity mainly to the generation of extra coherences induced by the superposition of the two SGC channels, and when the superposition is controlled by the interference between two SGC channels properly, the maximal Kerr nonlinearity does not only enter the electromagnetically induced transparency window as the spontaneous generated coherences intensify, but also gets enhanced about 10 times with very large SGC coefficients than that with no SGC effect.

2. The system and density-matrix equation

The closed, four-level N-type atomic system is shown in Fig. 1. $|2\rangle$ and $|1\rangle$ are two lower levels and they are closely lying near-degenerate. A probe field Ω_b is applied to the transition $|2\rangle \leftrightarrow |3\rangle$, while the other two laser fields Ω_a and Ω_d are used to drive transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, respectively, and all the Rabi frequencies (Ω_b, Ω_a, Ω_d) are considered real in our problem. γ_{31} and γ_{32} denote the spontaneous decay rates from levels $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$, respectively, $\gamma_3 = \gamma_{31} + \gamma_{32}$; γ_{42} and γ_{41}) denote the spontaneous decay rates from level $|4\rangle$ to levels $|2\rangle$ and $|1\rangle$, respectively, $\gamma_4 = \gamma_{41} + \gamma_{42}$. ω_a, ω_b and ω_d are carrier frequencies of the corresponding fields, and $\Delta_a = \omega_{31} - \omega_a$, $\Delta_b = \omega_{32} - \omega_b$ and $\Delta_d = \omega_{42} - \omega_d$ are detunings of the corresponding fields.

Because the two lower levels $|2\rangle$ and $|1\rangle$ are closely spaced so that the two transitions to the excited state interact with the same vacuum mode, spontaneously generated coherence can be present. This N-type system can be regarded as concluding two Λ-type systems, one is $|3\rangle - |1\rangle - |2\rangle$, the other is $|4\rangle - |1\rangle - |2\rangle$, so there are two spontaneous emission pathways respectively, they are, $|3\rangle$ to $|1\rangle$ and $|2\rangle$, and, $|4\rangle$ to $|1\rangle$ and $|2\rangle$, therefore, two SGC terms must be taken into consideration. Under the rotating-wave approximation, the systematic density matrix in the interaction picture involving the SGC can be written as

$$
\begin{aligned}
\dot{\rho}_{11} &= -i(\rho_{13} - \rho_{31})\Omega_a + 2(\gamma_{41}\rho_{44} + \gamma_{31}\rho_{33}) \\
\dot{\rho}_{12} &= i((\Delta_a - \Delta_b)\rho_{12} + \rho_{32}\Omega_a - \rho_{13}\Omega_b - \rho_{14}\Omega_d) \\
&\quad + 2(\eta_1\sqrt{\gamma_{31}\gamma_{32}}\rho_{33} + \eta_2\sqrt{\gamma_{41}\gamma_{42}}\rho_{44}) \\
\dot{\rho}_{13} &= i(\Delta_a\rho_{13} + (-\rho_{11} + \rho_{33})\Omega_a - \rho_{12}\Omega_b) - (\gamma_{31} + \gamma_{32})\rho_{13} \\
\dot{\rho}_{14} &= i((\Delta_a - \Delta_b + \Delta_d)\rho_{14} + \rho_{34}\Omega_a - \rho_{12}\Omega_d) - (\gamma_{41} + \gamma_{42})\rho_{14} \\
\dot{\rho}_{22} &= -i((\rho_{23} - \rho_{32})\Omega_b) + (\rho_{24} - \rho_{42})\Omega_d + 2(\gamma_{42}\rho_{44} + \gamma_{32}\rho_{33}) \\
\dot{\rho}_{23} &= i(\Delta_b\rho_{23} - \rho_{21}\Omega_a + (-\rho_{22} + \rho_{33})\Omega_b + \rho_{43}\Omega_d) - (\gamma_{31} + \gamma_{32})\rho_{23} \\
\dot{\rho}_{24} &= i(\Delta_d\rho_{24} + \rho_{34}\Omega_b) + (-\rho_{22} + \rho_{44})\Omega_d - (\gamma_{41} + \gamma_{42})\rho_{24} \\
\dot{\rho}_{33} &= i((\rho_{13} - \rho_{31})\Omega_a + (\rho_{23} - \rho_{32})\Omega_b) - 2(\gamma_{31} + \gamma_{32})\rho_{33} \\
\dot{\rho}_{34} &= -i((\Delta_b - \Delta_d)\rho_{34} - \rho_{14}\Omega_a - \rho_{24}\Omega_b + \rho_{32}\Omega_d) \\
&\quad - (\gamma_{31} + \gamma_{32} + \gamma_{31} + \gamma_{32})\rho_{34} \\
\dot{\rho}_{44} &= i(\rho_{24} - \rho_{42})\Omega_d - 2(\gamma_{41} + \gamma_{42})\rho_{44}
\end{aligned}
\tag{1}
$$

The above equations are constrained by $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$, and $\rho^*_{ji} = \rho_{ij}$. The effect of SGC is very sensitive to the orientations of the atomic dipole moments μ_{31} and μ_{32} (μ_{41} and μ_{42}). The

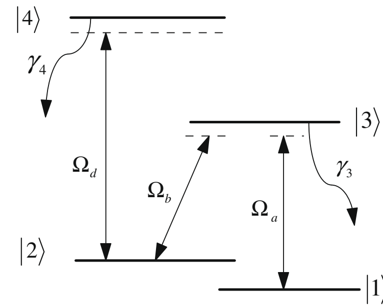

Fig. 1. Four-level N-type atomic system driven by three coherent fields.

parameter η_1 (η_2) denotes the alignment of the two dipole moments and is defined as $\eta_1 = \vec{\mu}_{31} \cdot \vec{\mu}_{32}/|\vec{\mu}_{31} \cdot \vec{\mu}_{32}| = \cos\theta_1(\eta_2 = \vec{\mu}_{41} \cdot \vec{\mu}_{42}/|\vec{\mu}_{41} \cdot \vec{\mu}_{42}| = \cos\theta_2)$, and θ_1 (θ_2) is the angle between the two induced dipole moments, it is always nonzero, though it could be small. The terms $\eta_1\sqrt{\gamma_{31}\gamma_{32}}(\eta_2\sqrt{\gamma_{41}\gamma_{42}})$ in Eq. (1) represent the quantum interference resulting from the cross coupling between spontaneous emission paths $|3\rangle - |1\rangle$ and $|3\rangle - |2\rangle(|4\rangle - |1\rangle$ and $|4\rangle - |2\rangle)$. If the two dipole moments $\vec{\mu}_{31}$ and $\vec{\mu}_{32}(\vec{\mu}_{41}$ and $\vec{\mu}_{42})$ are parallel, η_1 (η_2) = 1, and SGC is maximum, if the two dipole moments $\vec{\mu}_{31}$ and $\vec{\mu}_{32} vec\mu_{41}$ and $\vec{\mu}_{42}$) are orthogonal, η_1 (η_2) = 0, and Eq. (1) reduce to the equation for a four-level N-type atomic system without SGC. With the restriction that each field acts only on one transition, the Rabi frequencies are connected to the angle $\theta_1(\theta_2)$ and represented by $\Omega_{a(b)} = \Omega^0_{a(b)}\sin\theta_1 = \Omega^0_{a(b)}\sqrt{1-\eta_1^2}(\Omega_d = \Omega^0_d\sin\theta_2 = \Omega^0_d\sqrt{1-\eta_2^2})$. It should be noted that only for small energy spacing between the two lower levels are the interference terms in Eq. (1) significant; otherwise the oscillatory terms will average out to zero and thereby the SGC effect vanishes.

3. Analytical solutions obtained by iterative method

It is known that the response of the atomic medium to the probe field is governed by its polarization $P = \varepsilon_0(E_a\chi + E^*_a\chi^*)/2$,with χ being the susceptibility of the atomic medium. By performing a quantum average of the dipole moment over an ensemble of N atoms, we find $P = N(\mu_{32}\rho_{23} + \mu_{23}\rho_{32})$. In order to derive the linear and nonlinear susceptibilities, we need to obtain the steady-state solution of the density-matrix equations. In the present approach, an iterative method is used and the density-matrix elements are expressed as $\rho_{mn} = \rho^{(0)}_{mn} + \rho^{(1)}_{mn} + \rho^{(2)}_{mn} + \rho^{(3)}_{mn} + \cdots$ Assuming that the probe field is much weaker than the other two laser fields, the zeroth-order solution will be $\rho^{(0)}_{22} = 1$ and other elements are equal to zero. For simplicity, we set the parameters $\gamma_{31} = \gamma_{32} = \gamma_{41} = \gamma_{42} = \gamma$, $\Delta_a = \Delta_d = 0$ during calculation [20,34]. Under the weak-probe approximation, we get the matrix element ρ_{23} up to third order:

$$
\begin{aligned}
\rho^{(1)}_{23} &= -(8\gamma^2\Delta_b\Omega_b - 6i\gamma\Delta_b^2\Omega_b - \Delta_b^3\Omega_b + \Delta_b\Omega_a^2\Omega_b + 4i\gamma\Omega_b\Omega_d^2 + \Delta_b\Omega_b\Omega_d^2)/A \\
\rho^{(1)}_{12} &= -(8\gamma^2\Omega_a\Omega_b + 6i\gamma\Delta_b\Omega_a\Omega_b - \Delta_b^2\Omega_a\Omega_b + \Omega_a^3\Omega_b - \Omega_a\Omega_b\Omega_d^2)/A + 32i\gamma^3\Delta_b \\
\rho^{(1)}_{34} &= -(-2\gamma\Delta_b\Omega_b\Omega_d - i\Delta_b^2\Omega_b\Omega_d - i\Omega_a^2\Omega_b\Omega_d + i\Omega_b\Omega_d^3)/(-iA + 32\gamma^3\Delta_b)
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\rho^{(2)}_{12} = {} & ((2\gamma + i\Delta_b)\Omega_b(8\gamma^2 + 6i\gamma\Delta_b - \Delta_b^2 + \Omega_a^2 + \Omega_d^2)(4\gamma^2\eta_1(\rho^{(1)}_{23} \\
& - \rho^{(1)}_{32})\Omega_a^2 + i\gamma(\eta_1 + \eta_2)(\rho^{(1)}_{34} + \rho^{(1)}_{43})\Omega_a^2\Omega_d \\
& + (2\gamma^2(\eta_1 - \eta_2)(\rho^{(1)}_{23} - \rho^{(1)}_{32}) - i\gamma(\eta_1 + \eta_2)(\rho^{(1)}_{12} + \rho^{(1)}_{21})\Omega_a \\
& + 2(\eta_1 - \eta_2)(\rho^{(1)}_{23} - \rho^{(1)}_{32})\Omega_a^2)\Omega_d^2))/(4BC)
\end{aligned}
\tag{3}
$$

$$\rho_{22}^{(2)} = (\Omega_b(4\gamma^3(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a + \gamma(\rho_{34}^{(1)} + \rho_{43}^{(1)})(4\gamma^2 + 3\Omega_a^2)\Omega_d + \gamma(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a\Omega_d^2 + 2i(\rho_{32}^{(1)} - \rho_{23}^{(1)})(\gamma^2 + \Omega_a^2)(4\gamma^2 + \Omega_d^2)/8\gamma B \tag{4}$$

$$\rho_{23}^{(2)} = (\Omega_a\Omega_b(8\gamma^2 - 6i\gamma\Delta_b - \Delta_b^2 + \Omega_a^2 - \Omega_d^2)(4\gamma^2\eta_1(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a^2 + i\gamma(\eta_1 + \eta_2)(\rho_{34}^{(1)} + \rho_{43}^{(1)})\Omega_a^2\Omega_d + (2\gamma^2(\eta_1 - \eta_2)(\rho_{23}^{(1)} - \rho_{32}^{(1)}) - i\gamma(\eta_1 + \eta_2)(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a + 2(\eta_1 - \eta_2)(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a^2)\Omega_d^2))/(-2iBC) \tag{5}$$

$$\rho_{31}^{(2)} = \Omega_b(2\gamma^3(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a - i(\gamma^2(\rho_{34}^{(1)} + \rho_{43}^{(1)})\Omega_a\Omega_d + \Omega_a^2(\gamma^2 + \Omega_d^2)(\rho_{12}^{(1)} - 2\gamma^2\Omega_d^2\rho_{21}^{(1)})))/4\gamma B \tag{6}$$

$$\rho_{33}^{(2)} = \Omega_b(4i\gamma^2(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a^2 - \gamma(\rho_{34}^{(1)} + \rho_{43}^{(1)})\Omega_a^2\Omega_d + (\gamma(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a + 2i\rho_{23}^{(1)}(\gamma^2 + \Omega_a^2) - 2i\rho_{32}^{(1)}(\gamma^2 + \Omega_a^2))\Omega_d^2)/8\gamma B \tag{7}$$

$$\rho_{34}^{(2)} = ((2\gamma + i\Delta_b)\Omega_a\Omega_b\Omega_d(4\gamma^2\eta_1(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a^2 + i\gamma(\eta_1 + \eta_2) \times (\rho_{34}^{(1)} + \rho_{43}^{(1)})\Omega_a^2\Omega_d + (2\gamma^2(\eta_1 - \eta_2)(\rho_{23}^{(1)} - \rho_{32}^{(1)}) - i\gamma(\eta_1 + \eta_2)(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a + 2(\eta_1 - \eta_2)(\rho_{23}^{(1)} - \rho_{32}^{(1)})\Omega_a^2)\Omega_d^2))/(2BC) \tag{8}$$

$$\rho_{42}^{2} = \Omega_b(-2i\gamma^2\rho_{43}^{(1)}\Omega_a^2 + \gamma(i\gamma(\rho_{12}^{(1)} + \rho_{21}^{(1)})\Omega_a + 2\rho_{23}^{(1)}(\gamma^2 + \Omega_a^2) - 2\rho_{32}^{(1)}(\gamma^2 + \Omega_a^2))\Omega_d + i(\rho_{34}^{(1)} - \rho_{43}^{(1)})(\gamma^2 + \Omega_a^2)\Omega_d^2)/4\gamma B \tag{9}$$

$$\begin{aligned}\rho_{23}^{(3)} = {} & (\Omega_b(4i\gamma^2(\rho_{31}^{(2)} + \eta_1(\rho_{23}^{(2)} - \rho_{32}^{(2)}))\Omega_a^3(8\gamma^2 + \Omega_a^2) \\ & - \gamma\Omega_a^3(8\gamma^2 a_1 + 4i\gamma\rho_{42}^{(2)}\Omega_a + a_3\Omega_a^2)\Omega_d + \Omega_a(16i\gamma^4(2\rho_{31}^{(2)} \\ & + a_2(+8\gamma^3(a_1 + 2(\rho_{22}^{(2)} - \rho_{33}^{(2)}))\Omega_a + 2i\gamma^2(16\rho_{31}^{(2)} \\ & + (7\eta_1 - 9\eta_2)(\rho_{23}^{(2)} - \rho_{32}^{(2)}))\Omega_a^2 + \gamma a_1\Omega_a^3 + 2i(2\rho_{31}^{(2)} + a_2)\Omega_a^4)\Omega_d^2 \\ & + \Omega_a^3(\gamma a_3 - 4i\rho_{42}^{(2)}\Omega_a)\Omega_d^3 + (16\gamma(\gamma^2 + \Omega_a^2)(\rho_{22}^{(2)} - \rho_{33}^{(2)}) \\ & + \Omega_a(-2i\gamma^2(2\rho_{31}^{(2)} + a_2) - \gamma a_1\Omega_a + 2i(2\rho 31^{(2)} + a_2)\Omega_a^2))\Omega_d^4 \\ & + 4i\rho_{22}^{(2)}(\gamma^2 + \Omega_a^2)\Omega_d^5 + \Delta_b^2(4\gamma^2\Omega_a^2(6\gamma(\rho_{33}^{(2)} - \rho_{22}^{(2)}) - i(\rho_{31}^{(2)} \\ & + \eta_1(\rho_{23}^{(2)} - \rho_{32}^{(2)}))\Omega_a) + \gamma\Omega_a^2(-4i\gamma\rho_{42}^{(2)} + a_3\Omega_a)\Omega_d \\ & + (24\gamma(\gamma^2 + \Omega_a^2)(\rho_{33}^{(2)} - \rho_{22}^{(2)}) + \Omega_a(2i\gamma^2(2\rho_{31}^{(2)} + a_2) + \gamma a_1\Omega_a \\ & + 2i(2\rho_{31}^{(2)} + a_2)\Omega_a^2))\Omega_d^2 - 4i\rho_{42}^{(2)}(\gamma^2 + \Omega_a^2)\Omega_d^3) \\ & + 4i\Delta_b^3(\rho_{22}^{(2)} - \rho_{33}^{(2)})B + 2\Delta_b(2i\rho_{33}^{(2)}(8\gamma^2 + \Omega_a^2 + \Omega_d^2)D) \\ & + \gamma(12\gamma^2(\rho_{31}^{(2)} + \eta_1(\rho_{23}^{(2)} - \rho_{32}^{(2)}))\Omega_a^3 + \gamma\Omega_a^2(4\gamma\rho_{42}^{(2)} + 3ia_3\Omega_a)\Omega_d \\ & + 3\Omega_a(2\gamma^2(2\rho_{31}^{(2)} + a_2) - i\gamma a_1\Omega_a + 2(2\rho_{31}^{(2)} + a_2)\Omega_a^2)\Omega_d^2 \\ & + 4\rho_{22}^{(2)}(\gamma^2 + \Omega_a^2)\Omega_d^3) - 2i\rho_{22}^{(2)}(8\gamma^2 + \Omega_a^2 + \Omega_d^2)B)))/4B(-C \\ & + 16(\Delta_b(2\gamma^2 + \Omega_a^2 + \Omega_d^2) - \Delta_b^4))\end{aligned} \tag{10}$$

$$\begin{aligned}A = {} & (-16i\gamma^3\Delta_b - 20\gamma^2\Delta_b^2 + 8i\gamma\Delta_b^3 + \Delta_b^4 + 8\gamma^2\Omega_a^2 - 8i\gamma\Delta_b\Omega_a^2 \\ & - 2\Delta_b^2\Omega_a^2 + \Omega_a^4 + 8\gamma^2\Omega_d^2 - 8i\gamma\Delta_b\Omega_d^2 - 2\Delta_b^2\Omega_d^2 - 2\Omega_a^2\Omega_d^2 + \Omega_d^4) \\ B = {} & \gamma^2\Omega_d^2 + \Omega_a^2(\gamma^2 + \Omega_a^2) \\ C = {} & (-8\gamma\Delta_b^3 - i\Delta_b^4 + 8\gamma\Delta_b(2\gamma^2 + \Omega_a^2 + \Omega_d^2) + 2i\Delta_b^2(10\gamma^2 + \Omega_a^2 + \Omega_d^2) \\ & - i(\Omega_a^4 + 8\gamma^2\Omega_d^2 + \Omega_d^4 + \Omega_a^2(8\gamma^2 - 2\Omega_d^2)))) \\ D = {} & \gamma^2\Omega_a^2 + (\gamma^2 + \Omega_a^2)\Omega_d^2\end{aligned} \tag{11}$$

$$\begin{aligned}a_1 &= (\eta_1 + \eta_2)(\rho_{12}^{(2)} + \rho_{21}^{(2)}) \\ a_2 &= (\eta_1 - \eta_2)(\rho_{23}^{(2)} - \rho_{32}^{(2)}) \\ a_3 &= (\eta_1 + \eta_2)(\rho_{34}^{(2)} + \rho_{43}^{(2)})\end{aligned} \tag{12}$$

Therefore, the first- and third-order susceptibilities $\chi^{(1)}$ and $\chi^{(3)}$ can be attained according to following expressions:

$$\chi^{(1)} = \frac{-2N|\vec{\mu}_{23}|^2}{\varepsilon_0\hbar\Omega_p}\rho_{23}^{(1)} \tag{13}$$

$$\chi^{(3)} = \frac{-2N|\vec{\mu}_{23}|^4}{3\varepsilon_0\hbar\Omega_p^3}\rho_{23}^{(3)} \tag{14}$$

4. Effect of SGC on dynamics

In the expressions (13) and (14), where $\chi^{(1)}$ is the linear susceptibility and $\chi^{(3)}$ is the third-order susceptibility. The linear absorption proportional to the imaginary part of $\chi^{(1)}$, while the Kerr nonlinearity corresponds to the real part of the third-order susceptibility in optical media and the nonlinear absorption to the imaginary part. As is known, the angle θ_1 (θ_2) between the two induced dipole moments can be controlled by the applied fields, hence, $\eta_1(\eta_2)$ can be controlled. We show the linear absorption (dotted curve) and the refractive part of the third-order susceptibility (solid curve) as a function of the probe detuning in the following figures according to expressions (13) and (14) with different SGC coefficients.

From Fig. 3a, we can see that when $\eta_1 = \eta_2 = 0$, which demonstrates no interference between spontaneous emission channels, a couple of general linear absorption and Kerr nonlinearity curves occur [35] For small values of η_1 and η_2, because the change of the system behavior is small and the results are quite similar to those of $\eta_1 = \eta_2 = 0$, we do not show them here. With the spontaneous emission interference becoming strong, for example, in Fig. 2a, the refractive part of the third-order susceptibility is enhanced but the Kerr nonlinearity is very weak accompanied by strong absorption when $\eta_2 = 0$ and $\eta_1 = 0.99$. Although the refractive part of the third-order susceptibility evidently builds up when SGC effect becomes great in Fig. 2c, there still exists some absorption. Fortunately, in Fig. 3, when we control the SGC coefficients $\eta_1 = \eta_2 = 0.99$, it is easy to see that the maximum value of the Kerr nonlinearity at $\eta_1 = \eta_2 = 0.99$; is about 10 times that at $\eta_1 = \eta_2 = 0$, in addition, the maximum of Kerr nonlinearity enters the narrowing EIT window gradually [36] and therefore the corresponding linear absorption becomes negligible, since the linear absorption is very low, the nonlinear absorption given by the imaginary part of $\chi^{(3)}$ should be considered now. We can found that the variation of the nonlinear absorption (dashed–dotted curve) with SGC is very similar to that of linear absorption (dotted curve). Therefore, the enhanced Kerr nonlinearity gradually enters the "nonlinear EIT window" and the ratio of the real and imaginary parts of $\chi^{(3)}$ improves significantly when the SGC intensifies from $\eta_1 = \eta_2 = 0$ to $\eta_1 = \eta_2 = 0.99$. This means that in the case of optimal SGC, both enhanced Kerr nonlinearity and negligible linear and nonlinear absorption can be realized simultaneously.

5. Analysis of above results

By close inspection of the analytical expression of $\rho_{23}^{(3)}$, though Eq. (10) looks complex, we can clearly find that how the two SGC coefficients η_1 and η_2 affect the whole system. Density-matrix elements $\rho_{12}^{(2)}, \rho_{23}^{(2)}, \rho_{34}^{(2)}$ in Eq. (10) are the three extra coherence terms that introduced by SGC. The reason why there are three extra coherence terms is that N-type is a multilevel system, and then, there are two SGC channels, both of which are similar to that of

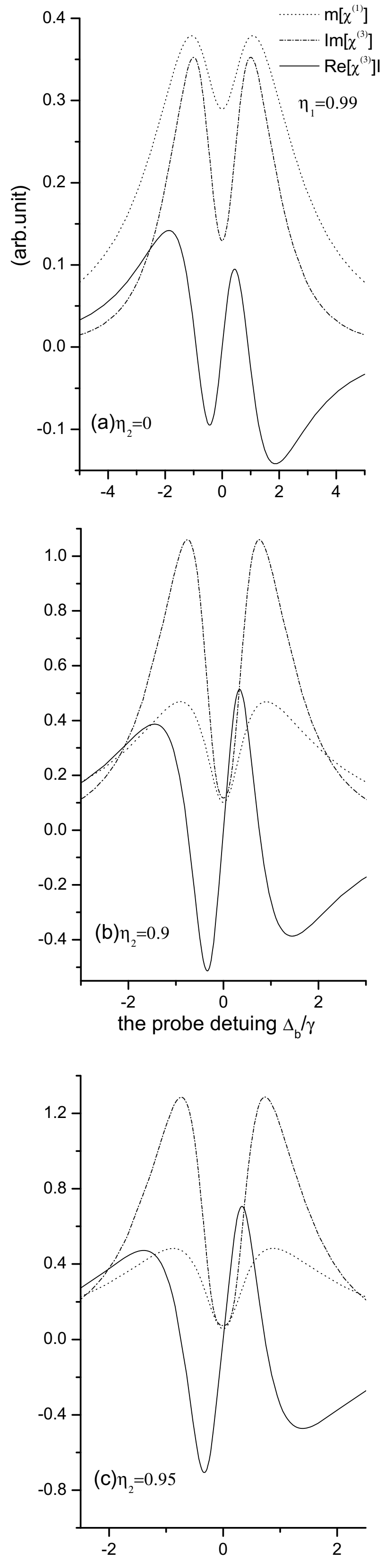

Fig. 2. The Kerr nonlinearity Re[χ(3)] (solid curve), linear absorption Im[χ(1)] (dotted curve), and nonlinear absorption Im[χ(3)] (dash-dotted curve) of the N-type system with different SGC. Parameters are 0 $\Omega_a^0 = 6.0\gamma,\ \Omega_d^0 = 1.0\gamma$.

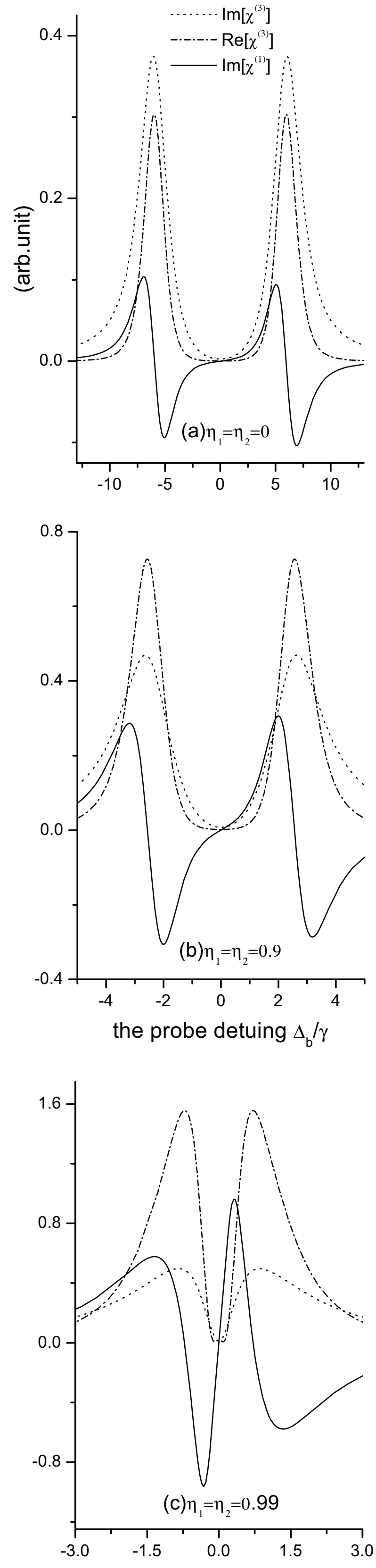

Fig. 3. The Kerr nonlinearity Re[χ(3)] (solid curve), linear absorption Im[χ(1)] (dotted curve), and nonlinear absorption Im[χ(3)] (dash-dotted curve) of the N-type system with different SGC. Parameters are $\Omega_a^0 = 6.0\gamma,\ \Omega_d^0 = 1.0\gamma$.

Λ-type three-level system. As the analytical expression for $\rho_{23}^{(3)}$ shows, the generations of all the extra terms are mainly because of the superposition of the two SGC channels. The extra term $\rho_{23}^{(2)}$ mainly multiplies $\eta_1 - \eta_2$, that is to say, $\rho_{23}^{(2)}$ is generated mainly

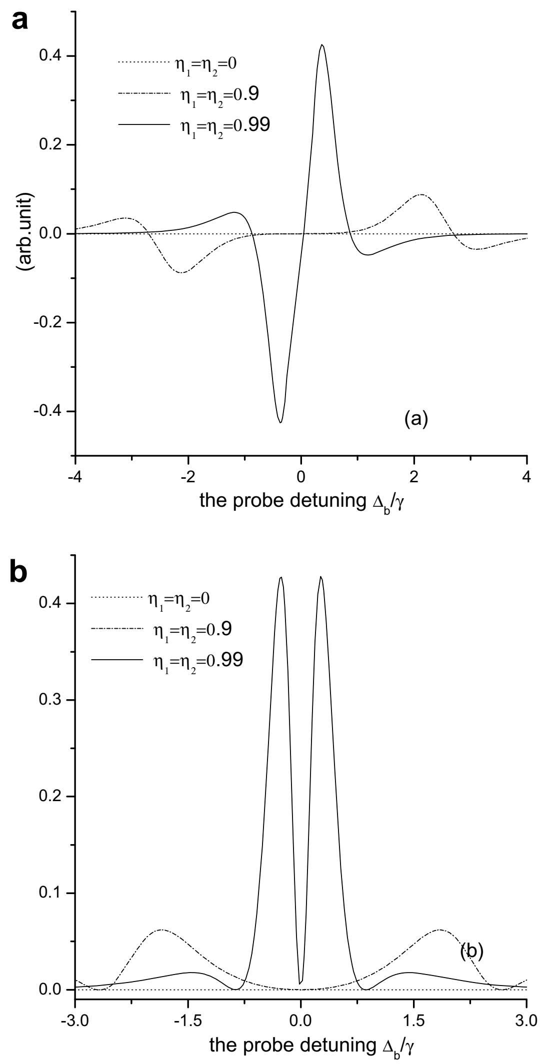

Fig. 4. (a) The real part of the extra coherence terms in $\rho_{23}^{(3)}$; (b) the imaginary part of the extra coherence terms in $\rho_{23}^{(3)}$. Parameters are the same as in Fig. 2.

because of the destruction superposition, besides, the other two extra terms $\rho_{12}^{(2)}$ and $\rho_{34}^{(2)}$ multiply $\eta_1+\eta_2$, which means, the generation of them are because of the construction superposition. Accordingly, by controlling the interference between SGC channels properly, we successfully attain the enhanced Kerr nonlinearity with negligible linear and nonlinear absorption under the condition of the strong SGC effects in multilevel systems.

Fig. 4 shows the real part (a) and the imaginary part (b) of all the extra coherence terms in $\rho_{23}^{(3)}$ as a function of the probe detuning. We surprisingly find that, the stronger the SGC is, the more remarkable the coherence becomes, besides, the curves of both the real part of the extra terms and the imaginary part (when $\eta_1(\eta_2)\neq 0$), are all very similar to those of Im $[\chi^{(3)}]$ and Re $[\chi^{(3)}]$ in Fig. 3, which means, just because of the extra coherence terms created by SGC that contribute to the enhancement of Kerr nonlinearity.

6. Conclusion

We have investigated the effect of SGC on the self-Kerr nonlinearity in a four-level N-type system. It was found that the interference between the two SGC channels can be controlled for enhancing the self-Kerr nonlinearity in this multilevel atomic system, and the maximum Kerr nonlinearity can be enhanced about 10 times at large SGC coefficient comparing with SGC effect nonexistence. The superposition of the two SGC channels induce three extra coherence terms (while in the general three-level systems, they are only one extra term [37]), therefore, give rise to the enhancement of Kerr nonlinearity enters the EIT windows as the SGC intensifies.

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