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Filamentation of a rotating electron beam in a magnetized plasma

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Abstract

Filamentation of a nonrelativistic rotating electron beam in a magnetized plasma is investigated by solving the kinetic equation and finding its dielectric permitivity. The period and the establishment time of the filamentation structure and threshold for instability development are obtained. It will be shown that only when the external magnetic field strength becomes smaller than a characteristic value, filamentation appears.

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1. Introduction

The processes of filamentation and their instabilities have been studied analytically and investigated in simulation experiments as well [1–11]. Furthermore, filamentation has been observed in the largest-scale cosmological plasmas, in the solar corona, as well as in various types of laboratory plasmas, such as the plasma focus and the magnetohydrodynamic (MHD) generator [12].

Filamentation in laser–plasma and beam–plasma systems as well being a fundamental issue has been the subject of great activity due to the numerous applications including X-ray lasers, laser-plasma produced particle accelerators, laser fusion, and ionospheric modification experiments in F and E regions [13–19]. The ordinary-mode filamentation instability in the aforementioned systems has been investigated in circumstances where a low-density electron beam is injected into a high-density preionized plasma under the special instability criterion [20–23]. In these systems instability produces beam filamentation, plasma heating, and ultimately, beam recombination into a single self-pinched neutralized filament without any current [1]. Furthermore, the essential feature of these systems is the filamentation and coalescence of fast electron beams and magnetic channel formation [24–26]. But, when a low-density electron beam is injected into a dense neutral gas under the special instability criterion, the beam is experimentally observed not to filament [27]. The absence of filamentation instability in

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circumstances where the beam–plasma system is not neutralized strongly suggests that the azimuthal self-magnetic field has a stabilizing influence on the filamentation instability [28]. Moreover, it was shown that the interchange instability corresponding to the lamination of the electron beam into separate current-carrying filaments is possible in a plasma–beam system in the absence of an external magnetic field and thermal motion of particles (see, for example, Refs. [29,30]).

It is well known that Buneman's instability represents the stimulated Čerenkov radiation of low-frequency fields in the current-driven plasmas [29,30]. It takes place when the velocity of the electron beam exceeds the wave phase velocity. It was shown that, in this case, neglecting thermal motion and collisions of particles, a purely transverse instability can be developed in the systems with sufficiently large lateral size [29,30]. Therefore, magnetic pressure in current-driven plasmas is higher than kinetic pressure and, as a consequence, the self-pressing (pinch effect) is possible. In fact, the same effect actually takes place when the filamentation instability is developed in a beam–plasma system and, as a result, the filamentation of a current-driven plasma arises. This mechanism being the cause of the filamentation can be understood by noting that it also takes place in collisional plasmas. Furthermore, in the presence of a sufficiently strong external magnetic field, the filamentation is not possible. Recently, the diffusive as well as ion-acoustic filamentation of nonrelativistic current-driven plasmas were investigated by the Lorentz transform of the dielectric tensor in the same method [31,32] as it is used in the present Letter. There, the period and the establishment time of the filamentation structure and the threshold for instability development were obtained in the diffusion and ion-acoustic frequency regions.

In most previous mentioned works the filamentation instability was studied when a straight beam streams in the plasma. On the other hand, filamentary instability of a rotating electron beam was investigated in Ref. [22]. In this Letter, filamentation of an annular rotating electron beam streaming in a collisional plasma was studied by fluid equations. Different from previous works, in the present Letter, filamentation instability of a rotating current-driven plasma, which is very important for the plasma confinement problem, is considered by the kinetic equation. It will be shown that under special circumstances the instability filamentation also takes place when a rotating electron beam flows in a plasma. This instability, like Buneman's instability, is connected to the stimulated Čerenkov radiation. Therefore, this instability which resists the layer current compression, should be taken into account. The aforementioned instability, developed in the rotating beam system allows us to determine the threshold for the period of cross structure, resulted from the instability development.

This Letter is presented in four sections. In Section 2, we formulate the problem. In Section 3, we study the rotating electron beam–plasma filamentation. Finally, in Section 4, a summary and conclusion is given.

2. Formulation of the problem

It is well known that the interaction of an electron beam with the plasma is strong when the Čerenkov or the cyclotron resonance condition is fulfilled [29,30]. In the case of a straight beam the Čerenkov interaction prevails since the second-order poles correspond to it whereas the cyclotron interaction is represented by first-order poles. However, the cyclotron interaction appears to be as strong as the Čerenkov interaction, when the beam electrons have a directed velocity component perpendicular to the external magnetic field in addition to the longitudinal one.

It follows from the resonance condition $\omega = k_z u$ for Čerenkov resonance and $\omega = k_z u \pm \Omega_e/\gamma$ for cyclotron resonance that the cyclotron interaction determines no limit for the phase velocity of the waves; it may be both smaller and larger than the beam velocity. Only the ratio between the cyclotron frequency of the electrons in the external magnetic field and the frequency of the electromagnetic wave is important, here. Therefore, to get the mechanism of the cyclotron interaction as clear as possible we consider the perturbations in a rotating beam–plasma system supposing $k_z = 0$, which excludes the Čerenkov resonance. The perturbations are assumed to propagate strictly across the magnetic field.

Now, we consider an electron beam with nonrelativistic velocity component perpendicular $(u_\perp)$ to the external magnetic field, in addition to longitudinal one, moving through a medium of resting particles, or, equivalently a plasma with a rotating electron beam injected into it. It should be noted that the injected electron beam induces charges and currents in the plasma which neutralize the charge and the current of the beam. They can be neglected when plasma density greatly exceeds the electron beam density. We assume the beam density to be smaller than the plasma density ($\omega_b\gamma^{1/2} \ll \omega_{pe}$), and transverse velocity be small ($u_\perp^2 \ll c^2$) as well.

For simplicity, we further restrict the investigation to a monoenergetic rotating beam interacting with a cold electron plasma.

The dielectric permitivity of this system can be written as [29,30]

$$\epsilon_{ij}(\omega,\vec{k}) = \epsilon_{ij}^0(\omega) + \delta\epsilon_{ij}(\omega,\vec{k}), \tag{1}$$

where $\epsilon_{ij}^0(\omega)$ is the dielectric permitivity of the cold electron plasma [29].

$$\epsilon_{ij}^0(\omega) = \begin{pmatrix} \epsilon_{\perp 0} & ig_0 & 0 \\ -ig_0 & \epsilon_{\perp 0} & 0 \\ 0 & 0 & \epsilon_{\parallel 0} \end{pmatrix}, \tag{2}$$

with

$$\epsilon_{\parallel 0} = 1 - \frac{\omega_{pe}^2}{\omega^2}, \qquad g_0 = -\frac{\omega_{pe}^2\Omega_e}{\Omega_e(\omega^2-\Omega_e^2)},$$
$$\epsilon_{\perp 0} = 1 - \frac{\omega_{pe}^2}{\omega^2-\Omega_e^2}, \tag{3}$$

and $\delta\epsilon_{ij}(\omega,\vec{k})$ is a correction caused by the electron beam and Ω_e is the electron Larmor frequency. To calculate $\delta\epsilon_{ij}(\omega,\vec{k})$, the particle distribution of the nonequilibrium plasma with anisotropic distribution functions of the particles of the type α of the form

$$f_{0\alpha} = f_{0\alpha}(p_\perp, p_z), \tag{4}$$

is assumed where p_z and $p_\perp$ is the momentum of particles along and across the z-axis, respectively, in the cylindrical coordinates. The external homogeneous magnetic field $\vec{B}_0$ is oriented along the z-axis. Examples of nonequilibrium plasmas of this type are the plasma with an anisotropic temperature, the plasma with a beam of charged particles moving parallel to the magnetic field or rotating around it. However, to calculate the dielectric permitivity explicitly, we need the exact form of the function $f_{0\alpha}(p_\perp, p_z)$. Taking into account the thermal effects, we assume [29,30]

$$f_{0\alpha}(p_\perp, p_z) = \frac{N_{0\alpha}\exp\left(-\frac{(p_\perp-p_{\perp 0})^2}{2m_\alpha T_{\perp 0}} - \frac{(p_\parallel-p_{\parallel 0})^2}{2m_\alpha T_{\parallel 0}}\right)}{2\pi m_\alpha p_{\perp 0}\sqrt{T_{\perp 0}T_{\parallel 0}}}, \tag{5}$$

describing a system of particles with average longitudinal $p_{\parallel 0}$ and transverse $p_{\perp 0} = p_{\theta 0} \gg \sqrt{m_\alpha T_{\perp 0}}$ momenta with respect to the direction of $\vec{B}_0$. The thermal spread of the momenta is defined by the temperatures $T_{\perp\alpha}$ and $T_{\parallel\alpha}$ assuming $T_{\parallel\alpha} \ll m_\alpha c^2$ and $T_{\perp\alpha} \ll m_\alpha c^2$. It should be noted that for our rotating beam the transverse direction assigned by $\perp$ is θ direction.

For an inverted population rotating beam in the cold plasma limit, $T_{\perp\alpha} = T_{\parallel\alpha} = 0$, we have [28–30]

$$f_{0\alpha} = \frac{N_{0\alpha}}{2\pi p_{\perp 0}}\delta(p_\perp - p_{\perp 0})\delta(p_\parallel - p_{\parallel 0}). \tag{6}$$

This distribution function describes a system of monoenergetic particles rotating perpendicularly to the external homogeneous magnetic field with the momentum $p_{\perp 0} = p_{\theta 0} = \text{const}$, and moving parallel to it with constant momentum $p_{\parallel 0}$. It describes a real system when the thermal spread of the momentum may be neglected, i.e., for $p_{\perp 0} \gg \sqrt{m_\alpha T_{\perp 0}}$ and $p_{\parallel 0} \gg \sqrt{m_\alpha T_{\parallel 0}}$. Such rotating beam arises in the cross electric and magnetic fields and in this case gravitational instability may take place as well.

Finally, by solving the Valsov equation and making use of expression (6) as the distribution function for the rotating electron beam, after some calculations for perturbation with $k_z \neq 0$, we find [29,30]

$$\delta\epsilon_{xx} = \sum_n \left[\frac{2}{z}n^2 J_n(z)J_n'(z)P_n + n^2 J_n^2(z)Q_n\right],$$
$$\delta\epsilon_{yy} = \sum_n \left\{\frac{1}{z}\left[z^2 J_n'^2(z)\right]' P_n + z^2 J_n^2(z)Q_n\right\},$$
$$\delta\epsilon_{zz} = -\frac{\omega_b^2}{\omega^2\gamma^2} + \sum_n \left\{\left[\frac{2k_z u_\parallel}{\omega - k_z u_\parallel}J_n^2(z) + \frac{u_\parallel^2}{u_\perp^2}2zJ_n(z)J_n'(z)\right]P_n + \frac{u_\parallel^2}{u_\perp^2}z^2 J_n^2(z)Q_n\right\},$$

$$
\begin{aligned}
\delta\epsilon_{xy} &= -\delta\epsilon_{yx} \\
&= -i\sum_n \left\{ \frac{n}{z}\left[zJ_n(z)J_n'(z)\right]' P_n \right. \\
&\qquad \left. + nzJ_n(z)J_n'(z)Q_n \right\}, \\
\delta\epsilon_{xz} &= \delta\epsilon_{zx} \\
&= \sum_n \left\{ \left[\frac{n\Omega_e k_z}{k_\perp(\omega - k_z u_\parallel)} J_n^2(z) \right. \right. \\
&\qquad \left. + \frac{u_\parallel}{u_\perp} 2nJ_n(z)J_n'(z) \right] P_n \\
&\qquad \left. + \frac{u_\parallel}{u_\perp} nzJ_n^2(z)Q_n \right\}, \\
\delta\epsilon_{yz} &= -\delta\epsilon_{zy} \\
&= i\sum_n \left(\left\{ \frac{\Omega_e k_z z J_n(z)J_n'(z)}{k_\perp(\omega - k_z u_\parallel)} \right. \right. \\
&\qquad \left. + \frac{u_\parallel}{u_\perp}\left[zJ_n(z)J_n'(z)\right]' \right\} P_n \\
&\qquad \left. + \frac{u_\parallel}{u_\perp} z^2 J_n(z)J_n'(z)Q_n \right). \qquad (7)
\end{aligned}
$$

Here we introduced the notations

$$
\begin{aligned}
P_n &\equiv \frac{\omega_b^2(\omega - k_z u_\parallel)}{\gamma\omega^2(\omega - k_z u_\parallel - n\Omega_e/\gamma)}, \\
Q_n &= \frac{\omega_b^2\gamma^{-3}\Omega_e^2(\omega^2 - k_z^2c^2)}{\omega^2c^2k_\perp^2(\omega - k_z u_\parallel - n\Omega_e/\gamma)^2}, \\
z &= \frac{k_\perp u_\perp \gamma}{\Omega_e}, \qquad \gamma = \left(1 - \frac{u_\parallel^2 + u_\perp^2}{c^2}\right)^{-1/2}. \qquad (8)
\end{aligned}
$$

Making use of Eqs. (1)–(3), (7) and (8), the stability of a rotating electron beam, streaming into a cold plasma, in the presence of an external magnetic field can be studied by the dispersion equation [29,30]

$$
D(\omega, \vec{k}) = \left| k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\epsilon_{ij}(\omega, \vec{k}) \right| = 0, \qquad (9)
$$

which will be studied in the next section.

3. Rotating electron beam–plasma filamentation

In this section, the instability of a rotating current-driven plasma which is very important for the plasma confinement problem is considered. It will be shown that under special conditions an instability takes place which makes the waves in the rotating electron beam–plasma system propagating in the direction perpendicular to the external magnetic field be unstable. In addition, it will be shown that the purely growing mode in the aforementioned system is the low-frequency version of the filamentation instability across the external magnetic field. To do this, we analyze the interaction of a rotating electron beam with electromagnetic wave propagating across the magnetic field, i.e., $k_z = 0$, $k_\perp = k$. It should be noted that the beam terms of the dielectric permitivity tensor contribute the second order poles at $\omega = k_z u_\parallel - n\Omega_e/\gamma$, if $u_\perp \neq 0$. Consequently the beam strongly interacts with a electromagnetic wave.

Let us start the stability problem from the general dispersion Eq. (9). The latter equation for the transverse oscillation can be written as

$$
\begin{aligned}
D(\omega, \vec{k}) &= k^4\epsilon_{xx} - \frac{k^2\omega^2}{c^2}\left[\epsilon_{xx}(\epsilon_{yy} + \epsilon_{zz}) + \epsilon_{xy}^2 - \epsilon_{xz}^2\right] \\
&\quad + \frac{\omega^4}{c^4}\left[\epsilon_{xx}\left(\epsilon_{yy}\epsilon_{zz} + \epsilon_{yz}^2\right) + \epsilon_{xy}^2\epsilon_{zz} \right. \\
&\quad \left. + 2\epsilon_{xy}\epsilon_{xz}\epsilon_{yz} - \epsilon_{xz}^2\epsilon_{yy}\right] = 0. \qquad (10)
\end{aligned}
$$

In the limit $u_\perp \ll c$ and when $(k_\perp u_\perp \delta)/(\Omega_e) \ll 1$, only the fundamental harmonic $n = \pm 1, 0$, of the cyclotron resonances is accounted for $\delta\epsilon_{ij}(\omega, \vec{k})$. Assuming $\omega_{pe}^2 \gg \omega_b^2 > \Omega_e^2$, from Eqs. (7) and (8), we obtain

$$
\begin{aligned}
\delta\epsilon_{xx} &= -\frac{\omega_b^2}{\gamma^2(\omega^2 - \Omega_e^2/\gamma^2)}, \\
\delta\epsilon_{yy} &= \frac{\omega_b^2}{\gamma(\omega^2 - \Omega_e^2/\gamma^2)} + \frac{\omega_b^2 k^2 u_\perp^2 \gamma}{\Omega_e^2\omega^2}, \\
\delta\epsilon_{zz} &= \frac{\omega_b^2}{\gamma^2\omega^2} - \frac{\omega_b^2 k^2 u_\parallel^2 \gamma}{\Omega_e^2\omega^2} \\
&\quad + \frac{\omega_b^2 k^2 u_\parallel^2 \gamma}{\Omega_e^2(\omega^2 - \Omega_e^2/\gamma^2)} + \frac{\omega_b^2 u_\parallel^2}{\gamma^2\omega^2}, \\
\delta\epsilon_{xy} &= -\delta\epsilon_{yx} = -i\frac{\omega_b^2\Omega_e}{\omega\gamma_2(\omega^2 - \Omega_e^2/\gamma^2)}, \\
\delta\epsilon_{xz} &= \delta\epsilon_{zx} = \frac{\omega_b^2 k u_\parallel}{\omega\gamma_2(\omega^2 - \Omega_e^2/\gamma^2)}, \\
\delta\epsilon_{yz} &= -\delta\epsilon_{zy} = -i\frac{\omega_b^2 k u_\parallel}{\Omega_e\omega^2}. \qquad (11)
\end{aligned}
$$

Analyzing the stability problem, we expand the dispersion equation (10) in the very low frequency region ($\omega \to 0$) to find the spatial structure. To show when filamentation instability takes place, firstly we should find the dispersion spectrum, i.e., $\omega = \omega(k)$. Then, when ω has positive imaginary part and simultaneously wave number has periodic character this means that filamentation instability takes place, otherwise only a modulation occurs. In order to indicate that in a cold plasma with a rotating electron beam a transverse structure with a characteristic period $l_0 = \pi/k_0$ exists, we should find the time needed for the establishment of this structure which can be determined from the time that is necessary for instability development. To obtain this value we consider a system close to the threshold ($k \approx k_0$) and the time of instability development $\tau = 1/\,\mathrm{Im}(\omega)$. In the limit of very low frequency region, i.e., the static limit $\omega \to 0$, for waves propagating across the external magnetic field, $k_z = 0$, this dispersion equation can be expanded as

$$D(0,\vec{k}) + \frac{1}{2}\omega^2 \frac{\partial^2 D(0,\vec{k})}{\partial \omega^2} = 0. \tag{12}$$

From Eqs. (1)–(3), (10)–(12) and $D(0,\vec{k}) = 0$ for a dense plasma in the static limit, i.e., $\omega \to 0$, we obtain

$$k_0^2 = \frac{\frac{\omega_{pe}^2}{c^2}\left(\frac{\omega_b^2 u_\perp^2 \gamma}{\Omega_e^2 c^2} - 1\right)}{1 + \frac{\omega_b^2 u_\parallel^2 \gamma}{\Omega_e^2 c^2}}. \tag{13}$$

Eq. (13) indicates that, when

$$\Omega_e^2 c^2 < \omega_b^2 u_\perp^2 \gamma, \tag{14}$$

k_0^2 is positive ($k_0^2 > 0$). Thus, in the rotating electron beam–plasma system a transverse structure with a characteristic period $l_0 = \pi/k_0$ can exist in the static limit. The establishment of this structure needs a time which could be determined from the instability development. To obtain this value let us consider a system close to the threshold ($k \approx k_0$). Then, expression (12) leads to

$$\omega^2 = -2\frac{D(0,\vec{k})}{\frac{\partial^2 D(0,\vec{k})}{\partial \omega^2}} = \frac{\Omega_e^2 c^2}{\omega_{pe}^2}\left(k^2 - k_0^2\right). \tag{15}$$

The first term describes the diffusive attenuation of inhomogeneous of a layer of thickness $\simeq 1/k$, while the second term describes the compression resulting from the magnetic field. The compression takes place when $k < k_0$. Eqs. (13) and (14) indicate that there is a magnetic threshold upper which a filamentation instability can occur. Therefore, when the magnetic field is larger than the threshold $m\omega_e u_\perp \sqrt{\gamma}/\mathrm{e}$, and/or when the inequality $\Omega_e^2 c^2 > \omega_b^2 u_\perp^2 \gamma$ holds, magnetic pressure in the electron beam–plasma system shows the self-pressing effect (pinch effect) and filamentation disappears.

4. Summary and conclusion

Using the total dielectric permitivity tensor of a rotating electron beam–plasma system in the laboratory frame by solving the kinetic equation, the general dispersion relation was obtained. Analyzing the dispersion relation and finding the frequency spectra, the filamentation of the aforementioned system was investigated by expanding the dispersion relation around the static case. It was shown that a cold magnetized plasma with a rotating electron beam is unstable in the direction perpendicular to the external magnetic field. Furthermore, a threshold for instability development was obtained. It was shown that close to the threshold, the rotating electron beam–plasma system will be subdivided into separate current filaments with a period of the order of l_0. Therefore, only when the magnetic field is smaller than the limit value $m\omega_e u_\perp \sqrt{\gamma}/\mathrm{e}$, the self-pressing effect (pinch effect) of magnetic pressure becomes negligible in the electron beam–plasma system and filamentation appears.

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